Phase transition in a nonequilibrium 2D degenerate system

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This letter analyzes how an external field opposing the attainment of thermodynamic equilibrium affects a 2D magnetic material with n-component spins ($n \geqslant 2$). In a nonequilibrium system with a purely dissipative dynamics, there is a second-order phase transition accompanied by the onset of a spontaneous magnetization. This behavior is different from that in an equilibrium degenerate magnetic material. The nonequilibrium phase transition falls in the universality class of an equilibrium transition in a uniaxial ferroelectric. The phase diagram of the system is analyzed. © *1995 American Institute of Physics.*

Phase transitions in nonequilibrium systems have attracted considerable interest over the past decade.[1-17] The universality classes in critical phenomena are key questions here. Researchers have encountered situations in which the universality class of a nonequilibrium system is the same as that of the equilibrium analog of the system.[3-5] They have also encountered several examples in which the critical behavior changes. In some cases the system belongs to one of the known equilibrium universality classes,[6-8] while in other cases a new nonequilibrium class arises.[9-12]

The critical behavior of the two-temperature Ising kinetic model has turned out to be surprising[13,14] (the same is true of the model of a fast ionic conductor in a random electric field, which belongs to the same universality class[15]). It was found in Ref. 8 that, despite the local nature of the dynamics and despite the absence of a long-range order, the model of Refs. 13 and 14 falls in the universality class of an equilibrium system with an infinite interaction range: a uniaxial ferroelectric.[18-20] The conclusions reached in Ref. 8 have been supported by numerical simulations.[17] These simulations have also established[18] that a particular condition which the similarity hypothesis imposes on the shape of the phase diagram near a bicritical point for a uniaxial ferroelectric is satisfied in the two-temperature Ising kinetic model.[19] It was also mentioned in Ref. 17 that the phase transition for a nonequilibrium system in the spherical approximation is retained not only for a space of dimensionality $d \geqslant 3$, as in the equilibrium case, but also in two-dimensional space. Since the spherical approximation is equivalent to a model with an infinite number of spin components, there is the possibility that the phase transition in the two-temperature kinetic model is retained for any finite and otherwise arbitrary number of components greater than one, in particular, in the $X-Y$ model and the Heisenberg model.

Our purposes in the present letter are to demonstrate that the lower critical dimensionality of the two-temperature kinetic model with vector spins is actually lower than that of its equilibrium analog and to clarify the structure of the phase diagram in the vector case.

We consider a model of n-component classical spins S_i, $i=1,...,n$, of unit length $(\Sigma_i S_i^2=1)$. These spins are at the sites of a square lattice. Neighboring spins interact ferromagnetically. The dynamics of the system consists of an exchange of the orientations of spins in neighboring sites. The probabilities for an exchange in the two perpendicular directions x and y correspond to the conditions of detailed balance for two different temperatures T_x and T_y. The vector total spin of the system is an integral of motion for a dynamics of this sort. We restrict the discussion below to universal properties, so the details of the dynamics are of no interest here. For example, we could consider a system in which elementary processes involve only conservation of the sum of the spins participating, or we could assume an interchange of not two spins but several spins belonging to a common horizontal or vertical line in the lattice.

Pursuing the universality hypothesis, we can treat a dynamics as simple as necessary to preserve the symmetries, the conservation laws, and the nature of the competing dynamic processes in the phase-transition problem. We write the equations of motion for the Fourier components of the spin, $S^i_{\mathbf{q}}(t)$, in the following form, which is a generalization of model B (Ref. 21):

$$\dot{S}^i_{\mathbf{q}}(t)=L_x S^i_{\mathbf{q}}+L_y S^i_{\mathbf{q}}+\eta^i_x(\mathbf{q},t)+\eta^i_y(\mathbf{q},t). \tag{1}$$

Diffusion in the directions $\alpha=x$ and $\alpha=y$ is controlled by corresponding operators L_α:

$$L_\alpha S^i_{\mathbf{q}}=-D_\alpha q^2_\alpha\left[(\tau_\alpha+c^x_\alpha q^2_x+c^y_\alpha q^2_y)S^i_{\mathbf{q}}+u_\alpha\sum_{j=1}^{n}\int d\mathbf{q}_1 d\mathbf{q}_2 S^j_{\mathbf{q}_1}S^j_{\mathbf{q}_2}S^i_{\mathbf{q}-\mathbf{q}_1-\mathbf{q}_2}\right]. \tag{2}$$

