Spatial distribution of incoherent radiation sources by correlation processing

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A new principle is proposed for displaying the spatial distributions of sources of incoherent radiation. The principle is based on correlation processing of signals received by spatially separated detectors. It was shown experimentally that tomographic-type information can be obtained in this manner. © *1995 American Institute of Physics.*

It is well known that a stationary interference pattern of incoherent radiation is observed when the path difference between the interfering rays is small compared to the coherence length of the radiation. This feature is used in measurements of the linear dimensions of gage blocks[1] in Fizeau or Kösters type interferometers. In the present paper we present a new principle, based on this phenomenon, for obtaining information on the spatial distribution of sources of incoherent electromagnetic and acoustic radiation or radiation of any other nature. While in Fizeau and Kösters interferometers the spatial dimension of objects is measured along one direction, the principle proposed here makes it possible to obtain information on three-dimensional spatial distributions of the emission intensity of incoherent radiation. The main requirement on the radiation parameters for such a probe is that the coherence length L_c must be small compared to the required spatial resolution Δr.

Consider a space in which radiation propagates with velocity v and attenuation α. To simplify the analysis, we assume that all sources of incoherent radiation are located inside a region V (Fig. 1). For simplicity, we represent the instantaneous emission amplitudes of the radiation sources by a real scalar source function $N(\mathbf{r},t)$, which will be discussed in greater detail below. Let the radiation detectors be located at different points $\mathbf{r}_i$ $(i=1,2,\dots,N)$ outside the region V. Near the sensitive elements of the detectors the amplitude of the radiation from the sources can then be determined by the expression

$$S_i(t)=\int_V N\left(\mathbf{r},t-\frac{|\mathbf{r}-\mathbf{r}_i|}{v}\right)\frac{\exp(-\alpha|\mathbf{r}-\mathbf{r}_i|)}{|\mathbf{r}-\mathbf{r}_i|}d^3r. \tag{1}$$

The signal detectors perform a linear conversion of the amplitude $S_i(t)$ of the radiation into an electric signal. Ordinarily, such a conversion occurs with limitation of the frequency (temporal and spatial) spectra. However, without loss of generality, assuming the detectors are ideal, the effects of the limitation of the temporal and spatial spectra can be taken into account by prescribing appropriately the properties of the source function $N(\mathbf{r},t)$ and by assuming that the directivity patterns of the radiation detectors are nearly

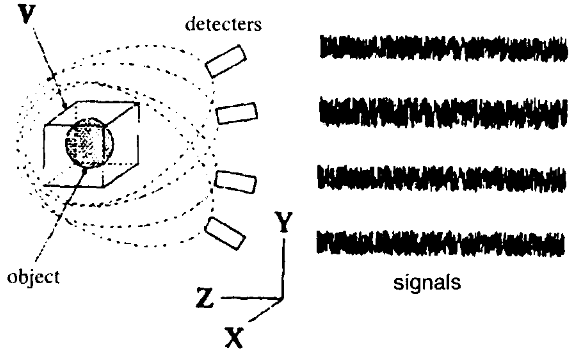

FIG. 1. Arrangement of the detectors with respect to an object and the form of the output signals. The dashed lines represent the directivity patterns of the detectors.

isotropic. In this case it can be assumed that the form of the electric signals at the outputs of the detectors is the same as at the input — $S_i(t)$. To analyze the signals $S_i(t)$, we employ the second-order correlation function:

$$C_{ij}(\tau_{ij})=\int_{-\infty}^{\infty} S_i(t)S_j(t-\tau_{ij})dt, \tag{2}$$

where τ_{ij} is the delay between the ith and jth signals. Substituting into Eq. (2) expression (1) for $S_i(t)$ and switching the order of integration, we obtain

$$\int_V d^3r\int_V d^3r' \frac{\exp[-\alpha(|\mathbf{r}-\mathbf{r}_i|+|\mathbf{r}'-\mathbf{r}_j|)]}{|\mathbf{r}-\mathbf{r}_i||\mathbf{r}'-\mathbf{r}_j|}\int_{-\infty}^{\infty} N\left(\mathbf{r},t-\frac{|\mathbf{r}-\mathbf{r}_i|}{v}\right)$$
$$\times N\left(\mathbf{r}',t-\frac{|\mathbf{r}'-\mathbf{r}_j|}{v}-\tau_{ij}\right)dt, \tag{3}$$

where the integral

$$I=\int_{-\infty}^{\infty} N\left(\mathbf{r},t-\frac{|\mathbf{r}-\mathbf{r}_i|}{v}\right)N\left(\mathbf{r}',t-\frac{|\mathbf{r}'-\mathbf{r}_j|}{v}-\tau_{ij}\right)dt \tag{4}$$

can be calculated with the spatial and temporal coherence parameters of the radiation which are determined by the source function $N(\mathbf{r},t)$.

