# $SU(3)$-Skyrmion with $B=2$ and large strangeness content

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The bound $SU(3)$-skyrmion with baryon number $B=2$ is found, with strangeness content close to 0.5. It is of the molecular (dipole) type and has a binding energy of several tens of MeV. © *1995 American Institute of Physics.*

The effective theory of chiral fields first proposed in Ref. 1 provides an attractive possibility of describing mesons and baryons starting from few basic ingredients. The existence of bound states of skyrmions—$SU(3)^2$ and $SU(2)^3$—opens up wide prospects for application of this theory in nuclear physics as well.[4–7] The predictions of the spectrum of strange and nonstrange dibaryons, obtained mainly by means of the collective-coordinates quantization procedure, can be used to check the concepts of the chiral soliton approach.[7,4]

A fundamental problem is to identify the lowest energy state in $SU(3)$ configuration space for each baryon number $B$. This configuration should be used as a starting one for quantization of the zero and nonzero modes to get the observable spectrum of physical states. This question is closely related to the problem of the existence of strange matter fragments investigated previously in the framework of other approaches.[8–10]

Here we investigate this problem in the $SU(3)$ extension of the model for the case of baryon number $B=2$. The most general configuration can be described by 8 functions of 3 variables (instead of 3 functions in $SU(2)$ case).

The expression for the soliton energy can be obtained from the well-known Skyrme Lagrangian[1] extended to $SU(3)$ in terms of 8 functions $L_i$ which determine the left Cartan–Maurer current $U^+d_iU$ (or, equivalently, $R_i$):

$$U^+d_iU=i\lambda_pL_{p,i}, \quad p=1,\dots 8.$$

The second order (kinetic) term is especially simple:

$$E_{\text{kin}}=\frac{F_\pi^2}{8}(L_1^2+L_2^2+\dots+L_8^2). \tag{1}$$

The Skyrme term is given by

  

$$E_{\rm Sk}=\frac{1}{2e^2}\Big\{[L_1L_2]^2+[L_2L_3]^2+[L_3L_1]^2+[L_4L_5]^2+[L_6L_7]^2+\frac{3}{4}([L_4L_8]^2$$
$$+[L_5L_8]^2+[L_6L_8]^2+[L_7L_8]^2)+([L_4L_6]^2+[L_4L_7]^2+[L_5L_6]^2+[L_5L_7]^2$$
$$+[L_1L_4]^2+[L_1L_5]^2+[L_1L_6]^2+[L_1L_7]^2+[L_2L_4]^2+[L_2L_5]^2+[L_2L_6]^2$$
$$+[L_2L_7]^2)+[L_3L_4]^2+[L_3L_5]^2+[L_3L_6]^2+[L_3L_7]^2\Big)/4+\frac{\sqrt{3}}{2}([L_8L_4]$$
$$\times([L_1L_6]+[L_3L_4]-[L_2L_7])+[L_8L_5]([L_1L_7]+[L_2L_6]+[L_3L_5]))$$
$$+\tfrac{3}{2}([L_1L_2]([L_4L_5]+[L_7L_6])+[L_2L_3]([L_4L_7]+[L_6L_5])+[L_1L_3]([L_6L_4]$$
$$+[L_7L_5])+[L_4L_5][L_6L_7])\Big\}. \qquad (2)$$

In the $SU(2)$ case only the first 3 terms remain in the expression for $E_{\rm Sk}$. The baryon (winding) number density in terms of $L_i$ is

$$B=\frac{1}{2\pi^2}\epsilon_{ijk}\int d^3r\Big\{L_{1i}L_{2j}L_{3k}+\frac{1}{2}[L_{1i}(L_{4j}L_{7k}-L_{5j}L_{6k})+L_{2i}(L_{4j}L_{6k}+L_{5j}L_{7k})$$
$$+L_{3i}(L_{4j}L_{5k}-l_{6j}L_{7k})]+\frac{\sqrt{3}}{2}L_{8i}(L_{4j}L_{5k}+L_{6j}L_{7k})\Big\}. \qquad (3)$$

The explicit expressions for $L_i$ depend on the ansatz for the $SU(3)$ matrix $U$. At first we used the following ansatz, similar to that which is often used for the description of zero modes of $SU(3)$ rotations in the procedure of quantization of rotated skyrmions, see e.g.:[7]

$$U=U_LU_4U_8U_R, \qquad (4)$$

where $U_L$ and $U_R$ describe $SU(2)$ skyrmions embedded into $SU(3)$. The left and right baryon numbers $B_L$ and $B_R$ can in general be arbitrary; $U_4=\exp(-i\nu\lambda_4)$, $U_8=\exp(-i\rho\lambda_8/\sqrt{3})$. In this case we have, after some chiral rotation which only affects the chiral symmetry breaking mass terms:

$$L_{1i}=c_\nu\lambda_{1i}+r_{1i}, \text{ etc.} \qquad (5)$$

Here $l_{k,i}$ and $r_{k,i}$ are the left and right $SU(2)$ Cartan–Maurer currents associated with $U_L$ and $U_R$.