The noise η_α satisfies the conditions of detailed balance with corresponding temperatures T_α:

$$\langle\eta^i_\alpha(\mathbf{q},t)\eta^j_{\alpha'}(\mathbf{q}',t')\rangle=2D_\alpha T_\alpha q^2_\alpha\delta_{\alpha\alpha'}\delta_{ij}\delta(\mathbf{q}+\mathbf{q}')\delta(t-t'). \tag{3}$$

In Eqs. (2) and (3), we have $\tau_\alpha=a_\alpha(T_\alpha-T^0_\alpha)$; the parameters D_α, u_α, c^x_α, c^y_α, a_α, and T^0_α are constants. The fact that the system is not at detailed equilibrium is expressed by the relation

$$\frac{L_x S^i_{\mathbf{q}}}{D_x T_x q^2_x}\neq\frac{L_y S^i_{\mathbf{q}}}{D_y T_y q^2_y}. \tag{4}$$

We obtain the simplest model in which the symmetries and conservation laws are the same as in the original formulation by assuming $D_\alpha=1$, $T_\alpha=1$, $c^x_\alpha=c^y_\alpha=1$, $u_\alpha=1$, and only $\tau_x\neq\tau_y$. Equation (1) simplifies, taking a form analogous to the dynamic equation of equilibrium model B:

$$\dot{S}^i_{\mathbf{q}}=-q^2\frac{\delta H}{\delta S^i_{-\mathbf{q}}}+iq_x\tilde{\eta}^i_x+iq_y\tilde{\eta}^i_y, \tag{5}$$

where $\tilde{\eta}_{x,y}$ is a δ-correlated noise, and the effective Hamiltonian is

$$H=\frac{\tau_y}{2}\int d\mathbf{q}\mathbf{S}_\mathbf{q}\mathbf{S}_{-\mathbf{q}}+\frac{\tau_x-\tau_y}{2}\int d\mathbf{q}\mathbf{S}_\mathbf{q}\frac{q_x^2}{q^2}\mathbf{S}_{-\mathbf{q}}$$

$$+\frac{1}{2}\int d\mathbf{q}q^2\mathbf{S}_\mathbf{q}\mathbf{S}_{-\mathbf{q}}+\frac{u}{4}\int d\mathbf{q}_1 d\mathbf{q}_2 d\mathbf{q}_3(\mathbf{S}_{\mathbf{q}_1}\mathbf{S}_{\mathbf{q}_2})(\mathbf{S}_{\mathbf{q}_3}\mathbf{S}_{-\mathbf{q}_1}-\mathbf{q}_2-\mathbf{q}_3). \quad (6)$$

Hamiltonian H is the same as that of a uniaxial ferroelectric.[18-20] We thus conclude that the two-temperature kinetic model with vector spins belongs to the universality class of a uniaxial ferroelectric (with n-component dipole moments), as has been observed in the scalar case.[8] Analyzing Eq. (1) without simplifying assumptions regarding the coefficients, we reach the same conclusion, although we cannot write the dynamic equation in the form in (5), which is typical of equilibrium systems. On the other hand, it can be shown that the "non-Hamiltonian" terms on the right side of the dynamic equation are unimportant near the point of the phase transition.

A long-range order reduces the lower critical dimensionality. The latter can be determined with the help of the Peierls–Landau arguments [we are assuming that the dynamic equations have the form in (5)]. We assume that there is an average spontaneous magnetization $\mathbf{S}$ in the system. At low temperatures, we can ignore fluctuations of the modulus of the magnetization. Accordingly, the spin is described as a function of the coordinates by

$$\mathbf{S}(\mathbf{r})=\mathbf{S}\sqrt{1-\delta S^2(\mathbf{r})/S^2}+\delta\mathbf{S}(\mathbf{r}), \quad (7)$$

where the vector $\sigma\mathbf{S}(\mathbf{r})$ is perpendicular to $\mathbf{S}$. Expanding Hamiltonian (6) in $\delta\mathbf{S}$, we find, in lowest order,

$$H=\frac{1}{2}\int d\mathbf{q}G_0^{-1}(\mathbf{q})\delta\mathbf{S}_\mathbf{q}\delta\mathbf{S}_{-\mathbf{q}}, \quad G_0(\mathbf{q})=1/\left(q^2+(\tau_x-\tau_y)\frac{q_x^2}{q^2}\right). \quad (8)$$

For definiteness we have assumed $\tau_x>\tau_y$ here. The magnitude of the fluctuations is determined by the equation

$$\langle\delta\mathbf{S}^2\rangle=T_y\int G_0(\mathbf{q})d\mathbf{q}. \quad (9)$$

Fluctuations do not destroy the long-range order if the integral in (9) converges. For a d-dimensional system subjected to a noise corresponding to a temperature T_x in one direction and subjected to a noise corresponding to T_y in the other directions, we find a lower critical dimensionality $d_c=1$ in the case $T_y<T_x$ or $d_c=3/2$ in the case $T_y>T_x$. These values are the same as those found in the limit $n\to\infty$ in Ref. 17.

