Evolution of a wave packet in a Josephson two-junction interferometer

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A system of two Josephson junctions in a superconducting ring is studied. The dynamics of the system can be described adequately by the regular equations of quantum mechanics taking into account the change in the external magnetic flux and the coupling with the heat reservoir. It is shown that a two-junction interferometer can be regarded as a device that performs a quantum-mechanical measurement of a coordinate. © *1995 American Institute of Physics.*

According to the orthodox point of view of the quantum-mechanical measurement process (see, for example, Refs. 1 and 2), a measuring device must be fundamentally *classical* and the measurement process cannot be described by the regular equations of quantum mechanics. This postulate makes it possible to circumvent the question of how the device can convert a pure (coherent) state of the quantum system being measured, for example, a linear superposition of two states

$$a|-\rangle+b|+\rangle, \tag{1}$$

into a statistical mixture of the states $|-\rangle$ and $|+\rangle$ with probabilities which are proportional to $|a|^2$ and $|b|^2$.

The main direction in searching for a solution to the problem[3–7] is to take into account the *irreversibility* of the measurement process because of the coupling of the measured variable with other degrees of freedom (heat reservoir) which do not participate directly in the measurement process. The instrumental irreversibility, as a necessary property of the measuring process, is in itself not new or controversial.[1,2] The question is whether or not it can be described by the regular equations of quantum mechanics.

Several important steps have been made in this direction. In Ref. 3 Peres studied a scheme in which the initial coherence of the states $|-\rangle$ and $|+\rangle$ vanishes as a result of the randomness of the initial phase of the wave function of the measuring instrument. Unfortunately, he did not propose any real experimental system that implemented the principles which he proposed. Conversely, in Ref. 4 Zimanyi and Vladar describe two specific models in which a degenerate electron gas plays the role of the measuring instrument. However, it remains unclear how the state of the electron gas is to be measured without again turning to sensitive instruments which again pose a measurement problem. The same criticism applies to Refs. 5–7.

At the end of the measuring process the difference between the indications of the instrument which correspond to the initial states actually must be so large that there is no

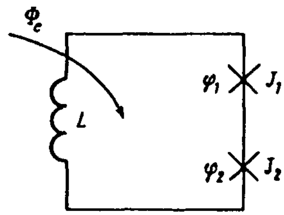

FIG. 1. Josephson two-contact interferometer: system of two junctions in a superconducting loop.

doubt that it can be read off classically. In other words, a quantum measuring device must have both the property of irreversibility and the property of *amplification.*

In the present study we consider a Josephson–junction interferometer[8] (Fig. 1) which satisfies the following two requirements:

1) The dynamics of the interferometer permits a quantitative quantum-mechanical description on the basis of the Caldeira–Leggett model,[9] where the dissipation is caused by the interaction of the measured variable with an ensemble of harmonic oscillators forming an equilibrium heat sink.

2) In this system the sign of the differential "stiffness" can be easily changed. This leads to signal amplification even in the presence of dissipation.[8]

Let each junction in Fig. 1 have a critical current I_c and capacitance C, and each junction is shunted externally by a normal metal conductor with resistance R. If the loop inductance L is sufficiently small ($\pi I_c L/\Phi_0 \ll 1$, where $\Phi_0 = h/2e$ is the magnetic-flux quantum), then the relation between the external magnetic flux $2x_e(t) = 2\pi\Phi_e(t)/\Phi_0$ and the *sum* phase $\varphi_1 + \varphi_2$ is linear.[8] We can therefore write an expression for the potential energy U of the system as function of the *difference phase* $2x = \varphi_1 - \varphi_2$ with a fixed external magnetic flux:

$$U(x) = -2E_c \cos[x_e(t)] \cos[x], \quad E_c = \Phi_0 I_c/2\pi. \tag{2}$$

Since for us the property of the system that the stiffness, i.e., the curvature of the potential $U(x)$ at the minimum value at $x \approx 0$, changes sign is important, we linearize the expression (2):

$$\tilde{U}(x) = m\omega_0^2 x^2/2 + \text{const}, \quad m\omega_0^2 = 2E_c \cos[x_e(t)]. \tag{3}$$

In the case of positive stiffness ($m\omega_0^2 > 0$), the system will behave with respect to small perturbations like a harmonic oscillator with damping $\gamma = 1/2RC$. Let the pure quantum state (1), where $|-\rangle$ and $|+\rangle$ are coherent states centered, respectively, at the points $(+x_i)$ and $(-x_i)$:

$$\psi(x) = a\psi_-(x) + b\psi_+(x), \tag{4}$$

$$\psi_\pm(x) = \frac{1}{(\pi\sigma_i^2)^{1/4}} \exp\left(-\frac{(x \mp x_i)^2}{2\sigma_i^2}\right),$$