The next step after the choice of ansatz is the choice of boundary conditions on the functions that appear in it. The choice $\nu_0=0$ corresponds to an incident strangeness content $SC=0$. The interference between the left and right $SU(2)$ currents is especially strong in this case.

For baryon number $B=2$ concentrated only in $U_L$ or $U_R$ we have found that the $SU(2)$ torus is a local minimum in $SU(3)$. For $B_L=B_R=1$ and for topological centers located at different points, a direct calculation shows that the point $\nu=0$ is also a local minimum of $SU(3)$ with respect to rotations into the "strange" direction.

The other parametrization of interest is

$$U=U_L(u,s)U(u,d)U_R(d,s) \tag{6}$$

with the $U$'s on the right-hand side located in different $SU(2)$ subgroups of $SU(3)$. One of the $SU(2)$ matrices, e.g., $U(u,d)$, depends on two functions, $a$ and $b$:

$$U(u,d)= \exp(ia\lambda_2)\exp(ib\lambda_3). \tag{7}$$

The total number of independent functions is again equal to 8. For this case we have, after some chiral rotation:

$$\begin{gathered} L_{1i}=s_a c_a l_{3i}, \quad L_{2i}=d_i a, \quad L_{3i}=(c_{2a}l_{3i}-r_{3i})/2+d_i b, \\ L_{4i}=l_{1i}c_a, \quad L_{5i}=c_a l_{2i}, \quad L_{6i}=l_{1i}s_a+r_{1i}(b), \\ L_{7i}=s_a l_{2i}+r_{2i}(b), \quad L_{8i}=\sqrt{3}(l_{3i}+r_{3i})/2, \end{gathered} \tag{8}$$

where $l_i$ and $r_i$ are related to $U(u,s)$ and $U(d,s)$.

The kinetic term is

$$E_{\mathrm{kin}}=\frac{F_\pi^2}{8}\{l_i^2+r_i^2+(1+s_a^2)l_{3i}r_{3i}+2s_a(l_{1i}r_{1i}(b)+l_{2i}r_{2i}(b))+(d_i a)^2+(d_i b)^2$$

$$+d_i b(c_{2a}r_{3i}-l_{3i})\}; \tag{9}$$

$$r_1(b)=r_1c_b-r_2s_b, \quad r_2(b)=r_2c_b+r_1s_b.$$

The baryon number density can also be rewritten in terms of the $SU(2)$ currents and the functions $a$ and $b$. The additivity of the $SU(2)$-baryon number density is then clear from this expression.

We start from the product ansatz with $(u,s)$ and $(d,s)$ hedgehogs in the most attractive orientation, similar to the $(u,d)-SU(2)$ case. The procedure of minimization with the help of the "hat" method applied previously for the case of $SU(2)$ skyrmions[3,7] yields some configurations which are local minima, at least with respect to local deformations of all the functions that appear in the energy functional, and to global rotations also.

The lines of equal mass density for the $B=2$ configuration, which obviously possesses a dipole form, are shown in Fig. 1 for values of the model parameters $F_\pi=186$ MeV and $e=4.12$, in the flavor-symmetric case. The binding energy of this configuration, representing the new local minimum, equals 73 MeV, its strangeness content $SC=0.494$, and the distance between topological centers of incident $SU(2)$ skyrmions $d=1.06$ fm, along the $z$ direction. When two hedgehogs are in different $SU(2)$ subgroups of $SU(3)$ and interact in one common degree of freedom, the attraction between them is not sufficient to form the torus-like bound state[3] in which their individuality is lost completely.

The flavor symmetry breaking (FSB) in the mass term can be taken into account easily in terms of the real parts of the diagonal matrix elements of $U$, denoted $v_1$, $v_2$, and $v_3$, and the kaon and pion masses:

$$M_{m.t.}=\tfrac{1}{8}F_\pi^2m_\pi^2(2-v_1-v_2)+\tfrac{1}{8}F_K^2m_K^2(1-v_3). \tag{10}$$

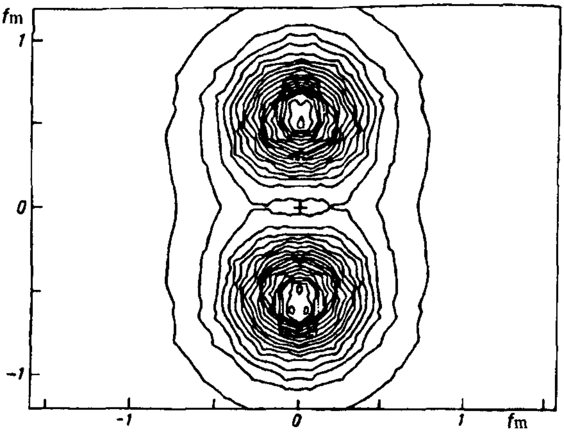


FIG. 1. Equal mass density lines for flavor symmetric (FS) case and distance between topological centers of solitons $d=1.06$ fm; $F_\pi=186$ MeV, $e=4.12$.

