Three-magnon decay of exchange spin wave

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Observation of the parametric excitation of spin waves by a short spin wave with a wave number $q \simeq 2\times 10^5$ cm^{-1} is reported. The experiment was carried out on a YIG film in which the anisotropy field varied smoothly along thickness. In this setting it was possible to linearly excite an initial wave and to observe intense emission at frequencies of parametrically generated waves. This emission can have an extremely narrow spectrum, with a linewidth of less than 1 kHz. © *1995 American Institute of Physics.*

Effects which arise during an instability of spin oscillations in magnetic materials have attracted research interest for several reasons. One is that several processes characteristic of various nonlinear media can be realized by comparatively simple experimental methods. Examples of these processes are parametric wave excitation, self-modulation and the formation of envelope solitons, and a transition to chaos. In this letter we report experimental observation of parametric excitation of spin waves in three-magnon decay of an exchange spin wave, by which we mean a short spin wave whose propagation stems from an exchange interaction.

Three-magnon interactions which lead to a first-order instability satisfy the conservation laws

$$\omega_1 = \omega_2 + \omega_3, \quad \mathbf{q}_1 = \mathbf{q}_2 + \mathbf{q}_3, \tag{1}$$

where ω_1 and $\mathbf{q}_1$ are the frequency and wave vector of the original wave (the pump wave), while waves 2 and 3 are parametrically excited waves. The values of $\mathbf{q}_{2,3}$ are typically quite high, on the order of 10^4–10^5 cm^{-1}. Previous experiments on the first-order parametric instability have used as the pump either stimulated oscillations of the magnetization or various types of magnetostatic waves and oscillations with $\mathbf{q}_1 \leqslant 10^3$ cm^{-1} (including a uniform precession of the magnetization, $\mathbf{q}_1 \simeq 0$). There has also been a study of the parametric excitation of spin waves upon the application of a microwave field (parallel pumping). In all these cases, the experiments have been restricted to processes in which the condition $\mathbf{q}_1 \ll \mathbf{q}_{2,3}$ holds. Consequently, such wave properties of the pump as its wavelength and propagation direction have been unimportant. The study we are reporting here demonstrates that these characteristics turn out to be exceedingly important for the decay of an exchange spin wave.

In general, conditions (1) are consistent with a parametric excitation of waves which differ in both frequency and propagation direction. If the wave vectors $\mathbf{q}_1$, $\mathbf{q}_2$, and $\mathbf{q}_3$ are parallel, we say that the process is a "collinear decay." The frequencies ω_2 and ω_3 must satisfy the relations $\omega_2=\omega_1/2+\Delta\omega$ and $\omega_3=\omega_1/2-\Delta\omega$, where $\Delta\omega$ is the detuning from the half-frequency. The process with $\Delta\omega=0$ is a "frequency-degenerate decay."

Let us analyze the conditions which would be necessary for experimental realization of the decay of an exchange spin wave. The dispersion of such a wave in an isotropic medium is described by

$$\omega^2=(\omega_H+Dq^2)(\omega_H+Dq^2+\omega_M\sin^2\alpha), \tag{2}$$

where $\omega_H=\gamma H_{\text{in}}$, $\omega_M=4\pi\gamma M_S$, H_{in} is the internal magnetic field, M_S is the saturation magnetization, α is the angle between the wave vector and the magnetization direction, and D is the nonuniform-exchange constant ($D=4.6\times10^{-9}$ Oe·cm^2 in YIG). Analyzing dispersion relation (2), we can show that if the initial wave is directed along the magnetization ($\alpha=0$) a decay is possible at frequencies[1] $\omega_1\geqslant3\gamma H_{\text{in}}$. The limiting frequency $\omega_1=3\gamma H_{\text{in}}$ corresponds to the case of degenerate collinear decay, with $\omega_2=\omega_3=\omega_1/2$, and $\mathbf{q}_2=\mathbf{q}_3=\mathbf{q}_1/2$. We can thus draw two conclusions. First, it can be seen from (2) that only sufficiently short waves, with $q_1^2\geqslant2\omega_1/3D$, can decay. Even at a comparatively low frequency, $\omega_1/2\pi=1000$ MHz, the wave number q_1 must exceed 2.2×10^5 cm^{-1}. Second, the parametrically excited waves must also have short wavelengths.

