"Strong" helicon in metals with open orbits

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(Submitted 4 May 1995)
Pis'ma Zh. Éksp. Teor. Fiz. **61**, No. 12, 980–984 (25 June 1995)

The effect of carrier trapping by the field of a large-amplitude rf wave on the electromagnetic properties of precious metals was investigated in a geometry in which open orbits are present. Collisionless absorption by carriers with such orbits is so effective in the linear regime that passage of a helicon through the metal is impossible. It is shown that the trapping of these carriers by the wave field suppresses absorption and makes helicon propagation possible. © *1995 American Institute of Physics.*

1. It is known that a constant magnetic field **H** does not limit the motion of carriers on open orbits and has very little effect on the contribution of such carriers to the nonlocal conductivity, which is found to be virtually identical to the conductivity under the conditions of the anomalous skin effect with $H=0$. Consequently, even when the number of carriers on open orbits is a small fraction of the total number of conduction electrons, the collisionless absorption of the wave by such carriers is so strong that helicon propagation is impossible. This situation occurs in precious metals in the geometry $\mathbf{H}\parallel[110]$, in which open orbits are present and helicons are not observed.[1] At the same time, we know[2–4] that the magnetic field of a large-amplitude wave can "trap" carriers which are responsible for collisionless absorption, causing it to decrease. It will be shown below that the trapping of carriers with open orbits by the magnetic field of a wave can result in such a large decrease of the dissipative conductivity of a metal that helicon propagation becomes possible in it.

2. We shall study the case in which the propagation vector **k** of the wave is directed along the constant magnetic field $\mathbf{k}\parallel\mathbf{H}\parallel[110]\parallel z$, and the open part of the Fermi surface of the metal is a cylinder described by the equation $p_x^2/2m_\perp+p_z^2/2m=\varepsilon_F$, where m and $m_\perp$ are, respectively, the longitudinal and transverse masses of the carriers on open orbits, and ε_F is the Fermi energy of these carriers. A standard calculation of the Fourier transform of the conductivity in the nonlocal limit gives

$$\sigma_{xx}^0(k)=\frac{2n_0e^2}{m_\perp v|k|},\quad n_0=\frac{p_B\sqrt{mm_\perp}\varepsilon_F}{2\pi^2\hbar^3},\tag{1}$$

where $v=(2\varepsilon_F/m)^{1/2}$, n_0 is the density of carriers with open orbits, and p_B is the size of the Brillouin zone in the direction of the p_y axis. The expression for σ_{xx}^0 does not depend on H, since a magnetic field $\mathbf{H}\parallel z$ does not limit the motion of carriers along the x axis. This expression can be obtained by multiplying the local conductivity $\sigma_{xx}^l=n_0e^2\tau/m_\perp$ (τ is the mean free time of the carriers) by the amount $2/kv\tau$, which is the relative fraction of effective carriers.

In the geometry $\mathbf{H} \parallel [110]$ the concentration n_0 of carriers with open orbits in copper is 4% of the concentration n of electrons with closed orbits. For strong magnetic fields, in which the displacement of the electrons over a cyclotron period is smaller than the wavelength of the rf wave, the electronic part of the conductivity includes the nondissipative Hall conductivity nec/H ($-e$ is the electron charge, and c is the speed of light) and the dissipative conductivity associated with the collisionless cyclotron absorption of the wave. The point is that in the geometry $\mathbf{H} \parallel [110]$ the helicon threshold is determined by electrons with the minimum displacement over a cyclotron period, $u_0 = 2\pi p_0 c/eH$, where $2\pi p_0 = |\partial S/\partial p_z|_{\min}$ and $S(p_z)$ is the area of the section of the electron Fermi surface by the plane p_z=const. In the range of fields above the helicon threshold, where $ku_0/2\pi < 1$, there is therefore a cyclotron absorption of the wave by electrons for which $ku/2\pi = 1$. This absorption is strong in the region where $ku_0/2\pi \sim 1$, and it decreases rapidly as ku_0 decreases. On the basis of what we have said above, the dispersion relation for the minus polarization in the linear regime can be written in the form

$$k^2c^2 = 4\pi\omega\left\{\frac{nec}{H}\left[1 + i\left(\frac{ku_0}{2\pi}\right)^4\right] + \frac{i}{2}\sigma_{xx}^0\right\}, \tag{2}$$

