Metal–insulator transition in a two-dimensional hole gas in Si/SiGe heterostructures. Quantum or classical percolation?

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We study the current–voltage characteristics in the magnetic-field-induced insulating state of a two-dimensional hole gas in low-mobility Si/SiGe heterostructures. The observed behavior of the $I-V$ curves is explained in terms of the breakdown of the insulating phase in the classical percolation model. Analysis of all the experimental results obtained on different two-dimensional systems shows that none of them is in obvious contradiction with classical percolation. © *1995 American Institute of Physics.*

The metal–insulator transition is an important topic in the physics of two-dimensional (2D) systems subjected to a magnetic field. In the 2D case, depending on the value of conductivity σ_{xy}, one can distinguish two types of insulating phase. (i) The Fermi level lies in the localized Landau states, and extended states below the Fermi level are available (as the temperature tends to zero, $\sigma_{xx}\rightarrow 0$ and σ_{xy} is quantized). Traditionally this is called the quantum-Hall-effect phase. (ii) The Fermi level lies in the lowest band of localized states (both σ_{xx} and σ_{xy} vanish with decreasing temperature). These two might, in principle, be of different origin since the ground state of an ideal 2D electron system in the extreme quantum limit is expected to be a Wigner crystal.

Intensive studies of the scaling behavior of the peaks in the resistivity ρ_{xx} in relatively low-mobility heterostructures[1–4] have suggested that it is possible to determine the critical exponent s for the localization length L near the metal–insulator transition in the quantum-Hall-effect phase. The unexpected result $s\approx 2.3$ (the classical percolation model predicts $s=4/3$[5]) has led to the appearance of a number of theoretical works taking into account quantum tunneling and interference effects in the percolation picture. Among these, some numerical investigations[6–9] have been performed for microscopic samples which are several orders of magnitude smaller than those used in experimental studies. Therefore, despite the coincidence between the calculated and experimental values of the critical exponent, their relationship would be tenuous were it not for the analytical results[10–12] substantiating the quantum percolation argument.

Recently, an approach has been formulated for determining the localization length in a 2D electron system, based on analysis of the nonlinear current–voltage characteristics in an insulating phase.[13–15] The experimental results for a high-mobility 2D electron gas

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in both Si MOSFETs and AlGaAs/GaAs heterostructures can be interpreted within the classical percolation model, in contrast to the conclusions of previous publications. The properties of all insulating phases, including the zero-Hall-conductivity one, have been found to be very similar. The authors also argue that the scaling behavior is explained by thermal broadening of the conductivity peaks. In recent investigations of the metal–insulator transition in high-mobility Si MOSFETs,[16] it has been found that the set of temperature dependences of the longitudinal resistance near the phase boundary in both the metallic and insulating phase can be scaled into one curve by using a single parameter $T_0=e^2/\varepsilon L$. The localization length L proves to diverge with a critical exponent $s\approx1.6$, which is close to that expected for classical percolation. Thus, the results obtained by these alternative techniques are in complete disagreement with the interpretation of scaling measurements. This discrepancy might be due to the different quality of samples used in different experiments.

Here we employ a method of current–voltage characterization to investigate the metal–insulator transition to the insulating phase with $\sigma_{xy}=0$ in low-mobility Si/SiGe heterostructures. We find that this transition also is described in terms of the model of classical percolation, which seems adequate for the currently available 2D layers in MOSFETs and heterostructures.

The samples used are grown by solid-source molecular beam epitaxy: detailed information on the growth method is given elsewhere.[17] The 2D holes are confined in an approximately triangular potential well at the surface of a $Si_{0.8}Ge_{0.2}$ layer and have a mobility of ~2000 cm^2/Vs at $T=42$ K and a density of 2.5×10^{11} cm^{-2}. Both the size quantization and the strain present in the SiGe layer are responsible for the splitting of light- and heavy-hole bands. The 2D hole gas (2DHG) originates from heavy holes with angular momentum projection $m_J=\pm3/2$. The sample geometry is a standard Hall bar with dimensions of 4.7×1.0 mm; the distance between potential probes is equal to 1.7 mm. At liquid helium temperatures there is no sign of parallel conduction in the samples. The experiments are performed in a ^{3}He/^{4}He dilution refrigerator with a base temperature ≈25 mK. We employ a four-terminal dc technique with a Keithley 617 electrometer as high-input-resistance amplifier. Currents through the sample do not exceed 1 nA; in this range hole heating effects are found to be negligible.

