Inapplicability of the narrow-width approximation in the decays $\phi\to\gamma a_0$ and $\phi\to\gamma f_0$

N. N. Achasov[1)] and V. V. Gubin
Laboratory of Theoretical Physics, S. L. Sobolev Institute of Mathematics, Siberian Branch of the Russian Academy of Sciences, 630090 Novosibirsk 90, Russia

(Submitted 20 June 1995)
Pis'ma Zh. Éksp. Teor. Fiz. **62**, No. 3, 182–187 (10 August 1995)

The narrow-width approximation, which is used in theoretical and experimental studies of the radiative decays $\phi\to\gamma a_0$ and $\phi\to\gamma f_0$, is not valid for the f_0 and a_0 resonances. How to properly use the corrections for the finite width is shown. © *1995 American Institute of Physics.*

The central problem of precharm hadron spectroscopy is now the problem of the scalar $f_0(975)$ and $a_0(980)$ mesons. The problem is that these states exhibit a whole series of properties which are unusual from the standpoint of the naive quark ($q\bar{q}$) model; see, for example, the reviews Refs. 1 and 2. At the same time, all interesting properties of these mesons can be understood[1–3] on the basis of the four-quark ($q^2\bar{q}^2$) MIT-bag model.[4] In addition to the $q^2\bar{q}^2$ nature of the $a_0(980)$ and $f_0(975)$ mesons, the fact that they can possibly be $K\bar{K}$ molecules is also discussed.[5] It is also possible that the $f_0(975)$ and $a_0(980)$ mesons are "witnesses" of the trapping of quarks.[6]

As a result of the efforts of theoreticians over a number of years[7,8] (see also the bibliography cited in Ref. 8), it has been established that the decays $\phi\to\gamma f_0\to\gamma\pi\pi$ and $\phi\to\gamma a_0\to\gamma\eta\pi$ can play the key role in determining the nature of the scalar $f_0(975)$ and $a_0(980)$ mesons.

A study of the decay $\phi\to\gamma f_0\to\gamma\pi\pi$ on the upgraded VEPP-2M e^+e^- collider with the help of the KMD2-2 detector in Novosibirsk has now been initiated.[2)] Moreover, in Novosibirsk a SND detector, which will be used to study these decays, is now operating on the same collider. Finally, startup of the DAPHNE ϕ factory, which will make it possible to study the $f_0(975)$ and $a_0(980)$ scalar mesons, apparently exhaustively, is expected to start operating soon in Frascati.

It is obvious that a natural condition for solving the puzzle of the scalar mesons is the formulation of theoretical predictions before the experimental data are analyzed. In all theoretical studies (see Ref. 8 and the bibliography cited there), with the exception of Ref. 7, the approximation of a narrow width of the $f_0(975)$ and $a_0(980)$ mesons has been used, since the apparent width of the mesons is 25–50 MeV. In addition, experimentors have used this approximation to obtain the upper limits of the intensities of the decays[9,10] $\phi\to\gamma a_0$ and$\phi\to\gamma f_0$. In this connection, we wish to call attention to the fact that the narrow-width approximation is invalid in the present case. We shall show that the predictions obtained for the intensities of the decays in the narrow-width approximation are too high by at least a factor of two.

We introduce the amplitudes

$$M(\phi\to\gamma R;m)=g_R(m)(e(\phi)\cdot e(\gamma)),\ \ R=a_0,f_0, \tag{1}$$

where $e(\phi)$ and $e(\gamma)$ are the three-dimensional polarization vectors of the ϕ meson and the γ ray in the rest system of the ϕ meson, and m is the invariant mass of the ab state of the two pseudoscalar mesons a and b into which the R meson decays.

In e^+e^- collisions the ϕ meson which is produced is transversely polarized with respect to the axis of the beams in the center-of-mass system. Therefore, if the polarization of the γ ray in the reaction $e^+e^-\to\phi\to\gamma R$ is ignored, the amplitudes (1) will lead to the angular distribution

$$W(\theta)=\frac{3}{8}\,(1+\cos^2\theta), \tag{2}$$

where θ is the angle between the direction of emergence of the γ ray and the axis of the beams.