where ω is the angular frequency of the wave. The second term in brackets describes the cyclotron absorption of the wave by electrons and the term with σ_{xx}^0 describes the absorption by carriers with open orbits. For fields H in the range where the imaginary terms on the right-hand side of Eq. (2) are small compared to the real terms, the approximate solution has the form

$$k = k_H + i\kappa_l, \tag{3}$$

$$k_H \simeq k_a/\sqrt{h}, \quad k_a = (4\pi\omega ne^2/c^2p_0)^{1/3},$$
$$h = H/H_1, \quad H_1 = k_a c p_0/e, \tag{4}$$

$$\kappa_l \simeq \frac{k_a}{2}\left(h^{-13/2} + \frac{n_0}{n}\alpha h\right), \quad \alpha = \frac{p_0}{m_\perp v}, \tag{5}$$

where H_1 characterizes the helicon threshold. These expressions are applicable for fields in which $h > 1$ and $\kappa_l \ll k_a/\sqrt{h}$. The damping κ_l reaches a minimum for fields $H_m \simeq 2H_1/\alpha^{2/15}$; the minimum value of κ_l is

$$\kappa_m \simeq 0.046\alpha^{13/15}k_a. \tag{6}$$

Setting $m_\perp = 10^{27}$ g, $p_0 = 0.6\hbar$ Å^{-1}, and $v = 3\times 10^7$ cm/s, we find that at a frequency of 1 MHz and sample thickness $d = 0.02$ cm the product $\kappa_m d \simeq 3.7$. It therefore follows that it is virtually impossible to observe a helicon signal transmitted through such a plate (the signal is attenuated by more than a factor of 40). This result agrees with experiment: Helicons are not observed in the geometry $\mathbf{H} \parallel [110]$.[1]

3. We will now show that the situation changes radically in the case of a strong nonlinearity. We shall derive an expression for the nonlinear conductivity with the help of a modified concept of "inefficiency." Let us consider the equation of motion of a carrier with an open orbit in a static magnetic field $\mathbf{H}$ and in the field of a large-amplitude rf

wave. We write it in the coordinate system moving along the z axis with the phase velocity ω/k of the wave. In this system there is no electric field, the magnetic field is stationary, and the equation of motion has the form

$$\dot{\mathbf{p}} = -\frac{e}{c}[\mathbf{v}(\mathbf{H}+\mathbf{H}_\omega(z))], \tag{7}$$

where the overdot indicates a time derivative, and $\mathbf{H}_\omega(z) = \{H_\omega \cos kz, H_\omega \sin kz, 0\}$ is the magnetic field of the wave. Since the component $v_y = \partial\varepsilon/\partial p_y \equiv 0$, the equations for $\dot{v}_x$ and $\dot{v}_z$ do not contain H:

$$\dot{v}_x = \frac{eH_\omega}{m_\perp c} v_z \sin kz, \quad \dot{v}_z = -\frac{eH_\omega}{mc} v_x \sin kz. \tag{8}$$

We are interested in carriers with $v_z \ll v$. Setting for them $v_x \simeq v_\perp \equiv \sqrt{2\varepsilon_F/m_\perp}$, we write the equation of motion along the z axis in the form

