Interference intensification of wave scattering by an amplifying disordered medium

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Wave propagation in a quasiballastic disordered amplifying medium is studied. It is shown that in such a medium weak localization occurs in backward reflection. The shape of the peak is calculated near the lasing threshold. It is shown that interference amplification of transmission in a direction corresponding to inversion of the wave-vector component tangent to the surface is possible. © *1995 American Institute of Physics.*

Weak localization in backward reflection from disordered media has been studied extensively .[1] In Ref. 2 it was shown how this effect is modified in an amplifying medium. A narrow peak appears in backward reflection near the lasing threshold of such a system. This peak can also be interpreted as fluctuation-induced amplification of weak localization near a phase transition in a disordered optical system. The feedback required for generation is provided by random multiple scattering,[3] which makes the propagation of radiation in the medium diffusive in nature . In a recent experiment[4] generation was observed in a medium that is different from the one studied in Refs. 2 and 3 in that the transverse size of the medium did not exceed the photon mean free path, so that photon passage through the plate was quasiballistic.

In the present paper we shall show that in the case of scattering by a thin medium of this kind a channel for the localization effect appears in the scattering of light. We shall estimate the magnitude of the backward reflection and the transmission near the generation threshold, and we shall study the necessary conditions for generation.

The appearance of a channel associated with weak localization can be explained as follows. Let a monochromatic plane wave with the wave vector $\mathbf{k}_0$ be incident on a plate. We shall study the interference of the waves traveling along different paths, as illustrated in Fig. 1. The first wave travels along the diffusion path i, starting at the point $\mathbf{a}$, and leaves the plate in the direction of the wave vector $\mathbf{k}_1$, scattering the last time at the point $\mathbf{b}$. Since we are considering elastic propagation, we have $|\mathbf{k}_1|=|\mathbf{k}_0|$. The second wave travels in the plate along the same path but in the opposite direction. The contribution of these waves to the scattering is proportional to the quantity

$$W_i(\mathbf{a},\ \mathbf{b})[1+\cos(\mathbf{k}_0+\mathbf{k}_1)(\mathbf{a}-\mathbf{b})]. \tag{1}$$

Here $W_i(\mathbf{a},\mathbf{b})$ is the probability that the wave travels along the path i from the point $\mathbf{a}$ to the point $\mathbf{b}$. In an ordinary sample this probability decreases exponentially when $|\mathbf{a}-\mathbf{b}|$ is greater than the mean free path, since the wave will exit the plate most likely because of scattering by defects. Under the conditions of negative absorption there is a finite (not

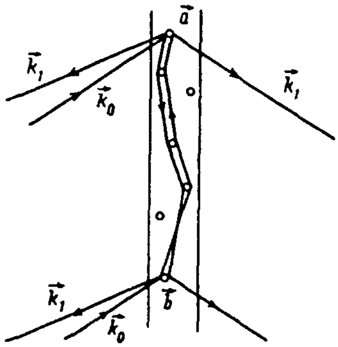

FIG. 1. Diagram illustrating the ith scattering path in a section of the amplifying medium. The open circles represent scatterers.

exponentially small) probability that a wave will travel in the plate a distance much longer than the mean free path, since escape through the lateral boundaries of the plate can be compensated for by coherent amplification.

Under these conditions interference enhancement of scattering from the plate arises in the direction $\mathbf{k}_0 = -\mathbf{k}_l$. The large random phase associated with the motion of the wave through the sample is cancelled in the direction of the wave which travels through the plate and has a component of the wave vector in the plane of the plate such that $(\mathbf{k}_0 + \mathbf{k}_l)_{\parallel} = 0$. The interference term in the expression (1) in this case is proportional to $\cos(\mathbf{k}_0 + \mathbf{k}_l)_{\perp}(\mathbf{a} - \mathbf{b})_{\perp}$ and is determined by the transverse components of the wave vectors. Amplification of the transmission in this direction is possible when the thickness d of the plate is of the order of the wavelength.