It follows that an experimental study would require the capability to excite and receive exchange spin waves with $q\sim10^5$ cm^{-1}, in order to (first) create the pump wave and (second) detect the decay products. There is a problem here: In the linear regime, short spin waves undergo essentially no interaction with an electromagnetic wave, because of the substantial difference (four or five orders of magnitude) in the scales of the wavelengths. This problem can be solved by working with spatially inhomogeneous media, e.g., by using YIG films in which the magnetization or uniaxial-anisotropy field H_a varies smoothly over thickness.[2,3] In such films, the local value of the wave vector of the exchange spin wave, $q(x)$, varies as this wave propagates along the x axis, which is directed along this thickness of the film. In the region with $q(x)\sim0$, an effective coupling of spin and electromagnetic waves arises. To explain the experiment, we assume a YIG film in an external magnetic field $\mathbf{H}_e$ which is directed along the normal to the surface. We assume that H_a depends on x, while the other parameters of the film are constants. We ignore the effect of a cubic anisotropy. The profile of the wave number along the coordinate x or the dependence of this wave number on the anisotropy field H_a can then be found from (2) under the assumptions $\alpha=0$ and $H_{\text{in}}(x)=H_e-4\pi M+H_a(x)$. Figure 1 shows plots of $q_1(H_a)$ at $\omega_1/2\pi=1000$ MHz (curve *1*), of $q_2(H_a)$ at $\omega_2=\omega_1/2$ (curve *2*), and $2q_2(H_a)$ (curve *3*). At point A, where q_1 vanishes, the pump wave is excited. The conditions for degenerate collinear decay are satisfied at point B. When the products of this decay reach point C, they have a q_2 on the order of zero, so we can expect radiation of electromagnetic waves with a frequency ω_2 from the sample. To the left of point B, where we have $2q_2>q_1$, there can be a parametric excitation of frequency-nondegenerate waves. The occurrence of this process can be inferred from the occurrence of emission at frequencies differing from $\omega_1/2$ by an amount $\pm\Delta\omega$.

For the experiments we used a YIG film in the (100) orientation with a thickness of 11 μm. The drop of the anisotropy field over the thickness of the film was about 350 Oe. The field $H_a(x)$ varied monotonically from 250 Oe at one surface to -100 Oe in a

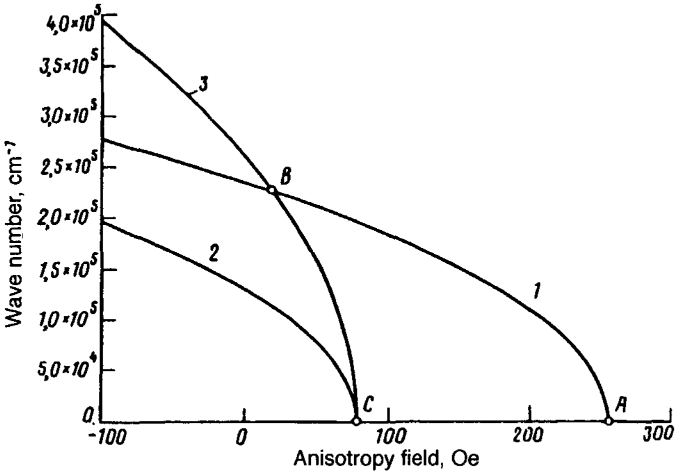

FIG. 1. Wave number versus the anisotropy field in a normally magnetized film for $H_e = 1850$ Oe and $4\pi M = 1750$ H. 1—$q(H_a)$ at $\omega/2\pi = 1000$ MHz; 2—$q(H_a)$ at $\omega/2\pi = 500$ MHz; 3—$2q(H_a)$ at $\omega/2\pi = 500$ MHz.

uniform layer 2 μm thick adjacent to the second surface. The spectrum of the spin-wave resonance of this test sample consists of several tens of intense absorption lines. It spans a frequency band about 1 GHz wide. The film is pressed against a stripline 50–500 μm wide, to which a microwave signal with a frequency $\omega_p/2\pi = 1000$ MHz is applied. The external field is adjusted to a strength such that the frequencies ω_p and $\omega_p/2$ lie within the spectrum of the spin-wave resonance. Emission from the sample is detected by the same stripline and then sent through a directional coupler to a spectrum analyzer. The spectrum of this emission was studied for various strengths and directions of the external magnetic field.

