Angular dependence of the probability of specular reflection of conduction electrons from the surface of a Bi sample

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A method, based on transverse electron focusing (EF) [V. S. Tsoĭ, JETP Lett. **19**, 70 (1974)] has been developed. This method is used to measure the dependence of the probability q of specular reflection of conduction electrons from the surface of a bismuth sample on the angle of incidence θ. It can also be used to determine the irregularities of the interphase boundary (internal surfaces) in situ. © *1995 American Institute of Physics.*

The use of conduction electrons (characteristic excitations of a solid) as probe irradiation for structure analysis of a surface has several advantages over external irradiation (see Refs. 1–3). The development of an experimental base for this purpose is important because it opens up the possibility of performing structure analysis on internal surfaces in situ. It should be noted that the fundamental possibilities of solving this problem have been demonstrated in theoretical studies of the resonance absorption of Rayleigh sound by surface electrons,[4] electron focusing (EF),[5,6] and the Sondheimer effect.[7]

The simple [in the case of a free particle (wave)] experimental problem of measuring the angular dependence $q(\theta)$ of the probability of specular reflection from a rough surface becomes complicated in the case of conduction electrons. Few measurements of $q(\theta)$ for conduction electrons have therefore been performed.[8,9] In Ref. 8 the function $q(\theta)$ in the case of reflection of bismuth electrons from sample surfaces parallel to the bisector and trigonal planes was reconstructed, by means of electron focusing, from the dependence of the monotonic behavior of the collector voltage on the external magnetic field H. In Ref. 9 the dependence $q(\theta)$ in the case of reflection of antimony electrons from a whisker surface oriented parallel to the trigonal plane was determined from measurements of the amplitude of the quantum oscillations of the magnetoresistance in thin samples.[10]

The development of this method was made possible by the solution of the problem of creating a system of contacts for observing electron focusing with an intercontact spacing $L \sim 1-10$ μm (a description of the system will be published separately). For such small values of L, drift electron focusing (DEF) can be observed in bismuth with the contact lines L and H oriented close to the direction of the C_1 axis. Th spacing L must be decreased, since for $H \| C_1$ the velocity of the electrons in the central section is more than an order of magnitude higher than the velocity of the electrons at the reference point.[11] Therefore, in the absence of anomalous anisotropy of scattering, with a fixed electron

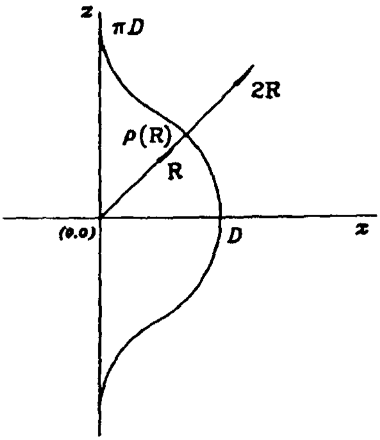

FIG. 1. The region of the surface which is accessible to electrons injected into a metal at the point (0, 0) and incident on the surface without reflections is bounded by the z axis and the magnetoid curve. D — Larmor diameter, $H\|z$.

relaxation time, the mean free path of electrons near the reference point, i.e., these are the electrons that form the DEF line, is more than an order of magnitude shorter than the mean free path in the perpendicular direction.

In the case of DEF the ratio of the amplitudes of the lines determines the probability of specular reflection of electrons with nonzero velocity component along the magnetic field H.[5,6] The probability of specular reflection of electrons incident on the surface at different angles θ but reflected from the same section of the surface of the sample midway between the contacts is measured for different directions of H. Omitting the simple mathematical calculations, we simply present the computational results for the DEF for the case of a spherical Fermi surface (FS). Figure 1 shows a set of points (magnetoid) on the surface of the sample, on which the electrons emerging from the emitter located at the origin of the coordinates are focused with H oriented along the z axis. The electrons injected into the metal return to the surface under the action of the Lorentz force. The region of the surface accessible to such electrons is bounded by the z axis and the magnetoid. The density of the electrons which are incident on the surface is singular on the magnetoid. The singularity, which arises in a magnetic field, in the density of the nonequilibrium electrons which emerge from the emitter and are incident on the surface, is the reason why electron-focusing lines appear (see Ref. 6). Focusing of the electrons incident on the surface at different angles θ (the electrons have a different momentum component normal to the surface) occurs as L is positioned along different directions relative to H. For $L\|H$ the electrons near the reference point, which are incident on the surface at the angle $\theta\approx 0$, are focused. When the contact line is perpendicular to H, the electrons incident on the surface at the angle $\theta\approx 90°$ are focused. We designate by φ the angle between H and L. As H rotates in the plane of the surface of the sample (φ changes), θ increases from 0 to 90°. If for fixed φ the DEF is observed in the field H_0, then electrons are focused, after being specularly reflected by the surface,

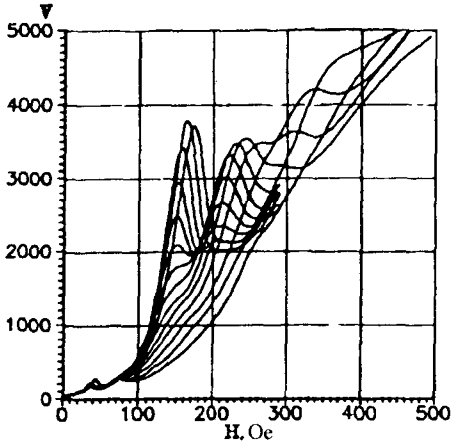

FIG. 2. DEF spectra for different directions of H. The angle between H and L varies from 20° up to 85° in steps of 5°. The contact line $L \| C_1$, $L = 14$ μm.

on the collector in a field $2H_0$, and the probability of specular reflection with a prescribed angle of incidence $\theta(\varphi)$ of the electrons on the surface is determined from the ratio of the amplitudes of the DEF lines in the fields H_0 and $2H_0$.

