Shot noise in mesoscopic diffusive conductors in the electron-temperature limit

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The shot noise power of a diffusive conductor in the presence of strong inelastic electron–electron scattering is calculated. It is shown that inelastic electron–electron scattering does not suppress the shot noise strongly, and the noise power is $(\sqrt{3}/2)e|\bar{I}|$. © *1995 American Institute of Physics.*

Electric current noise in small devices has recently been attracting considerable attention. The contribution to this noise that is proportional to the current itself and persists down to zero temperatures is known as shot noise. In the absence of correlations between the carriers, as is the case, for instance, in a classical tunnel junction, the noise power has a form analogous to the Poisson process ("full shot noise") $P_{\text{tun}} = 2e|V|G + 2e\bar{I}$, where $\bar{I}=GV$ is the average current and G is the conductance. All correlations reduce the shot noise, so that P remains below the full shot noise level. In perfect ballistic metallic systems the shot noise is completely suppressed by the Fermi correlation.[1] This clearly is not the situation in imperfectly transmitting systems, for which a general expression for the shot noise power (current-noise spectral density) P in the case of full quantum coherence[2] was recently derived. In particular, the shot noise power in a disordered diffusive conductor with purely elastic electron scattering was shown[3] to be reduced with respect to the "full shot noise" level by a reduction factor $\gamma = 1/3$. All these papers dealt mainly with *elastic* electron scattering, and it is generally believed that all *inelastic* scattering leads to strong suppression of the shot noise.

In the present work we show that inelastic electron–electron scattering, even if very efficient, does not suppress the shot noise strongly but leads only to some change in its magnitude. As we will show, the results obtained are in excellent agreement with experiment,[4] where the shot noise of a two-dimensional diffusive mesoscopic conductor was studied in the limit of strong electron–electron interaction.

Let us consider a diffusive conductor with a length L. In order to find an expression for the shot noise in this system we will use the semiclassical kinetic theory due to Gantsevich, Gurevich, and Katilius,[5] which permits one to express the correlation function of the fluctuating distribution function $F(\mathbf{r},\mathbf{p},t)$ in terms of the average over the temporal fluctuations $\langle F\rangle \equiv \bar{F}(\mathbf{r},\mathbf{p},t)$. This average is the solution of the Boltzmann equation, which we, for simplicity, take in one-dimensional form, assuming homogeneity in the other two directions:

$$\frac{\mathbf{d}}{\mathbf{d}t}\bar{F}(x,\mathbf{p},t) = -\hat{\mathrm{I}}^{\text{col}}_{\mathbf{p}}\{\bar{F}(x,\mathbf{p},t)\}, \tag{1}$$

where the derivative $(\mathbf{d}/\mathbf{d}t) \equiv (\partial/\partial t) + v_x(\partial/\partial x) + e\mathbf{E}(\partial/\partial\mathbf{p})$ takes into account the semiclassical motion of an electron with momentum $\mathbf{p}=m\mathbf{v}$ in the electric field $\mathbf{E}$. $\hat{I}^{\rm col}_{\mathbf{p}} = \hat{I}^{\rm im}_{\mathbf{p}} + \hat{I}^{ep}_{\mathbf{p}} + \hat{I}^{ee}_{\mathbf{p}}$ is the collision operator, which includes terms responsible for impurity scattering (im) and the electron–phonon (ep) and electron–electron (ee) interactions. We assume a stationary Fermi distribution f_F in the leads of the conductor as the boundary conditions for Eq. (1):

$$\bar{F}(\pm(L/2),\mathbf{p},t)=f_F(\varepsilon\mathbf{p}-\mu_\pm,T_{\rm bath}),\quad \mu_\pm=\mu_0\pm\frac{eV}{2}, \tag{2}$$

where μ_0 is the chemical potential of the electron gas in the absence of the bias, and $T_{\rm bath}$ is the thermal bath temperature. Note that boundary conditions of this form imply that the channel length L is much larger than the channel width.

It is rather difficult to solve Eq. (1). In order to simplify the problem, we consider the situation in which the electron–electron scattering is efficient enough to make a description in terms of an *electron temperature* possible. Also we assume that the diffusion of a particle is governed mainly by impurity scattering, which is the case if the effective time of electron–impurity collisions τ_p is much shorter than that of electron–electron collisions, $\tau_{\rm ee}$. In this approximation the part of the electron distribution that is even under the transformation $\mathbf{p}\to-\mathbf{p}$ has the Fermi form along the entire sample:

$$\bar{F}(x,\varepsilon\mathbf{p})=f_F[\varepsilon\mathbf{p}-\mu(x),T_{\rm el}(x)], \tag{3}$$

with a chemical potential $\mu(x)$ and temperature $T_{\rm el}(x)$ which depend on the coordinate and on the applied voltage, while the part that is odd under this transformation is assumed small. Then the problem reduces to calculation of the electron temperature profile $T_{\rm el}(x)$ along the sample. The most direct way to do this is to derive the equation for the total energy of a quasiparticle gas, which is uniquely related to $T_{\rm el}$.