$$\ddot{z} = -\frac{\omega_0^2}{k} \sin kz, \tag{9}$$

where

$$\omega_0^2 = kv_\perp \Omega, \quad \Omega = eH_\omega/mc. \tag{10}$$

The first integral of Eq. (9) has the form

$$\dot{z}^2 = v_{z0}^2 + \frac{2\omega_0^2}{k^2} \cos kz. \tag{11}$$

It follows from Eq. (11) that the carriers for which $v_{z0} > V \equiv \sqrt{2}\omega_0/k$ execute an infinite motion along the z axis and the carriers for which $v_{z0} < V$ are trapped by the field of the wave and execute an oscillatory motion with a frequency of the order of ω_0. The former carriers are said to be itinerant and the latter are said to be trapped. The absorption of the wave depends on the quantity $\omega_0\tau$. For $\omega_o\tau \ll 1$ carriers are not trapped and we have a linear regime. In the opposite case, $\omega_0\tau \gg 1$, a fraction V/v of the carriers is trapped by the wave. These are the carriers that are mainly responsible for the absorption of the wave.[2,4] The relation $v_x\dot{v}_x = -v_z\dot{v}_z$ follows from Eqs. (8). This relation means that the oscillations of a particle along the z axis are accompanied by modulation of the velocity v_x. The oscillating correction to v_x is phase-shifted by $\pi/2$ with respect to the field $H_{\omega y}(z)$. The situation is reminiscent of the motion of a particle in a high-frequency ($\omega\tau \gg 1$) electric field. If the oscillations are undamped, then the particle does not absorb energy from the field. The absorption is associated with the damping of the oscillations, which is due to the collisions of particles with the scatterers. The absorption is $(1+\omega^2\tau^2)$ times weaker than in the case of a low-frequency field. Similarly to this case, the absorption of the wave by trapped carriers in the nonlinear regime should be $(1+\omega_0^2\tau^2)$ times weaker. For this reason, a formula for the nonlinear conductivity can be obtained from the expression for the local conductivity $\sigma_{xx}^l = n_0 e^2 \tau/m_\perp$ by multiplying it by the fraction V/v of trapped carriers and dividing by $\omega_0^2\tau^2$. In other words, within a numerical factor of order 1, the conductivity determined by the carriers on open orbits has in the case of strong nonlinearity $\omega_0\tau \gg 1$ the form

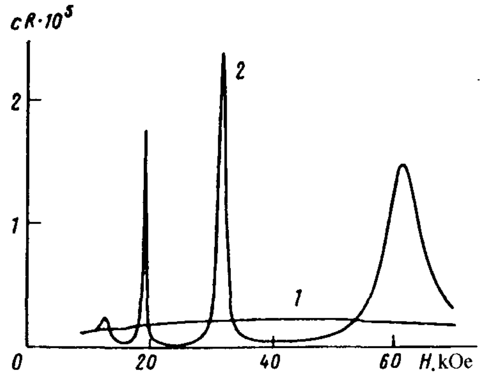

FIG. 1. $R(H)$ computed for a copper plate in the linear (1) and nonlinear (2) regimes.

$$\sigma_{xx} \approx \frac{2n_0e^2}{m_\perp kv\omega_0\tau}. \tag{12}$$

Comparing Eq. (12) with Eq. (1) we see that the absorption is $\omega_0\tau$ times smaller in the nonlinear regime. This result is similar to the results for magnetic Landau damping of helicons in metals,[2] for nonlinear cyclotron damping of a hole doppleron in cadmium,[3] and for the nonlinear skin effect.[4]

It follows from what we have said above that in the case of strong nonlinearity helicon damping is

$$\kappa = \frac{k_a}{2}\left(h^{-13/2} + \frac{n_0}{n}\frac{\alpha h}{\omega_0\tau}\right). \tag{13}$$

We assume that the electrons with large displacements over a cyclotron period (electrons with high v_z), which give rise to cyclotron absorption, are not trapped by the wave field, and this absorption remains the same as in the linear regime.

At large amplitudes of the exciting field the helicon damping caused by carriers on open orbits is therefore suppressed, and passage of the helicon through the sample becomes possible. In the case where only one electromagnetic mode is present in the metal, the surface impedance of the plate with antisymmetric excitation is determined by the formula (see, for example, Ref. 5)

$$Z = -\frac{8\pi i\omega}{c^2k}\tan\frac{kd}{2}. \tag{14}$$

Substituting here $k = k_H + i\kappa$, it is easy to derive the following expression for the surface resistance of the plate:

$$R = \frac{8\pi\omega}{c^2(k_H^2+\kappa^2)}\frac{k_H \sinh(\kappa d) - \kappa \sin(k_H d)}{\cosh(\kappa d) + \cos(k_H d)}. \tag{15}$$

For $\kappa d < 1$ and $k_H d = \pi(2s+1)$, where $s = 1,2,3,\ldots.$, the function $R(H)$ has sharp peaks which are determined by the excitation of standing helicon waves in the plate. The height of the peaks is inversely proportional to κd. The computational results for $R(H)$ for 1 MHz, $H_{\omega}=200$ Oe, $n=3\times 10^{22}$ cm^{-3}, $\tau=3\times 10^{-9}$ s, and $d=0.2$ mm are presented in Fig. 1. In the calculation we set $m=m_{\perp}$; the helicon damping reaches a minimum for the field $H_m=23.5$ kOe ($h_m=2.8$) and is equal to $\kappa_m\simeq 8$ cm^{-1}. We see that the nonlinear effect is very strong.

This work was supported by a grant (N3F000) from the International Science Foundation and the Russian government.

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Translated by M. E. Alferieff