It was found that, as the power applied to the sample is raised to ~ 1 mW, signals with frequencies close to $\omega_p/2$ appear in the spectrum (Fig. 2). This emission is presumably due to a parametric excitation of spin waves. Let us look at some of the experimental results.

1. Figure 2, a and b, demonstrates the possibility of a parametric excitation of spin waves with a frequency equal to precisely half the pump frequency. The emission spectrum can be exceedingly narrow (Fig. 2a). In this case the observed linewidth is determined by the resolution (1 kHz) of the spectrum analyzer, and there are no significant traces of noise in the emission. The emission intensity is high; it can reach 100 μW at an input power of 80 mW.

2. The spectra in Fig. 2, c and d, show that the decay can be frequency-nondegenerate (Fig. 2c), and that it can also occur simultaneously in several channels (Fig. 2d). In this case the emission intensity is much lower than in the case of the degenerate decay. These spectra have some noise. The maximum value of the frequency detuning $\Delta\omega/2\pi$ is no more than 2–3 MHz.

3. Emission was observed only when the external magnetic field was oriented at

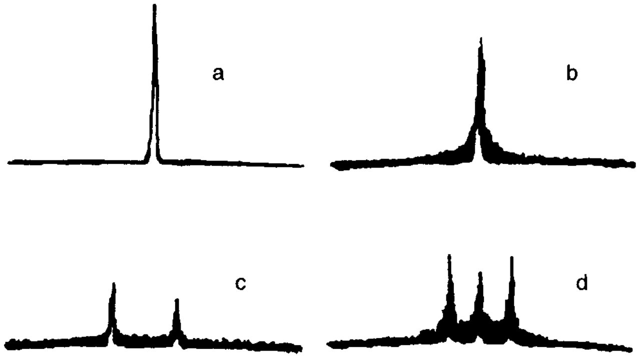

FIG. 2. Typical emission spectra. The power applied to the sample was 80 mW. The various spectra correspond to various strengths and directions of the external field. For all spectra, the horizontal sweep is 100 kHz, and the central frequency is 500 MHz. The gain of the receiver for spectrum a is 20 dB lower than that for spectra b–d.

some angle φ from the normal to the film. The interval of values $\varphi_{\min} \leqslant \varphi \leqslant \varphi_{\max}$ in which the emission was detected was extremely narrow, about 1°. Figure 3 shows values of $\varphi_{\min}$ and $\varphi_{\max}$ for various values of H_e. By varying φ or H_e slightly, one can cause a pronounced restructuring of the emission spectrum, to the point that the spectrum disappears completely. Accordingly, emission was not actually observed at every single point between $\varphi_{\min}$ and $\varphi_{\max}$ in Fig. 3.

Three-magnon processes were analyzed theoretically in Ref. 1 for a spatially homogeneous medium. The assumption of homogeneity makes it difficult to directly compare

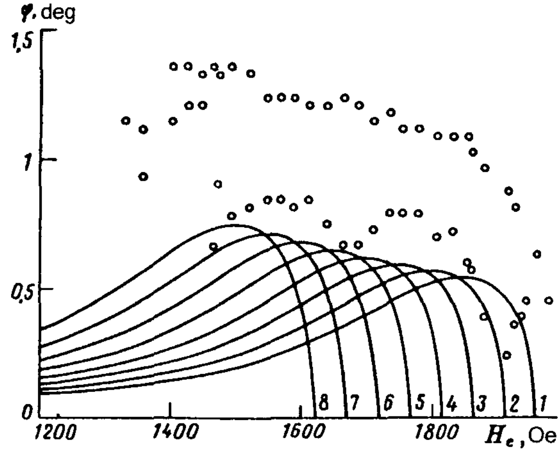

FIG. 3. Points—Experimental values of $\varphi_{\max}$ and $\varphi_{\min}$; curves—theoretical values of $\varphi_i(H_e)$ for layers with $H_a = -100 + 50(i-1)$ Oe. The curve label is the value of i.