In general, the correlation parameters of the radiation can be very diverse. To simply the analysis, we shall therefore study the general case of thermal radiation, which is also of interest for practical applications. The correlation length of the thermodynamic fluctuations of the temperature of a medium is determined by the standard expression $L_t=\sqrt{2\chi/\omega}$, where χ is the thermal diffusivity of the medium, and ω is the angular frequency of the fluctuations.[2] Even for relatively low frequencies — of the order of 1 MHz — and high (for the condensed state) thermal diffusivity $\chi=1$ cm^2/s, we obtain $L_t\approx 6\times 10^{-3}$ cm. For an entire series of probe problems which are useful in practice this value is less than the required spatial resolution Δr. Therefore, to simplify the analysis, we will restrict the discussion to the case in which the required spatial resolution $\Delta r > L_t$. In this case the radiation emitted from different points (separated from one another by a distance greater than L_t) $\mathbf{r}$ and $\mathbf{r}'$ in space may be assumed to be uncorrelated and I as a function of $\mathbf{r}$ and $\mathbf{r}'$ is proportional to $\delta(\mathbf{r}-\mathbf{r}')$. The effect of the temporal coherence of the radiation on I can be resolved in a similar manner. If we restrict the analysis again to the case $\Delta r > L_c$, where L_c is the coherence length defined as $L_c = v/\Delta f$ (v is the propagation velocity and Δf is the band width of the radiation), then I can be represented as a function of τ_{ij}, taking into account the delay:

$$\sim \delta\left(\frac{|\mathbf{r}-\mathbf{r}'|}{v} - \frac{|\mathbf{r}'-\mathbf{r}_j|}{v} - \tau_{ij}\right).$$

On the basis of what we have said above, the integral (4) can thus be estimated as follows:

$$I \sim \left\langle N^2(\mathbf{r})\right\rangle \delta(\mathbf{r}-\mathbf{r}')\, \delta\left(\frac{|\mathbf{r}-\mathbf{r}_i|}{v} - \frac{|\mathbf{r}'-\mathbf{r}_j|}{v} - \tau_{ij}\right), \tag{5}$$

where the brackets indicate time averaging.[2)] Substituting expression (5) into the initial expression (3) and performing the integration over dr', we obtain

$$\int_{V_{ij}} d^3r \frac{\exp\{-\alpha(|\mathbf{r}-\mathbf{r}_i| + |\mathbf{r}-\mathbf{r}_j|)\}}{|\mathbf{r}-\mathbf{r}_i||\mathbf{r}-\mathbf{r}_j|} \langle N^2(\mathbf{r})\rangle = k_i k_j C_{ij}(\tau_{ij}); \quad i,j = 1,2,\ldots,n, \quad i \neq j, \tag{6}$$

where k_i is the sensitivity coefficient for the ith channel, and V_{ij} is the region of space (hyperboloid) satisfying the equation

$$|\mathbf{r}-\mathbf{r}_i| - |\mathbf{r}-\mathbf{r}_j| = v\tau_{ij}. \tag{7}$$

The meaning of this expression is obvious — the value of the correlation function is determined by the sources located on the surface V_{ij} such that the path difference from points on V_{ij} up to the detectors located at the points $\mathbf{r}_i$ and $\mathbf{r}_j$ is equal to $v\tau_{ij}$. The presence of n detectors at different points in space makes it possible to obtain $n!/2(n-2)!$ different combinations of signal pairs and therefore different correlation functions $C_{ij}(\tau_{ij})$, which form the system of equations (6), where the unknown function is $\langle N^2(\mathbf{r})\rangle$. The system obtained is a Fredholm system of the first kind, and equations of this type are usually difficult to solve.