According to gauge invariance, the amplitudes of the decays are proportional to the tensor of the electromagnetic (electric) field, i.e., the energy of the photon (ω) at low energies,

$$g_R(m)\to\omega\cdot\text{const}, \tag{3}$$

if $m\to m_\phi$ and $\omega=(1/2)m_\phi(1-m^2/m_\phi^2)\to 0$.

The width of the decay in the approximation of a narrow width of the scalar resonance is

$$\Gamma(\phi\to\gamma R;m)=\frac{1}{3}\,\frac{|g_R(m)|^2}{4\pi}\,\frac{1}{2m_\phi}\left(1-\frac{m^2}{m_\phi^2}\right). \tag{4}$$

The physically measurable partial widths are

$$\Gamma(\phi\to\gamma R\to\gamma ab)=\frac{2}{\pi}\int_{m_a+m_b}^{m_\phi} m\,dm\,\frac{m\Gamma(R\to ab;m)\Gamma(\phi\to\gamma R;m)}{|D_R(m)|^2} \tag{5}$$

for $ab=\pi\pi$ and $\pi^0\eta$ and for $ab=K^+K^-$ and $K^0\bar{K}^0$ they are

$$\Gamma(\phi\to\gamma(a_0+f_0)\to\gamma K^+K^-)$$

$$=\frac{2}{\pi}\int_{2m_{K^+}}^{m_\phi} m^2\Gamma(f_0\to K^+K^-;m)\Gamma(\phi\to\gamma f_0;m)$$

$$\times\left|\frac{1}{D_{f_0}(m)}+\frac{g_{a_0}(m)g_{a_0K^+K^-}}{g_{f_0}(m)g_{f_0K^+K^-}}\,\frac{1}{D_{a_0}(m)}\right|^2 dm,$$

$$\Gamma(\phi\to\gamma(a_0+f_0)\to\gamma K^0\bar{K}^0)$$

$$=\frac{2}{\pi}\int_{2m_{K^0}}^{m_\phi} m^2\Gamma(f_0\to K^0\bar{K}^0;m)\Gamma(\phi\to\gamma f_0;m)$$

$$\times\left|\frac{1}{D_{f_0}(m)}+\frac{g_{a_0}(m)g_{a_0K^0\bar K^0}}{g_{f_0}(m)g_{f_0K^0\bar K^0}}\frac{1}{D_{a_0}(m)}\right|^2 dm, \tag{6}$$

where $1/D_R(m)$ is the scalar-meson propagator.

The width of the decay of a scalar R resonance into the ab state consisting of two pseudoscalar mesons with invariant mass m can be written in the form

$$\Gamma(R\to ab;m)=\frac{g^2_{Rab}}{16\pi}\frac{1}{m}\rho_{ab}(m),$$

$$\rho_{ab}(m)=\sqrt{(1-m_+^2/m^2)(1-m_-^2/m^2)},\quad m_\pm=m_a\pm m_b. \tag{7}$$

The identity of the final particles in the case $\pi^0\pi^0$ is taken into account in the definition of $g_{f_0\pi^0\pi^0}$.

The isotopic symmetry

$$g_{f_0K^+K^-}=g_{f_0K^0\bar K^0},\quad g_{a_0K^+K^-}=-g_{a_0K^0\bar K^0} \tag{8}$$

must be taken into consideration in Eq. (6).

Two factors make the narrow-width approximation invalid for scalar mesons in the case at hand.

The first (and principal) factor is associated with the soft (on the scale of the strong-interactions) photons. It follows from Eqs. (3)–(6) that the first shoulder of the resonance is suppressed by at least the factor $(\omega/\omega_0)^3$, where $\omega_0=m_\phi(1-M_R^2/m_\phi^2)/2$, and M_R is the mass of the resonance. It will be shown below that this results in the suppression of the integral contribution from the right-hand shoulder of the resonance by a factor of 5 for decays in the $\pi\pi$ and $\pi\eta$ channels and by at least a factor of 50 for decays in the K^+K^- and $K^0\bar K^0$ channels. Therefore, the physically measurable widths (5) are virtually completely determined by "half" of the resonance — the left-hand shoulder in the $\pi\pi$ and $\pi\eta$ channels. This alone overestimates by a factor of 2 the results obtained in the narrow-width approximation $[\Gamma(\phi\to\gamma R;M_R)]$.