The magnetoresistance ρ_{xx} exhibits Shubnikov–de Haas oscillations with periodic minima as a function of the reciprocal magnetic field. Following the standard procedure, we determine the values of both the filling factor corresponding to each oscillation minimum and the hole density. The oscillation numbers turn out to be odd, which can be interpreted as the coincidence of the cyclotron energy and spin splitting. As compared to conventional quantum oscillations, there is another distinction in the oscillations observed here: an enormous peak in the magnetoresistance between filling factors of 1 and 3. At minimum temperatures the typical values of ρ_{xx} in this region of magnetic field far exceed the Hall resistance ρ_{xy}, so that both the conductivities $\sigma_{xx}=\rho_{xx}/(\rho_{xx}^2+\rho_{xy}^2)$ and $\sigma_{xy}=\rho_{xy}/(\rho_{xx}^2+\rho_{xy}^2)$ tend to zero with decreasing temperature. This means that in the vicinity of the ρ_{xx} peak the 2DHG is an insulating phase; i.e., with increasing magnetic field there occurs a metal–insulator–metal transition. This is very similar to the re-entrant behavior of the metallic and insulating phases observed in Si MOSFETs and AlGaAs/GaAs heterostructures.

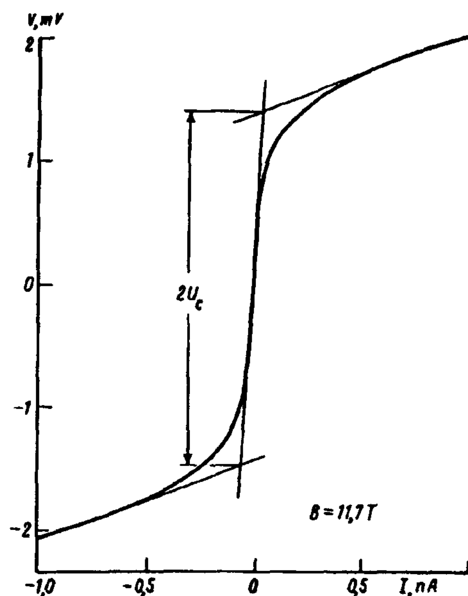

FIG. 1. Typical current–voltage curve in an insulating phase. A temperature $T=60$ mK was chosen to make observable the linear interval of the $I-V$ characteristics. The procedure for determining the critical voltage is shown.

In a tilted magnetic field, the peak in magnetoresistance increases strongly, which implies that the metal–insulator phase boundary is pushed to higher hole densities. As a result, in this case one can move more deeply into the insulating phase when sweeping the magnetic field. In the present paper, the measurements are carried out at an angle $\Theta=60°$ between the magnetic field direction and the normal to the interface.

In the insulating phase we observe strongly nonlinear current–voltage characteristics (Fig. 1), while in the metallic phase these are linear, at least for the range of currents used. Two quantities determine the shape of the $I-V$ characteristics: the critical voltage, U_c, corresponding to the onset of nonlinearities, and the resistance in the linear regime. We establish the type of metal–insulator transition by studying the behavior of the $I-V$ curves in the insulating phase near the metal–insulator phase boundary.

The resistance in the linear interval of $I-V$ curves shows an activational temperature dependence. The activation energy, E_a, is determined from an Arrhenius plot (inset in Fig. 2). As in the case of high-mobility Si MOSFETs, the value of the pre-exponential factor is close to 100 kΩ. As is seen from Fig. 2, the activation energy changes linearly with magnetic field. The slope of this linear dependence is inversely proportional to the density of states at the metal–insulator transition point B_c, which can be found by extrapolating the straight line to zero activation energy (Fig. 2).

The critical voltage proves to be roughly a parabolic function of magnetic field (Fig. 3). This function goes to zero at the same magnetic field B_c, within experimental uncer-

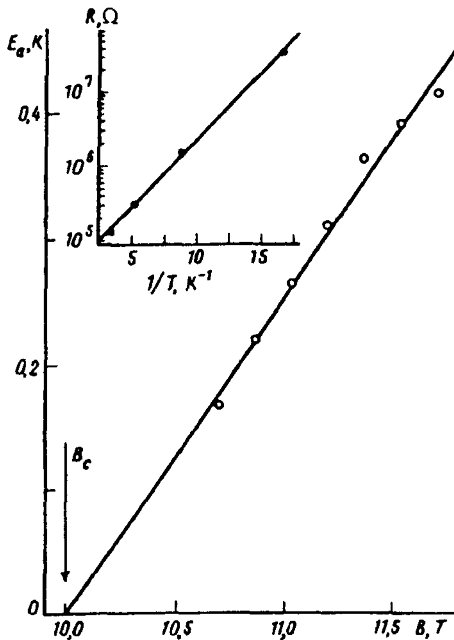

FIG. 2. Dependence of the activation energy on the magnetic field. The metal–insulator transition point is indicated by the arrow. Inset: Arrhenius plot of the resistance in the linear interval of the $I-V$ characteristics.

tainty. Thus, in a similar way to high-mobility Si MOSFETs and AlGaAs/GaAs heterostructures, both the nonlinearity of the $I-V$ characteristics and the activational behavior of the resistance in the linear regime disappear at the same value of the magnetic field, which is considered to be the point of the metal–insulator transition.