Let us first neglect the electron–phonon interaction. Making use of the relaxation-time approximation for the impurity scattering $\hat{I}^{\rm im}\{F\}=F/\tau_p$, we can obtain an equation for $\bar{F}(x,\varepsilon_p)$. Then we multiply all the terms in this equation by the quasiparticle energy $\varepsilon_p-\mu$ and sum over all ε_p. The final equation for the total energy of the quasiparticle gas takes the form

$$D\frac{\partial^2}{\partial x^2}\sum_{\varepsilon_p}(\varepsilon_p-\mu)\bar{F}(x,\varepsilon_p)+(eE)^2D\sum_{\varepsilon_p}(\varepsilon_p-\mu)\frac{\partial^2\bar{F}(x,\varepsilon_p)}{\partial\varepsilon_p^2}=0, \tag{4}$$

where $D=v_F^2\tau_p/3$ is a diffusion coefficient. Both terms of Eq. (4) have a straightforward physical meaning. The first one is a diffusion term, while the second is a Joule source of heat. In deriving Eq. (4) we have taken into account that electron–electron collisions conserve the total energy of the quasiparticle gas, and therefore $\Sigma_{\varepsilon_p}(\varepsilon_p-\mu)\hat{I}^{\rm ee}_{\mathbf{p}}\bar{F}(x,\varepsilon_p)=0$. During the summation in Eq. (4) yields an equation for $T_{\rm el}(x)$:

$$\frac{\pi^2}{6}\frac{\partial^2}{\partial x^2}[T_{\rm el}(x)]^2+(eE)^2=0. \tag{5}$$

Making use of the boundary conditions $T_{\rm el}(x = \pm L/2) = T_{\rm bath}$ and $E+V/L$ [compare with Eq. (2)], we finally obtain an expression describing the electron temperature profile along the sample. In the most interesting case $eV \gg T_{\rm bath}$ this expression takes the form:

$$T_{\rm el}(x)=\frac{\sqrt{3}}{2\pi}\, e|V| \sqrt{1-\left(\frac{2x}{L}\right)^2}. \tag{6}$$

At this point we have found the explicit form of the nonequilibrium distribution function. Now we will calculate the shot noise power in the conductor in a way similar to that used by Nagaev.[3] The current density at a point $\mathbf{r}$ of the conductor is $\mathbf{j}(\mathbf{r},t) = (e/\Omega_0)\Sigma \mathbf{pv}F(\mathbf{r},\mathbf{p},t)$ where $F(\mathbf{r},\mathbf{p},t)$ is the electron distribution function that has not been averaged over temporal fluctuations and $\Omega_0 = SL$ is the total volume of the sample (S is the sample cross section). The total current through the conductor is $I(t) = (S/\Omega_0)\int \mathbf{d}^3\mathbf{r} j_x(\mathbf{r},t)$. Therefore, the zero-frequency shot noise power is equal to

$$P=4\int_0^\infty \mathbf{d}t\langle \delta I(t)\,\delta I(0)\rangle$$

$$=4\frac{e^2}{S^2L^4}\int \mathbf{d}^3\mathbf{r}\mathbf{d}^3\mathbf{r}'\sum_{\mathbf{pp}'} v_x v_x' \int_0^\infty \mathbf{d}t\langle \delta F(\mathbf{r},\mathbf{p},t)\,\delta F(\mathbf{r}',\mathbf{p}',0)\rangle, \tag{7}$$

where $\delta F = F - \bar{F}$. According to the kinetic theory[5] and within the electron temperature approximation we are using throughout this paper

$$\int_0^\infty \mathbf{d}t\langle \delta F(\mathbf{r},\mathbf{p},t)\,\delta F(\mathbf{r}',\mathbf{p}',0)\rangle = \Omega_0 \delta(\mathbf{r}-\mathbf{r}')\,\tau_p \bar{F}(\mathbf{r},\mathbf{p})[1-\bar{F}(\mathbf{r}',\mathbf{p}')]\,\delta\mathbf{pp}'. \tag{8}$$

After integration in Eq. (7) with allowance for Eq. (3) we arrive at the result

$$P=\frac{4G}{L}\int_{-L/2}^{L/2}\mathbf{d}x\int_0^\infty \mathbf{d}\varepsilon_p \bar{F}(x,\varepsilon_p)[1-\bar{F}(x,\varepsilon_p)] = \frac{4G}{L}\int_{-L/2}^{L/2}\mathbf{d}x T_{\rm el}(x). \tag{9}$$

Here $G = e^2 D(\nu_F/\Omega_0)(S/L)$ is the total conductance, ν_F is the density of electron states at the Fermi level. Taking into account the previously obtained profile $T_{\rm el}(x)$ [Eq. (6)], we arrive at the final expression for the shot noise power in a diffusive conductor in the presence of strong electron–electron scattering:

$$P_{ee}=\frac{\sqrt{3}}{2}e|V|G. \tag{10}$$

The obtained value is only $4/\sqrt{3}$ times smaller than the "full shot noise" level $P_{\rm tun}$, which corresponds to a reduction factor $\gamma = 0.43$. Therefore, inelastic electron–electron scattering, even if very efficient, does not suppress shot noise strongly but leads only to some change in its magnitude. This result is in excellent agreement with a recent experiment by Liefrink *et al.*[4] where a value $\gamma \approx 0.45$ was found in the limit of weak electron–phonon scattering.

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