The second factor is associated with the corrections for the finite width in the scalar-meson propagators. We will use the general Breit–Wigner formula. If $m>2m_{K^+}$, $2m_{K^0}$, then

$$\frac{1}{D_R(m)}=\frac{1}{m_R^2-m^2-i(\Gamma_0(m)+\Gamma_{K\bar K}(m))m}, \tag{9}$$

$$\Gamma_{K\bar K}(m)=\frac{g^2_{RK^+K^-}}{16\pi}\left(\sqrt{1-4m^2_{K^+}/m^2}+\sqrt{1-4m^2_{K^0}/m^2}\right)\frac{1}{m}.$$

If $2m_{K^+}<m<2m_{K^0}$, we then have

$$\frac{1}{D_R(m)}=\left[m_R^2-m^2+\frac{g_{RK^+K^-}^2}{16\pi}\sqrt{4m_{K^+}^2/m^2-1}-i\frac{g_{RK^+K^-}^2}{16\pi}\right.$$
$$\left.\times\sqrt{4m_{K^0}^2/m^2-1}-i\Gamma_0(m)m\right]^{-1}. \tag{10}$$

When $2m_{K^+},\ 2m_{K^0}>m$,

$$\frac{1}{D_R(m)}=\left[m_R^2-m^2+\frac{g_{RK^+K^-}^2}{16\pi}\left(\sqrt{4m_{K^+}^2/m^2-1}+\sqrt{4m_{K^0}^2/m^2-1}\right)-i\Gamma_0(m)m\right]^{-1}, \tag{11}$$

where the width $\Gamma_0(m)$ of the decay of a scalar R resonance in the $\pi\eta$ or $\pi\pi$ channels is determined by expression (7).

Since the scalar resonances are below the $K\bar{K}$ thresholds, the position of the peak in the cross section or in the mass spectrum does not coincide with m_R; this can be easily verified by using expressions (9)–(11). For this reason, the mass in the Breit–Wigner expressions (9)–(11) must be renormalized:

$$m_R^2=M_R^2-\frac{g_{RK^+K^-}^2}{16\pi}\left(\sqrt{4m_{K^+}^2/M_R^2-1}+\sqrt{4m_{K^0}^2/M_R^2-1}\right), \tag{12}$$

where M_R^2 is the squared physical mass ($M_{a_0}=980$ MeV and $M_{f_0}=975$ MeV), and m_R^2 is the squared bare mass. Therefore, the physical mass is greater than the bare mass. This fact is especially important in the case of strong coupling of scalar mesons with the $K\bar{K}$ channels, as is the case in the four-quark or molecular models. On the other hand, this factor is disregarded in the fitting of experimental data and in the theoretical studies, with the exception of Refs. 1, 7, and 11. It should be noted that expressions (9)–(11) are applicable only in the resonance region. For example, at $m^2=0$ these expressions do not have the correct analytical properties. Expressions in which this deficiency is eliminated can be found in Refs. 1, 7, and 11.

Three sets of coupling constants between the scalar mesons and the $K\bar{K}$ channels are now under consideration:

1) superallowed coupling constants on the basis of the Okubo–Zweig–Iizuka (OZI) rule in the four-quark model:[1,4,7,11]

$$g_{f_0K^+K^-}^2/4\pi=g_{a_0K^+K^-}^2/4\pi=2.3\ \text{GeV}^2; \tag{13}$$

2) coupling consants in the molecular model:[5,8]

$$(g_{f_0K^+K^-}^2/4\pi)=(g_{a_0K^+K^-}^2/4\pi)=0.6\ \text{GeV}^2; \tag{14}$$

we note that in this model $M_R-m_R=24$ MeV; and

3) OZI-allowed constantly in the $q\bar{q}$ model

$$g_{a_0K^+K^-}^2/4\pi=g_{f_0K^+K^-}^2/4\pi\simeq 0.2\ \text{GeV}^2. \tag{15}$$

In the case of the superallowed constants (13) all effects associated with the finite width of the scalar resonances were taken into account in Ref. 7. We will therefore not consider them here.