To find the reflection coefficient, it is necessary to calculate the sum of the transition probabilities $W_i(\mathbf{a},\mathbf{b})$ along all paths i traversed in the plate and to integrate over the coordinates of $\mathbf{a}$ and $\mathbf{b}$. The average transition probability is determined from the equation shown graphically in Fig. 2. The dashed lines in the figure describe the scattering which detemines the mean free path l_{imp}. The solid lines represent Green's functions. We shall study this equation for $l \gg d$ and substitute into it the squared modulus, averaged over the cross section of the plate, of the Green's function. For uniform amplification the Green's function is given by the expression $(4\pi r)^{-1}\exp\{(ik - 1/2l)r\}$, where $l^{-1} = l_{\text{imp}}^{-1} - l_a^{-1}$ is the difference in the reciprocals of the mean free path l_{imp} and the amplification length l_a.

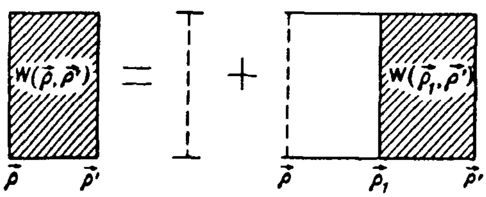

FIG. 2.

The transition probability, averaged over the cross section of the plate, at distances greater than l is given by the solution of the diffusion equation, to which the equation shown in Fig. 2 reduces in this case ($\boldsymbol{\rho}$ is the coordinate vector in the plane of the plate):

$$\{1-A_0-A_1\nabla_\rho^2\}W(\boldsymbol{\rho},\boldsymbol{\rho}')=\frac{4\pi}{l_{\mathrm{imp}}d}\,\delta(\boldsymbol{\rho},-\boldsymbol{\rho}'), \tag{2}$$

where

$$A_f=\frac{1}{4\pi l_{\mathrm{imp}}d}\int_0^d \partial z\partial z_1\int \partial\boldsymbol{\rho}(\rho/2)^{2f}(\rho^2+(z-z_1)^2)^{-1}\exp\left\{-\frac{\sqrt{\rho^2+(z-z_1)^2}}{l}\right\},$$

and $1-A_0=0$ is the equation for the critical amplification length l_{0a}. This equation can always be solved, since A_0 increases logarithmically as $l\to\infty$. For $2l_{\mathrm{imp}}\gg d$ the solution is $l_0^{-1}\equiv l_{\mathrm{imp}}^{-1}-l_{0a}^{-1}\propto d^{-1}\exp(-2l_{\mathrm{imp}}/d)$. In the case $2l_{\mathrm{imp}}\approx d$ the critical gain length is of the order of the thickness of the plate. The second coefficient in Eq. (2) is $A_1\approx l^2d/8l_{\mathrm{imp}}$.

The sum over paths of the quantity (1) is expressed in terms of the Fourier transform of the solution of Eq. (2), so that ultimately we obtain the following relation near the lasing threshold with $l>d$ for the reflection coefficient $R(\mathbf{k}_0,\mathbf{k}_1)$ which is defined as the ratio of the flux scattered in the direction $\mathbf{k}_1$ to the flux incident on the plate:

$$R(\mathbf{k}_0,\mathbf{k}_1)\approx\frac{8\pi}{\Delta}+\frac{8\pi}{\Delta+l_0^2/4(\mathbf{k}_0+\mathbf{k}_1)_\parallel^2},\quad \Delta=\frac{l_0-l}{l}. \tag{3}$$

The first term in Eq. (3), which corresponds to the first term in Eq. (1), describes diffusive scattering. We note that under the conditions of muliple scattering the amplitude of the wave increases, so that near the lasing threshold the intensity of the scattered light is much higher than that of the incident wave.

In conclusion, we note that expressions (1) and (3) hold when the wave propagates in the direction $(\mathbf{k}_0+\mathbf{k}_1)_\parallel=0$ and the thickness of the plate does not exceed the wavelength. In this limit the length l_0, which determines the inverse width of the peak, increases and the peak itself becomes narrower exponentially. The interference effect in transmission decreases exponentially when the thickness is greater than the wavelength.

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