The real part of the (3,3) diagonal matrix element of $U$, which defines the strangeness content of the configuration, is equal to

$$v_3=f_1q_1-f_4q_4+s_a[s_b(f_2q_3-f_3q_2)-c_b(f_3q_3+f_2q_2)], \tag{11}$$

where $\bar{U}_L=f_1+i(\bar{\tau}_1f_2+\bar{\tau}_2f_3+\bar{\tau}_3f_4)$, $\bar{U}_R=q_1+i(\tilde{\tau}_1q_2+\tilde{\tau}_2q_3+\tilde{\tau}_3q_4)$, $\bar{U}_L$ and $\bar{U}_R$ are $SU(2)$ parts of $U_L$ and $U_R$, and $\bar{\tau}_i$ and $\tilde{\tau}_i$ are the corresponding Pauli matrices.

$$SC=(1-v_3)/(3-v_1-v_2-v_3). \tag{12}$$

The lines of equal mass density are shown in Fig. 2 for the case of flavor symmetry breaking. This configuration has a binding energy of about 60 MeV for the same parameters of the model, but it quite naturally has smaller dimensions: the distance between

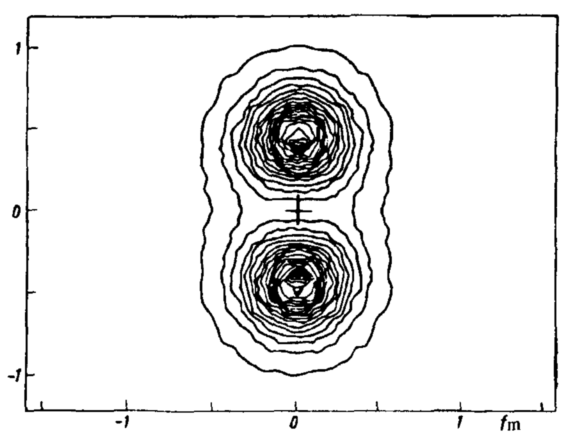


FIG. 2. Equal mass density lines for the case of FSB, $d=0.86$ fm on the start.

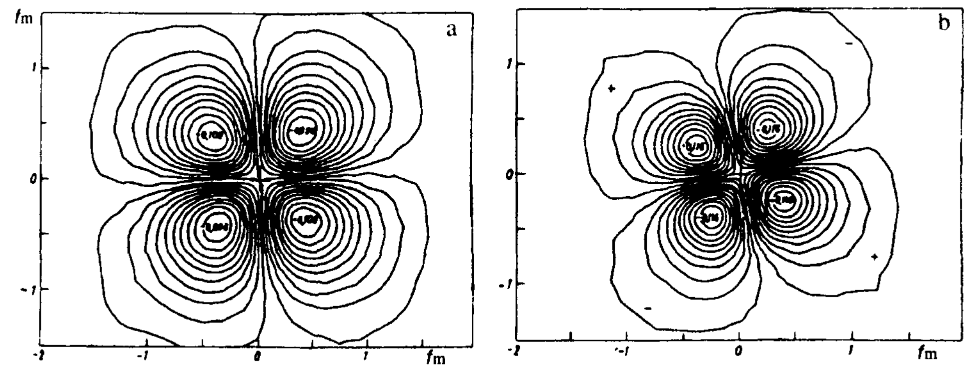


FIG. 3. Lines of equal values of the function $a$, which gives the relative local orientation of the two deformed strange hedgehogs in isospace, in the plane $z=0$: a) FS, b) FSB cases.

maxima of mass density is about 0.8 fm. The strangeness content of this configuration is close to 0.5 also: $SC=0.499$. So, the values of binding energy do not differ considerably from the flavor symmetric case, within the accuracy of the calculation, about 5–10 MeV. The strangeness content of the configuration does not decrease in the FSB case in comparison with the FS case. Note that $SC=1/3$ for the $SU(3)$ hedgehog,[2] as it should for strange matter.

The function $a$ giving the local relative orientation in isospace of the two $B=1$ solitons $(u,s)$ and $(d,s)$ is shown in Fig. 3a and 3b for the FS and FSB cases in the plane $z=0$. The decrease in size of the configuration with the FSB terms included is well illustrated.

The quantization of the dipole-type configuration should be performed in a manner similar to the quantization of the torus-like configurations, leading to the other branch of predictions for the spectrum of strange dybaryons, additional to that of Ref. 7. The correct quantum numbers of the states should be established after this procedure, e.g., the electric dipole moment of physical states should be equal to zero.

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