In what follows we set $\Gamma_0(M_R)=50$ MeV, which corresponds to an effective width of $\simeq 25$ MeV and relative intensity of the decay in the $K\bar{K}$ channels $BR(f_0(a_0)\to K\bar{K})=0.33(0.35)$ in the molecular model (14). In the case (15) the effective width is $\simeq 40$ MeV and $BR(f_0(a_0)\to K\bar{K})=0.15(0.16)$. In the region of the resonance the relation $M_R\Gamma_0(M_R)=m\Gamma_0(m)$ holds well.

In the molecular model[5,8] the integral over the right-hand shoulder of the resonance $f_0(a_0)$ in the $\pi\pi(\pi\eta)$ channel is

$$\frac{2}{\pi}\int_{M_R}^{m_\phi}\left(\frac{\omega}{\omega_0}\right)^3\frac{m^2\Gamma_0(m)}{|D_R(m)|^2}\,dm\simeq 0.0951(0.0813), \tag{16}$$

instead of 0.5 as in the narrow-width approximation.

On the left-hand shoulder of the resonance, as usual, it is natural to assume $\Gamma(\phi\to\gamma R;m)$ is a constant. This is confirmed in specific models.[7] The integral over the left-hand shoulder of the resonance is

$$\frac{2}{\pi}\int_{(m_a+m_b)}^{M_R}\frac{m^2\Gamma_0(m)}{|D_R(m)|^2}\,dm\simeq 0.2827(0.2618), \tag{17}$$

instead of 0.5 as the narrow-width approximation. Therefore, the integral over the resonance is equal to 0.3778(0.3431), instead of 1 in the narrow-width approximation.

The decays into the $K\bar{K}$ state have virtually no effect on this picture. The decay into the $K\bar{K}$ state occurs only on the right-hand shoulder of the resonance:

$$\frac{2}{\pi}\int_{2m_{K^+}}^{m_\phi}\left(\frac{\omega}{\omega_0}\right)^3\frac{m^2\Gamma(f_0(a_0)\to K^+K^-;m)}{|D_R(m)|^2}\,dm\simeq 0.0022(0.0041), \tag{18}$$

$$\frac{2}{\pi}\int_{2m_{K^0}}^{m_\phi}\left(\frac{\omega}{\omega_0}\right)^3\frac{m^2\Gamma(f_0(a_0)\to K^0\bar{K}^0;m)}{|D_R(m)|^2}\,dm\simeq 0.0004(0.0007). \tag{19}$$

We obtain 0.0026(0.0048) for the sum of the contributions (18) and (19).

In summary, the predictions in the narrow-width approximation are higher by the factor $1/0.38(1/0.35)\simeq 2.6(2.9)$. It should be noted that we disregarded the interference between a_0 and f_0 in the decays $\phi\to\gamma(a_0\pm f_0)\to\gamma K\bar{K}$ [see Eq. (6)]. In the model studied in Ref. 8, the inteference should be constructive in the K^+K^- channel and destructive in the $K^0\bar{K}^0$ channel. As are result, the contribution of the K^+K^- channel is four times larger than (18), and the contribution of the $K^0\bar{K}^0$ channel is 100 times smaller than (19). We note that in the sum $BR(\phi\to\gamma(a_0+f_0)\to\gamma K^+K^-)+BR(\phi\to\gamma(a_0-f_0)\to\gamma K^0\bar{K}^0)$ the interference does not vanish because of the large difference in the threshold energies of the K^+K^- and $K^0\bar{K}^0$ channels ($2m_{K^0}-2m_{K^+}=8$ MeV). However, since the contribution of the $K\bar{K}$ channel is negligible, our result remains essentially the same.

We now call attention to one other fact. Taking into account the loose structure of the molecule by means of a wave function should give additional suppression. In any case, one gets that impression at first glance. However, clarification of this point is not a subject of this letter.