We see that a phase transition occurs in the two-dimensional case. The critical exponents describing the transition in a uniaxial ferroelectric can be calculated in second order in $\epsilon=3-d$ by the method proposed in Ref. 20. The index of anomalous dimensionality, η, and the index ν, which determines the renormalization of the smaller of the two temperatures τ_x, τ_y (we assume $\tau_y<\tau_x$ everywhere below), are given by

$$\eta=\frac{4}{9}\frac{n+2}{(n+8)^2}\epsilon^2+O(\epsilon^3) \quad (10)$$

and

$$\nu=\frac{1}{2}+\frac{n+2}{4(n+8)}\epsilon+\frac{n+2}{2(n+8)^3}\epsilon^2\left[\frac{10}{9}(2n+7)+\frac{(n+2)(n+8)}{4}+(7n+20)\ln\frac{2}{\sqrt{3}}\right]+O(\epsilon^3). \tag{11}$$

An anisotropy of the system leads to an anisotropic scaling: A change in scale by a factor of μ along the y direction corresponds to a change in scale by a factor of $\mu^{1+\Delta}$ along the x direction. The anisotropy index is $\Delta=1-\eta/2$. The other critical indices can be expressed in terms of η, ν, and Δ by means of the scaling laws. For example, the correlation length is described by the indices $\nu_y=\nu$ and $\nu_x=\nu(1+\Delta)$, while the order parameter (in the $d=2$ case) is described by the index $\beta=\frac{\nu}{2}(\Delta+\eta)$. The diffusion nature of the dynamics has the consequence that the dynamic indices z_x, z_y are determined by the relations $z_y=z=4-\eta$, $z_x=z/(1+\Delta)=2$.

The phase diagram of this system has two lines of second-order phase transitions, symmetric with respect to the straight line $T_x=T_y$. The equations of these lines can be found at low temperatures by comparing the correlation length of the equilibrium system with the length scale over which the deviation from equilibrium becomes important. At small values of the difference (T_x-T_y), this length scale is proportional a power of the temperature difference, $L\propto 1/|T_x-T_y|^{\sigma}$, where $\sigma=$const. The correlation length of the equilibrium system for $n\geqslant 3$ has a temperature dependence[22] $\xi\propto\exp(\frac{\text{const}}{T})$. Since there is no spontaneous magnetization under the condition $\xi<L$, the equation of the transition lines for $n\geqslant 3$ is

$$\frac{T_x+T_y}{2}\sim\frac{1}{\ln|T_x-T_y|}. \tag{12}$$

In the $n=2$ case, the system is in a Kosterlitz–Thouless phase on the line $T_x=T_y<T_c$, where T_c is the point of the Kosterlitz–Thouless transition. Near this line, the term proportional to $(\tau_x-\tau_y)$ in Hamiltonian (6) becomes important. There is accordingly a long-range order on both sides of this line in the nonequilibrium system. At temperatures $T>T_c$ the correlation length in the $X-Y$ model satisfies[22]

$$\xi\propto\exp\left(\frac{\text{const}}{\sqrt{\mathrm{T}-\mathrm{T}_c}}\right).$$

We thus find the equation of the lines of transition from ordered phases to the disordered phase near the point $T_x=T_y=T_c$:

$$\frac{T_x+T_y}{2}-T_c\propto\frac{1}{\ln^2|T_x-T_y|}. \tag{13}$$

We note in conclusion that the phase transition which we have analyzed is possible only in an anisotropic nonequilibrium system. It does not occur in the equilibrium case (even if the system is anisotropic) or in a degenerate magnetic material which is driven from equilibrium without a disruption of spatial isotropy.

The effective long-range order which we have dealt with in this problem is also manifested in other nonequilibrium systems with a conserved order parameter. This situation could, in particular, explain the classical value of the index, $\beta = 1/2$, which was found in Refs. 9–12 for *driven diffusive systems* and for *hybrid driven diffusive systems*. The behavior of models with an order parameter which is not conserved is sharply different from that in the case in which the order parameter is conserved. The universality classes of such systems are the same as those of their equilibrium analogs.[3–5] On the other hand, we do not yet have a general criterion which would explain just when a nonequilibrium system falls in a Hamiltonian class with a short-range order, in a Hamiltonian class with a long-range order, and in a non-Hamiltonian class.

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