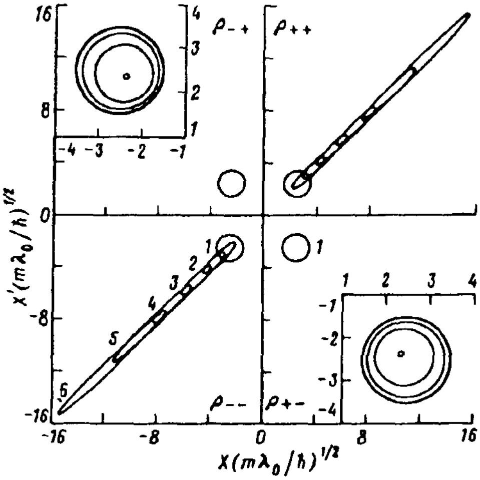

FIG. 2. Evolution of the macroelements of the density matrix in an inverted potential with dissipation. The sections of the moduli of the macroelements (at half maximum) were constructed at the times $\lambda_0 t = 0$, 2, 4, 6, 8, and 10. System parameters: $\alpha_i = 5$; $\sigma_i^2 = 1$; $(\gamma/\lambda_0)^2 = 10$; $\Omega/\lambda_0 = 50$; $\hbar\lambda_0/2kT = 100$. Insets: Evolution of the macroelements ρ_{-+} and ρ_{+-} at times up to t_c, where the time t_c corresponds to the decay of these elements (for our values of the parameters $t_c \simeq 5\times 10^{-3}\lambda_0^{-1}$). Here the sections were constructed at the times $t/t_c = 0$, 0.25, 0.5, 0.75, and 1.

be produced[2)] in this oscillator at $t=0$. For $x_i^2 \gg \sigma_i^2$ the lines of constant modulus of the initial density matrix $\rho(x,x',0) = \psi(x)\psi^+(x')$ consist of four nonoverlapping circles (see Fig. 2).

We now change the flux x_e rapidly (over a time $\delta t \ll \omega_0^{-1}$), i.e., we change the sign of the stiffness of the potential (3) $\omega_0^2 \to -\lambda_0^2$, and we write out the system density matrix, averaged over the degrees of freedom of the heat reservoir, explicitly in the $\eta-\xi$ coordinate representation ($\eta = x + x'$, $\xi = x - x'$). For $t>0$ this density matrix is related linearly to the initial density matrix:[9]

$$\rho(\eta,\xi,t) = (1/2)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d\eta_i d\xi_i J(\eta,\xi,t;\eta_i,\xi_i,0)\rho(\eta_i,\xi_i,0). \tag{5}$$

We shall find the propagator $J(\eta,\xi,t;\eta_i,\xi_i,0)$ of the inverted damped oscillator by the general scheme proposed by Caldeira and Leggett[9] by using notation $\lambda = (\lambda_0^2 + \gamma^2)^{1/2}$:

$$J(\eta,\xi,t;\eta_i,\xi_i,0) = F^2(t)\,\exp(i[K_2\eta\xi + K_1\eta_i\xi_i - L\eta_i\xi - N\eta\xi_i] - [A\xi^2 + B\xi\xi_i + C\xi_i^2]), \tag{6}$$

where

$$K_1 = \frac{m}{2\hbar}(\lambda \coth \lambda t + \gamma), \quad K_2 = \frac{m}{2\hbar}(\lambda \coth \lambda t - \gamma),$$

$$L=\frac{m\lambda}{2\hbar}\frac{e^{-\gamma t}}{\sinh\lambda t},\quad N=\frac{m\lambda}{2\hbar}\frac{e^{\gamma t}}{\sinh\lambda t},$$

$$\begin{pmatrix}A(t)\\B(t)\\C(t)\end{pmatrix}=\frac{m\gamma}{\pi\hbar}\int_0^{\Omega}d\nu\nu\coth(\hbar\nu/2kT)\begin{pmatrix}A_\nu(t)\\B_\nu(t)\\C_\nu(t)\end{pmatrix},$$

$$\begin{pmatrix}A_\nu(t)\\B_\nu(t)\\C_\nu(t)\end{pmatrix}=\begin{pmatrix}e^{-2\gamma t}\\2e^{-2\gamma t}\\1\end{pmatrix}\int_0^t\int_0^t d\tau ds\begin{pmatrix}\sinh\lambda\tau\sinh\lambda s\\\sinh\lambda\tau\sinh\lambda(t-s)\\\sinh\lambda(t-\tau)\sinh\lambda(t-s)\end{pmatrix}\frac{\cos\nu(\tau-s)}{\sinh^2\lambda t}e^{\gamma(\tau+s)}.$$

Here Ω ($\gg\lambda_0$) is the maximum frequency in the distribution of the oscillators forming the heat reservoir,[9] and T is the temperature of the heat reservoir.