These results are now explained in the framework of classical percolation. In this picture, metallic clusters in a 2DHG are separated at saddle points by potential barriers with height equal to an activation energy E_a that is defined as the difference between the percolation threshold E_c and the Fermi level. In a sufficiently small electric field F the clusters are equipotential regions, and so the barrier height decreases by eLF, where L is the cluster dimension (similar ideas for doped semiconductors were developed in Ref. 18). If the energy eLF becomes equal to the activation energy (i.e., the barrier height vanishes) localization in the 2D system breaks down and the conductivity grows abruptly, leading to nonlinear $I-V$ curves.[13–15] Since near the threshold E_c the cluster dimension diverges as $L(E) \propto |E-E_c|^{-s}$, the critical electric field for the breakdown is written $F_c \propto E_a^{s+1} \propto |B-B_c|^{s+1}$. By comparing the latter expression with the experimental data we obtain $s \approx 1$, in satisfactory agreement with the theoretical value $s=4/3$. Thus, the breakdown of an insulating phase in low-mobility Si/SiGe heterostructures as well can be described by the model of classical percolation.

The data obtained here, alongside the conclusions of Refs. 13–15, allow us to suggest that classical percolation should be universal for 2D systems. Obviously, this is in

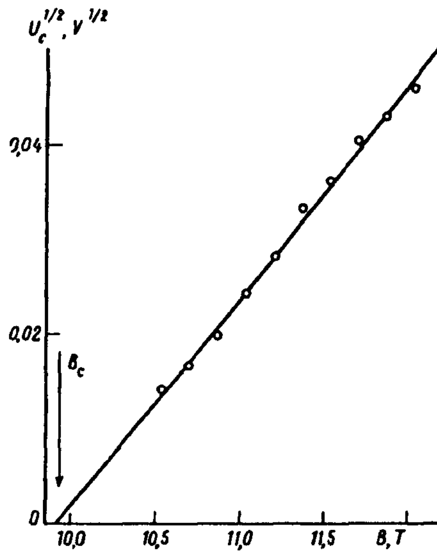

FIG. 3. Square root of the critical voltage as a function of magnetic field at $T \approx 25$ mK. The value of the magnetic field B_c corresponding to the metal–insulator transition coincides with that in Fig. 2 to within the experimental uncertainty.

contradiction with both the results of scaling measurements[1-4] and the theoretical statements.[6-12] To understand the reasons for the discrepancy let us first consider the theories[10-12] in more detail. The idea used there is to modify the classical percolation picture by making allowance for quantum tunneling at the saddle points of a random potential. The tunneling gives rise to an increase in the cluster dimension and, as a result, to a change in the critical exponent. We note that phase coherence, which is necessary in the quantum percolation model, takes place only if the tunneling processes are dominant. This implies, at very least, temperatures close to zero and a temperature-independent conductivity. Otherwise, if the temperature is not sufficiently low either the variable-range hopping at a saddle point of thermal activation to the percolation threshold is expected to be preferable to tunneling. Scattering events break the phase coherence, thereby leading back to the classical picture. It is easy to check experimentally which mechanism dominates: the results indicate that, at the minimum available temperatures, the conductivity in an insulating phase is always due to variable-range hopping. Therefore the theories[10-12] and also numerical calculations[6-9] describe a limiting case that currently is not achievable in experiments.

The majority of experimental papers on the scaling behavior deal with measurements on the half-width $\Delta\nu$ of peaks in ρ_{xx} as a function of temperature. The data are described by a power-law temperature dependence $\Delta\nu \propto T^{\kappa}$; however, the exponent obtained is not universal, varying from $\kappa = 0.21$[4] to ≈ 1.[2] Moreover, when plotted on a linear scale the same data follow a linear temperature dependence with a finite peak width at zero temperature, which is consistent with the thermal broadening of σ_{xx} (or ρ_{xx}) peaks in the classical percolation picture (see Ref. 14). These studies also suffer from an inherent

disadvantage: the authors perform scaling measurements on Hall bar samples and neglect the influence of edge channels on the peaks in ρ_{xx}. This results in weakening of the temperature dependence of $\Delta\nu$ and an underestimate of the value of the exponent κ; however, it is difficult to make rigorous estimates of the effect. Similar problems exist with the interpretation of the measurements of $\Delta\nu$ on samples of different dimensions.[3] Hence, for the time being there are no experimental results which would strongly disagree with the model of classical percolation.

In summary, we have investigated the metal–insulator transition in the zero-Hall-conductivity insulating phase of a 2D hole system in Si/SiGe heterostructures. The behavior of the nonlinear $I-V$ characteristics in the insulating phase has allowed us to establish the percolation nature of the metal–insulator transition and to determine the critical exponent $s\approx 1$ for the localization length. The experimental data are similar to those obtained on high-mobility 2D systems in Si MOSFETs and AlGaAs/GaAs heterostructures and can be explained in the framework of classical percolation. This is in contradiction to the interpretation of scaling measurements.

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