that theory with the results of the present experiments on an inhomogeneous film. Still, some of the conclusions found in Ref. 1 provide a qualitative explanation of why the emission is observed in only a narrow interval of angles φ.

First, the exchange spin waves excited and received by the stripline propagate along the normal to the surface of the film, since the maximum possible projection of $\mathbf{q}$ onto the plane of the film is determined by the width of the strip. This maximum possible value is 10^2-10^3 cm^{-1}, much smaller than the wave number of the exchange spin wave ($\sim 10^5$ cm^{-1}). It can thus be concluded that only the products of a collinear decay can cause emission and can thus be detected. Specifically, if the angle between $\mathbf{q}_2$ and $\mathbf{q}_1$ exceeds 0.01 rad, then the component of $\mathbf{q}_2$ in the plane of the film is greater than 10^3 cm^{-1}.

A deviation of $\mathbf{H}$ from the normal to the surface causes the magnetization to rotate in a nonuniform way over thickness; i.e., the angle β, between the magnetization and the normal, becomes a function of the coordinate. In this case the angle α, between the wave vector and the magnetization, also depends on x: $\alpha=\beta(x)$. To analyze this behavior we first need to find $\beta(x)$. We simplify the problem, assuming that the film consists of n homogeneous layers which differ in the value of the uniaxial-anisotropy field. We ignore the exchange interaction between these layers. We can then find the angle β_i, in layer i, by numerically solving the static problem of finding the equilibrium magnetization direction.[4] Replacing (2) by the dispersion relation for exchange spin waves in an anisotropic medium,[5] we can then construct the dependence $\varphi_i(H_e)$ for which conservation laws (1) hold in the given layer for the degenerate collinear decay of a spin wave which is propagating at an angle β_i with respect to the magnetization. Figure 3 shows that these conditions hold over a broad range of the magnetic field H_e, but the maximum values of φ_i do not exceed 0.8°. Consequently, the absence of emission at large values of φ can be explained on the basis that conservation laws (1) are violated under these conditions. We do not find a quantitative agreement here: The experimental values of $\varphi_{\max}$ reach 1.3°. Nevertheless, the agreement between theory and experimental can be judged completely satisfactory, since we ignored the cubic anisotropy and the exchange interaction in solving the static problem.

It can be seen from Fig. 3 that conditions (1) also hold in the case $\varphi_i=0$. Nevertheless, an emission is not observed in this case. The apparent reason for this result is that, as follows from Ref. 1, the threshold for the collinear decay goes off to infinity if all three wave vectors are oriented parallel to or perpendicular to the magnetization. In a normally magnetized film, the noncollinear decay has a lower threshold, but this process does not result in emission.

It can also be seen from Fig. 3 that the maximum value of the field H_e at which emission does arise agrees well with the calculation. At lower values of H_e, at which the external field is much smaller than the saturation field, a theoretical analysis runs into difficulties because of the possible formation of domains.

In summary, it can be concluded that the basic experimental results can be explained in the model discussed here. It follows that the emission which is observed is indeed due to a parametric excitation of waves by a short spin wave. We wish to stress that the emission spectra have several features which distinguish them from the parametric excitation of spin waves in YIG films and spheres which have been studied previously.[6,7] On the one hand, the spectrum reveals no emission at frequencies significantly different from $\omega_p/2$. On the other, the emission may be unusually stable and narrow-band even when the pump level is substantially above the threshold. The apparent reason for the latter result is a spatial localization of the process in the inhomogeneous medium. The propagation of the decay products out of the instability zone and the subsequent emission of these products can stabilize the amplitude of the parametrically excited wave.

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