It is not difficult to perform a similar calculation for an ellipsoidal Fermi surface, which satisfactorily describes the real Fermi surface of bismuth.[11] We note that to calculate the trajectory accurately, it is necessary to know the velocity of the electrons at an arbitrary point on a real Fermi surface, which is not known very accurately. This circumstance, however, is of no fundamental importance. Actually, the character of the reflection is determined by two parameters: the change $\Delta p_\perp$ in the normal component of the momentum on specular reflection (the tangential component is conserved) and the sizes of the irregularities; $\Delta p_\perp c/eH_0 = L \sin \varphi$.

The experiment was set up as follows: An emitter and a collector were positioned so that $L \sim 10$ μm and $L \| C_1$. The focusing spectrum was recorded for different directions of the magnetic field — the voltage on the collector was measured as a function of H. The angle φ was varied from 20° up to 85° in steps of 5°. The spectra are shown in Fig. 2, where the focusing lines in fields $H > 100$ Oe are due to focusing of electrons after intervalley scattering.[12] The observation of these lines made it possible to check, to a high degree of accuracy, the satisfaction of the condition $L \| C_1$ (Refs. 12 and 13). The focusing lines with much smaller amplitudes in the fields H_0 and $2H_0$ ($H_0 \sim 40$ Oe) are DEF lines. Figure 3 shows these lines on a scale normalized to H_0 and A_0 (A_0 is the amplitude of the first DEF line) after compensating for the monotonic variation. Figure 3 illustrates the obvious increase in the amplitude of the second line (probability of specular reflection) as φ decreases.

The anomalously large magnitude of the transfer electron-focusing lines compared to the DEF lines is due to the cylindrical shape of the Fermi surface of bismuth. The cylindrical shape of the Fermi surface has the effects that, first, the relative fraction of the

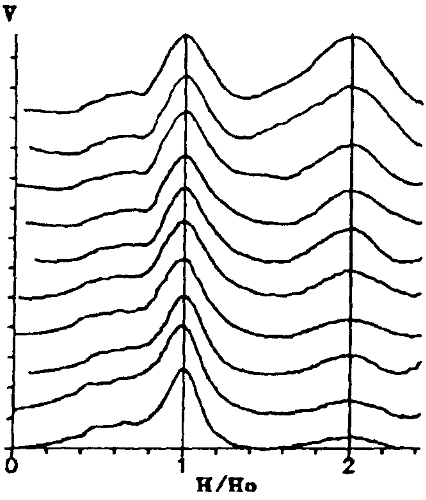

FIG. 3. Normalized DEF spectra of Fig. 2 for different directions of H. The angle between H and L varies from 20° up to 65° in steps of 5°. The monotonic dependence is subtracted out of the curve; the curves are normalized to H_0 and A_0 and they are shifted arbitrarily along the V axis.

focused electrons is large and, second, the diffuseness of the scattering is not so effective.[12]

The interference of the waves reflected from different sections of a rough surface results in suppression of the specularity of reflection. For a surface with random irregularities with a short correlation length $\mathscr{L} \ll 4\pi\eta$ (η is the rms height of the irregularities), the probability distribution of specular reflection is determined by Ziman's universal

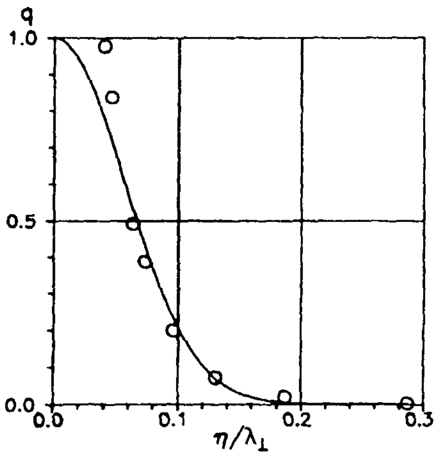

FIG. 4. Probability $q(\eta/\lambda_\perp)$ of specular reflection. Dots — Experiment; solid line — calculation according to Ziman's formula. The rms height of the irregularities is $\eta = 0.0338\ \mu\text{m}$.

formula:[14,15] $q=\exp\{-(4\pi\eta/\lambda_{\perp})^2\}$, where $\lambda_{\perp}$ is the component of the electron wavelength normal to the surface. The ratio $\eta/\lambda_{\perp}$ determines the probability of specular reflection. Figure 4 shows the measured dependence $q(\eta/\lambda_{\perp})$ (points) and the function calculated using Ziman's formula for $\eta=0.0338$ μm (solid line). The fact that even with diffuse reflection the amplitude of the second line is different from zero is taken into account in the measured dependence.[3,12] The small contribution of diffuse reflection to the amplitude of the second line was set equal to $(1-q)\alpha$. The quantity α was determined from the condition that for large angles of incidence $q=0$. As one can see from the figure, Ziman's formula describes the experimental data well. An appreciable deviation occurs only for small values of $\eta/\lambda_{\perp}$. Apparently, the discrepancy is due to the shadow effect (see, for example, Ref. 16): At some angle of incidence the humps of the irregularities screen the valleys and the effective height of the irregularities becomes less than for electrons incident on the surface at a large angle. The shadow effect is disregarded in Ziman's model. For the method used to prepare the samples—growth of oriented single crystals in a sectional, optically polished quartz mold[17]—the heights of the irregularities are probably determined by the irregularities of the quartz mold.

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