Light absorption by superlattices in crossed electric and magnetic fields: strong-field limit

S. N. Molotkov
Institute of Solid-State Physics, Russian Academy of Sciences, 142432 Chernogolovka, Moscow Region, Russia

(Submitted 21 June 1995; resubmitted 17 July 1995)
Pis'ma Zh. Éksp. Teor. Fiz. **62**, No. 4, 318–323 (25 August 1995)

It is shown that in crossed electric and magnetic fields in the strong electric field limit (Wannier–Stark ladder regime) a magnetic field of any magnitude plays the role of a small perturbation. The electric field destroys the complicated structure of the spectrum in a magnetic field. The role of the magnetic field reduces to a weak modulation of the Wannier–Stark ladder as a function of the level number. It is also shown that in contrast to the weak-field limit, the interband light absorption coefficient is not exponentially small as the electric field increases. The predicted behavior can be directly checked experimentally on semiconductor superlattices.

Interband light absorption in the region of the fundamental absorption edge in semiconductors in crossed electric and magnetic fields was first investigated by Aronov.[1] The influence of a magnetic field on the Franz–Keldysh effect[2,3] (absorption at frequencies below the intrinsic absorption edge) was investigated by Aronov and Pikus.[4]

The weak-field case is always realized in the case of absorption in a bulk uniform semiconductor. The weakness of the electric field means that the volume bands are not split into discrete levels (Wannier–Stark quantization[5–7]). In this case the effective–mass approximation or the effective–mass approximation within the two-band scheme is sufficient to calculate absorption.[4] The magnetic field is also considered to be ~~weak,~~ and the Larmor energy is much smaller than the width of the allowed band (the magnetic-field flux per crystal cell is much smaller than one quantum).

A characteristic feature of the interband light absorption coefficient (I) in weak, crossed fields is the exponential dependence on the parameter α

$$I \propto \exp(-\alpha^2),$$

where $\alpha = eEl_m/\hbar\omega_c$, $\omega_c = eH/c(m_e+m_h)$ is the cyclotron frequency, $m^*_{e,h}$ are the effective masses of the electron and hole, and $l_m = (\hbar c/eH)^{1/2}$ is the magnetic length. Absorption vanishes in the limit $E \to \infty$. This limiting behavior is a result of the effective–mass approximation.

In the present letter we show that in the strong-field limit the absorption coefficient does not approach zero and it does not have any special smallness in the limit $E \to \infty$. The strong-field limit cannot be realistically achieved for a uniform semiconductor, but it can be easily achieved in semiconductor superlattices (see, for example, Ref. 8), where the width of a miniband (t) can be made quite small ($t \ll edE$, where d is the period of the superlattice). It turns out that in the strong-electric-field limit a magnetic field of any strength (even when the Larmor energy is of the order of the miniband width) is a small perturbation.

In the strong-field limit the effective-mass approximation is inadequate, since information is required about the spectrum over the entire allowed band. The tight-binding method is useful for these purposes. We shall study the following model. Let the initial spectrum of the superlattice be such that the width of the first minibands in the valence band and the conduction band is less than the energy splitting from the neighboring minibands. This makes it possible to use the one-band approximation to describe the spectrum in the minibands. The strong-field limit in this case means that the fields are strong on the energy scale of a miniband. The phenomena associated with the multiband nature (magnetic breakdown, interband tunneling) require a separate analysis.

For simplicity we shall study a two-dimensional superlattice (superlattice in two directions, miniband along both the x and y axes; this fact will be employed below). The existence of a third coordinate perpendicular to the plane is not fundamental, since the problem is uniform in this direction. Let each isolated quantum well have one quantum-size level $\varepsilon_{c,v}^{(0)}$ for the conduction band and the valence band. Hops between wells are described by the overlap integrals $t_{c,v}$. This quantity determines the width of a miniband and the sign of the effective mass near the extrema. Let a uniform electric field be directed along the x axis and the magnetic field directed along the normal to the plane (z axis). The electric field is taken into account via the shifts of the levels in the wells. The change in the hopping integral under the action of an electric field is ignored, since it does not change the results fundamentally. The action of a magnetic field on the wave function is taken into account by introducing the magnetic-translation operator,[9-15] whose action reduces to the following:

$$\hat{T}_{\mathbf{d}}\psi(\mathbf{r})=\psi(\mathbf{r}+\mathbf{d})\exp\left(\frac{i}{2}\,\mathbf{r}\left(\frac{e\mathbf{H}}{\hbar c},\,\mathbf{d}\right)\right). \tag{1}$$

The Landau gauge is used for the vector potential: $\mathbf{A}=H(0,x,0)$.