The density matrix at an arbitrary time can be written in the form

$$\rho(\eta,\xi,t)=|a|^2\rho_{--}(\eta,\xi,t)+|b|^2\rho_{++}(\eta,\xi,t)+ab^*\rho_{-+}(\eta,\xi,t)+a^*b\rho_{+-}(\eta,\xi,t),\tag{7}$$

where we have introduced the "macroelements," which do not depend on the coefficients a and b,

$$\rho_{\pm\pm}(\eta,\xi,t)=\frac{1}{(\pi\sigma_\eta^2)^{1/2}}\exp\left(-\frac{(\eta\mp\alpha_\eta)^2}{4\sigma_\eta^2}-\frac{\xi^2}{4\sigma_\xi^2}+i\beta\eta\xi\mp i\theta_\xi\xi\right),\tag{8}$$

$$\rho_{\pm\mp}(\eta,\xi,t)=\frac{1}{(\pi\sigma_\eta^2)^{1/2}}\exp\left(-\frac{\eta^2}{4\sigma_\eta^2}-\frac{(\xi\mp\alpha_\xi)^2}{4\sigma_\xi^2}+i\beta\eta\xi\mp i\theta_\eta\eta-\kappa\right),\tag{9}$$

and we use the notation

$$\sigma_\eta^2=\delta^2/4N^2,\quad \alpha_\eta=\alpha_iK_1/N,\quad \delta^2=\sigma_i^{-2}+4C+4\sigma_i^2K_1^2,\quad \alpha_i=2x_i,$$

$$\sigma_\xi^2=\tfrac{1}{4}(A+\sigma_i^2L^2-\delta^{-2}(B-2\sigma_i^2K_1L)^2)^{-1},$$

$$\alpha_\xi=\frac{\alpha_i}{2\sigma_i^2}\frac{2\sigma_i^2K_1L-B}{\delta^2(A+\sigma_i^2L^2-\delta^{-2}(B-2\sigma_i^2K_1L)^2)},$$

$$\beta=K_2+2\delta^{-2}N(B-2\sigma_i^2LK_1),$$

$$\theta_\xi=\alpha_i(L+2\delta^{-2}K_1(B-2\sigma_i^2LK_1)),\quad \theta_\eta=\alpha_iN/\sigma_i^2\delta^2,$$

$$\kappa=\frac{\alpha_i^2}{4\sigma_i^2}\left(1-\frac{1}{\sigma_i^2\delta^2}-\frac{(B-2\sigma_i^2K_1L)^2}{\sigma_i^2\delta^4(A+\sigma_i^2L^2-\delta^{-2}(B-2\sigma_i^2K_1L)^2)}\right).\tag{10}$$

In the absence of dissipation ($\gamma=0$) the moduli of the macroelements of the density matrix have equal peaks at the four points

$$(\pm\alpha/2,\pm\alpha/2),\quad (\mp\alpha/2,\pm\alpha/2)$$

in the (x,x') plane, but the distance α of the centers of these macroelements from the origin of the coordinates and the width σ of the peaks grows exponentially with time:

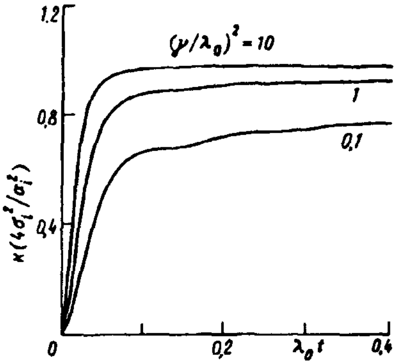

FIG. 3. The coefficient κ plotted as a function of time for different values of the dissipation in the system $(\gamma/\lambda_0)^2=0.1$, 1, and 10 and the parameters $\Omega/\lambda_0=50$ and $\hbar\lambda_0/2kT=100$.

$$\alpha=\alpha_\eta=\alpha_\xi\propto \exp\lambda_0 t,\quad \sigma=\sigma_\eta=\sigma_\xi\propto \exp\lambda_0 t,\quad t\gg\lambda_0^{-1}.$$

This growth reflects the "signal amplification" occurring because of the negative stiffness of the system. It is important that in the absence of dissipation the parameter κ is equal to zero and the off-diagonal macroelements[9] have the same amplitude as the diagonal elements. This corresponds to complete conservation of the initial coherence of the states in the inverted system with no dissipation.