The situation is essentially the same in the case of the $q\bar{q}$ model [Eq. (15)]. The integral over the right-hand shoulder of the resonance $f_0(a_0)$ in the $\pi\pi(\pi\eta)$ channel is

$$\frac{2}{\pi}\int_{M_R}^{m_\phi}\left(\frac{\omega}{\omega_0}\right)^3\frac{m^2\Gamma_0(m)}{|D_R(m)|^2}\,dm \simeq 0.1124(0.0997), \tag{20}$$

instead of 0.5 as the narrow-width approximation. The integral over the left-hand shoulder of the resonance is

$$\frac{2}{\pi}\int_{(m_a+m_b)}^{M_R}\frac{m^2\Gamma_0(m)}{|D_R(m)|^2}\,dm \simeq 0.3835(0.3609), \tag{21}$$

instead of 0.5 as the narrow-width approximation. Therefore, the integral over the resonance is equal to 0.4959(0.4598), instead of 1 in the narrow-width approximation.

The contribution of the $K\bar{K}$ states is

$$\frac{2}{\pi}\int_{2m_{K^+}}^{m_\phi}\left(\frac{\omega}{\omega_0}\right)^3\frac{m^2\Gamma(f_0(a_0)\to K^+K^-;m)}{|D_R(m)|^2}\,dm \simeq 0.0015(0.0026), \tag{22}$$

$$\frac{2}{\pi}\int_{2m_{K^0}}^{m_\phi}\left(\frac{\omega}{\omega_0}\right)^3\frac{m^2\Gamma(f_0(a_0)\to K^0\bar{K}^0;m)}{|D_R(m)|^2}\,dm \simeq 0.0003(0.0005). \tag{23}$$

We obtain 0.0018(0.0031) for the sum of the contributions (22) and (23). Therefore, the predictions in the narrow-width approximation in (15) are higher by a factor of 2.

We wish to make one final remark. It is obvious from what we have said above that the upper limits $BR(\phi\to\gamma a_0)$ and $BR(\phi\to\gamma f_0)$ presented in Refs. 9 and 10, strictly speaking, make no sense, since they assume that

$$BR(\phi\to\gamma a_0\to\gamma\pi\eta)=BR(\phi\to\gamma a_0)BR(a_0\to\pi\eta),$$

$$BR(\phi\to\gamma f_0\to\gamma\pi\pi)=BR(\phi\to\gamma f_0)BR(f_0\to\pi\pi), \tag{24}$$

and are found to be lower by at least a factor of 2. In addition, it is assumed in this case that

$$BR(\phi\to\gamma R\to\gamma K\bar{K})=BR(\phi\to\gamma R)BR(R\to K\bar{K}), \tag{25}$$

and the result is at least 50 times too high. We need only mention $BR(\phi\to\gamma a_0\to\gamma\pi\eta)$ and $BR(\phi\to\gamma f_0\to\gamma\pi\pi)$.

Partial support for this work was provided by the Russian Fund for Fundamental Research, grant 94-02-05 188.

[1]e-mail: achasov@math.nsk.su

[2]Report by E. P. Solodov at DAPHNE'95 in Frascati. N. N. Achasov is grateful to E. P. Solodov for many discussions.

[1] N. N. Achasov, S. A. Devyanin, and G. N. Shestakov, Usp. Fiz. Nauk **142**, 361 (1984) [Sov. Phys. Usp. **27**, 161 (1984)].
[2] N. N. Achasov, Nucl. Phys. B. (Proc. Suppl.) **21**, 189 (1991).
[3] N. N. Achasov and G. N. Shestakov, Usp. Fiz. Nauk **161**, 53 (1991) [Sov. Phys. Usp. **34**, 471 (1991)].
[4] R. L. Jaffe, Phys. Rev. D **15**, 267, 281 (1977).
[5] J. Weinstein and N. Isgur, Phys. Rev. D **41**, 2236 (1990).
[6] F. E. Close, Yu. L. Dokshitzer, V. N. Gribov *et al.*, Phys. Lett. B **319**, 291 (1993).
[7] N. N. Achasov and V. N. Ivanchenko, Nucl. Phys. B **315**, 465 (1989).
[8] F. E. Close, N. Isgur, and S. Kumano, Nucl. Phys. B **389**, 513 (1993).
[9] Particle Data Group, Phys. Rev. D **50**, 1468, 1469 (1994).
[10] R. R. Akhmetshin *et al.*, Preprint No. PITT 94-05, University of Pittsburgh.
[11] N. N. Achasov and G. N. Shestakov, Z. Phys. C **41**, 309 (1988).

Translated by M. E. Alferieff