This difficulty can be overcome by using a different method for processing the received signals, specifically, by replacing the second-order correlation integral (2) by a fourth-order correlation integral of the type

$$C_{ijkl}(\tau_{ij}, \tau_{ik}, \tau_{il}) = \int_{-\infty}^{\infty} S_i(t) S_j(t+\tau_{ij}) S_k(t+\tau_{ik}) S_l(t+\tau_{il})\, dt, \tag{8}$$

where τ_{ij}, τ_{ik}, and τ_{il} are the delay times of the j, k, and l channels with respect to the ith channel. In this case, the application of a result which is similar to the one presented previously makes it possible to transform expression (8) to the form

$$\int_{V_{ijkl}} \langle N^4(\mathbf{r})\rangle \frac{\exp\{-\alpha(|\mathbf{r}-\mathbf{r}_i| + |\mathbf{r}-\mathbf{r}_j| + |\mathbf{r}-\mathbf{r}_k| + |\mathbf{r}-\mathbf{r}_l|)\}}{|\mathbf{r}-\mathbf{r}_i||\mathbf{r}-\mathbf{r}_j||\mathbf{r}-\mathbf{r}_k||\mathbf{r}-\mathbf{r}_l|} d^3r, \tag{9}$$

where V_{ijkl} is a region in the space V satisfying the system of equations

$$|\mathbf{r}-\mathbf{r}_i|-|\mathbf{r}-\mathbf{r}_j|=v\tau_{ij},\quad |\mathbf{r}-\mathbf{r}_i|-|\mathbf{r}-\mathbf{r}_k|=v\tau_{ik},\quad |\mathbf{r}-\mathbf{r}_i|-|\mathbf{r}-\mathbf{r}_l|=v\tau_{il}. \tag{10}$$

Equations (10), as we have already mentioned, describe hyperbolic surfaces. The region V_{ijkl} with noncoincident detector positions $\mathbf{r}_i,\mathbf{r}_j,\mathbf{r}_k$, and $\mathbf{r}_l$ is therefore determined by the intersection of hyperbolic surfaces whose parameters are determined by the delays τ_{ij},τ_{ik}, and τ_{il}. For the case in which the detectors are located in the same plane, for example, the XY plane, the pattern of intersections is obviously symmetric relative to this plane. Since the probed region lies in the half-space $Z>0$, only the solutions of the system (10) with $Z>0$ need be taken into account. Therefore, the region V_{ijkl} is a single point. Hence, it follows directly that

$$\langle N^4(\mathbf{r})\rangle = k_ik_jk_kk_l|\mathbf{r}-\mathbf{r}_i||\mathbf{r}-\mathbf{r}_j||\mathbf{r}-\mathbf{r}_k||\mathbf{r}-\mathbf{r}_l|$$
$$\times C_{ijkl}(\tau_{ij},\tau_{ik},\tau_{il})\exp\{\alpha(|\mathbf{r}-\mathbf{r}_i|+|\mathbf{r}-\mathbf{r}_j|+|\mathbf{r}-\mathbf{r}_k|+|\mathbf{r}-\mathbf{r}_l|)\}, \tag{11}$$

i.e., the value of $\langle N^4(\mathbf{r})\rangle$ at any point in the region V is determined by the correlation function $C_{ijkl}(\tau_{ij},\ \tau_{ik},\ \tau_{il})$. The coefficients k_i, k_j, k_k and k_l are again determined by the sensitivities of the i-, j-, k-, and lth channels. Given different values of the delay times τ_{ij},τ_{ik}, and τ_{il}, it is possible to examine the points of the region of space under investigation and to obtain the three-dimensional spatial intensity distributions of the sources of incoherent radiation. In the case where more than four detectors are present, $n!/4!(n-4)!$ images can be formed from different points of view — using different combinations of the four signals $S_i(t)$.

To check the possibilities of the principle proposed above for displaying information, we performed a mathematical simulation of the process of receiving signals and reconstructing from them the spatial distribution of the intensity of the radiation sources. The simulation consisted of summing, in accordance with expression (1), the signals arriving at the detectors from a space filled with statistically independent radiators. It was found that the reconstructed distributions are close to the initial distributions. Compared with the initial distributions, however, there is some smoothing of the spatial differentials and random (not repeating from one experiment to another) spatial noise is present in the reconstructed distributions. Analysis showed that the smoothing is associated with the limited nature of the spectrum of the signals employed. Additional calculations showed that a reduction of the sampling period reduces the smoothing effect, but it also leads to a corresponding increase in the amount of information that must be processed. The calculations showed that the spatial noise appears when the averaging used in the correlation processing is inadeqaute — the noise decreases approximately as $1/\sqrt{N}$, where N is the number of counts.