Figure 2 shows the evolution of the macroelements in the presence of dissipation $(\gamma>0)$. In this case the lines of constant magnitude of the diagonal macroelements have the form not of circles but rather ellipses, since for $t\gg\lambda^{-1}$ the variance along the η axis (σ_η) and the position of the centers of the macroelements along this axis (α_η) grow as $\exp(\lambda-\gamma)t$, while the corresponding parameters along the ξ axis (σ_ξ and α_ξ) approach constant values.

It is of fundamental importance that an additional exponential factor $\exp(-\kappa)$ appears in the amplitudes of the off-diagonal macroelements (9) for $\gamma>0$. We shall show that for $t\gg(\lambda-\gamma)^{-1}$ and $\alpha_i^2\gg\sigma_i^2$ the quantity κ is much greater than 1. Indeed, the asymptotic expression for κ in this limit has the form

$$\kappa(\infty)=\frac{\alpha_i^2}{4\sigma_i^2}\left(\frac{A(\infty)[4C(\infty)+4\sigma_i^2K_1^2(\infty)]-B^2(\infty)}{A(\infty)[\sigma_i^{-2}+4C(\infty)+4\sigma_i^2K_1^2(\infty)]-B^2(\infty)}\right),\tag{11}$$

where

$$K_1(\infty)=(m/2\hbar)(\lambda+\gamma),\quad A_\nu(\infty)=[(\lambda+\gamma)^2+\nu^2]^{-1},$$

$$C_\nu(\infty)=[(\lambda-\gamma)^2+\nu^2]^{-1},\quad B_\nu(\infty)=\frac{2[-(-\lambda^2+\nu^2+\gamma^2)\cos\nu t+2\lambda\nu\sin\nu t]}{(-\lambda^2+\nu^2+\gamma^2)^2+4\lambda^2\nu^2}.$$

It follows from these formulas that the expression in parentheses in Eq. (11) is of the order of unity and is virtually independent of the time (Fig. 3). We see that the asymptotic value of κ is always of the order of α_i^2/σ_i^2, so that if the initial wave packets overlap slightly[3)] $\alpha_i^2\gg\sigma_i^2$, then $\kappa(\infty)\gg 1$. In this case, the off-diagonal elements of the density matrix actually completely decay even at short times ($t \ll \lambda_0^{-1}$, see inset in Fig. 2). Therefore, because the dissipation in the system is finite, the off-diagonal macroelements of the density matrix at long times are exponentially small compared with the diagonal elements. This corresponds to a breakdown of the quantum coherence of the initial state (1) as a result of the irreversible character of the interaction of the degree of freedom x of interest to us with the coordinates of the heat sink.

It is physically obvious that the destroyed coherence cannot be restored spontaneously, so that the further evolution of the centers of the packets is governed by the laws of classical dynamics with damping and leads to the establishment of stationary positions of the packets at the points where the potential (2) is minimum. The stationary state of the system will then consist of a classical mixture of states in each of the two minima of the potential with the probabilities $|a|^2$ and $|b|^2$. It is fundamentally important that for a specific system the distance between the minima of the potential U is of the order of Φ_0, i.e., it is completely "macroscopic" [modern SQUID magnetometers make it possible to perform flux measurements with a sensitivity of $10^{-6}\Phi_0$ and a measurement time of 1 s (Ref. 8)].

Therefore, the *specific* Josephson-junction system which we studied exhibits the properties of a quantum measuring device and is completely described by the standard laws of quantum evolution.

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[2)]Such a pure state can be obtained, for example, by means of subbarrier tunneling, by connecting the experimental system into an additional superconducting circuit.[10]

[3)]For $\alpha_i^2 \approx \sigma_i^2$ the off-diagonal macroelements do not decay, even in the limit $t \rightarrow \infty$. In this case the residual coherence remains because of the overlapping of the wave packets near the origin of the coordinates ($x \simeq x' \simeq 0$). For our purposes, only the limit $\alpha_i^2 \gg \sigma_i^2$, in which a "definite" state of the quantum object (for example, $a=1$, $b=0$) definitely leads to one of the final states of the device, is of interest.

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