To check whether or not the proposed principle for displaying information is realizable, we studied experimentally the process of obtaining the spatial intensity distribution of the sources of the acoustic radiation. In the experiment we used four microphones to receive acoustic signals emitted by a dynamic loud speaker which was excited by the noise electric signal. The excitation power was sufficient to ensure that the signal-to-noise ratio in the receiving channels exceeded 10. The characteristic size of the radiator was about 50 cm.

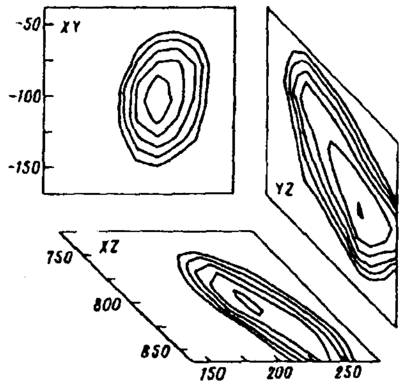

FIG. 2. Results of an experimental study of the spatial distribution of the intensity of an acoustic source of incoherent radiation (sections by planes parallel to the XY, XZ, and YZ planes). The outermost contours of the sections correspond to half-maximum.

Recording the signals consisted of transforming them into a digital form with the aid of an eight-register ADC, operating in a mode in which four channels are queried successively with a timing frequency of about 24 kHz. Signals with a frequency spectrum of width of about 500 Hz were recorded. This was determined mainly by the frequency bandwidth of the radiator. In the measurement process the signals were recorded into the main memory of a computer. After the final measurements, they were then transferred for storage to a magnetic disk. The process of constructing one section, consisting of $\sim 10^3$ spatial points, on the basis of realizations of signals received from 10^5 time points, takes about 10 min on a AT486 type computer.

Figure 1 shows the arrangement of the region investigated relative to the detectors. The signals obtained from the outputs of the detectors are displayed on the right side of Fig. 1. The distance from the plane containing the detectors to the radiator was about 8 m; three detectors were located at the vertices of a triangle (close to an equilateral tirangle with approximately 6 m sides). A fourth detector was located at the center of the triangle.

Figure 2 shows in the form of sections by planes parallel to the XY, XZ, and YZ planes (in order to reflect the tomographic character of the picture) the typical spatial dependence of the emission from a radiator. The outermost contours of the sections correspond to the half-maximum height. As one can see, the distribution obtained has the character of a spatially localized maximum, reminiscent of an ellipsoid. The extent was about 50 cm in the X direction, 70 cm in the Y direction, and about 100 cm in the Z direction. The maximum extent of the sections at half-maximum was about 120 cm and was observed in the YZ plane. Analysis of the reasons for the broadening of the observed distribution showed that, as in the computational experiments, the degradation of the spatial resolution was mainly associated with the limitation of the frequency spectrum of the signals, which leads to long coherence lengths of the received signals. Estimates showed that under the experimental conditions the coherence length of the signals was about 1 m, which is close to the spatial resolution obtained by us.

In conclusion, we shall discuss an important advantage which distinguishes the information display principle described above from existing principles: Four radiation detectors are sufficient to display in three-dimensional space the distributions of the emission sources with a large ($\gg 4$) number of resolved elements, while the existing principles of passive information display, one of which, for example, is the basis of vision, require a number of detectors equal to or greater than the required number of resolved elements. The physical basis for this difference is, in our opinion, that the principle presented above can be implemented only with incoherent signals, which do not have a repetition period and whose autocorrelation functions have one maximum, whereas the existing principles can also be implemented with periodic signals whose autocorrelation functions are periodic.

The most likely areas of application of the principle described above, in our view, are: acoustic and radiometric thermometry and applications in which it is necessary to determine the location of nonperiodic disturbances, such as earthquake foci, acoustic emission signals preceding the collapse of structures, lightning discharges, and other effects.

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2)It should be noted that the procedure presented can be extended to the case of partial coherence of the radiation, where the integral I can no longer be expressed in terms of a delta function but rather it has a more complicated form.[3]

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