The wave function then satisfies the effective Schrödinger equation

$$(\varepsilon_{c,v}^{(0)}+edmE-\varepsilon)\psi_{c,v}(x,y)+t_{c,v}[\psi_{c,v}(x+d,y)+\psi_{c,v}(x-d,y)$$

$$+\exp\left(-i\frac{eHdx}{\hbar c}\right)\psi_{c,v}(x,y+d)+r\ \exp\left(i\frac{eHdx}{\hbar c}\right)\psi_{c,v}(x,y-d)]=0, \tag{2}$$

where the coordinates x and y assume discrete values at the sites $x=md$ and $y=nd$, where d is the period of the superlattice (assumed to be the same along the x and y axes).

It has been shown[14,15] that in the absence of an electric field the features of the spectrum are determined by the dimensionless parameter $\alpha=ed^2H/2\pi\hbar c$ — the number of magnetic-flux quanta in a unit cell. The magnetic-translation operator affects only the x coordinate, so that along the y axis it is natural to assume that

$$\psi_{c,v}(md,nd)=\exp(ik_y nd)A_{c,v}(m). \tag{3}$$

The effective Schrödinger equation assumes the form

$$A_{c,v}(m+1)+A_{c,v}(m-1)+[2\cos(2\pi m\alpha-k_yd)+f_{c,v}m]A_{c,v}(m)=\varepsilon A_{c,v}(m), \quad (4)$$

where $f_{c,v}=edE/t_{c,v}$, $\varepsilon=\varepsilon-\varepsilon_{c,v}^{(0)}/t_{c,v}$, and for convenience we set $\hbar=1$. In the absence of an electric field ($f_{c,v}=0$) the recurrence relations for arbitrary α were investigated numerically in Ref. 15. We were able to show analytically that for small α (weak magnetic fields) the spectrum consists of a series of equidistant Landau levels. For small α, treating the index m as a continuous variable, we obtain the Mathieu differential equation instead of a difference equation ($k_y=0$ — at the extremum of the bands)

$$\frac{1}{4}\frac{d^2A(z)}{dz^2}+(\gamma-4q\cos(2z))A(z)=0, \quad (5)$$

where

$$z=\frac{m}{(\pi\alpha)^2},\quad \gamma=\left(\frac{1}{2}-\frac{\varepsilon}{2t_{c,v}}\right)\frac{1}{(\pi\alpha)^2},\quad q=\frac{1}{8(\pi\alpha)^2}\gg 1$$

— a weak magnetic field. Using the asymptotic representation for the eigenvalues of the Mathieu equation for large values of the parameter q,[16] we have

$$\gamma_\nu\approx -4q+\nu\sqrt{2q}. \quad (6)$$

The spectrum consists of a series of equidistant levels. In the absence of a magnetic field the spectrum of electrons and holes ($k_y=0$) has the form

$$\varepsilon=\varepsilon_{c,v}^{(0)}+2t_{c,v}\cos(k_xd). \quad (7)$$

Near the extrema of the conduction and valence bands $k_x\approx 0$, we have

$$\varepsilon=\varepsilon_{c,v}^{(0)}+2t_{c,v}\mp t_{c,v}d^2k_x^2.$$

The sign of the overlap integrals $t_{c,v}$ determines the sign of the effective mass of the carriers, so that for the conduction band $t_c<0$ ($m_c^*=1/t_cd^2>0$) and for the valence band $t_v>0$ ($m_v^*=1/t_vd^2<0$). The quantity $\varepsilon_g^{(0)}=\varepsilon_c^{(0)}+2t_c-(\varepsilon_v^{(0)}+2t_v)$ plays the role of the band gap in the absence of external fields.

We obtain for the spectrum of electrons and holes in a weak magnetic field the expressions

$$\varepsilon_c\approx -2|t_c|+\nu\frac{eH}{m_c^*c},$$

$$\varepsilon_v\approx 2t_v+\nu\frac{eH}{m_v^*c}.$$

Unfortunately, the energy of the zero-point vibrations is not reproduced because of the asymptotic character of the expansion.

The spectrum of the system can also be found exactly in the absence of a magnetic field, since the recurrence relations with $\alpha=0$ are identical to the recurrence relations for Bessel functions.[16,17] Thus

$$\varepsilon_{c,v\nu}=\varepsilon_{c,v}^{(0)}+edE\nu+2t_{c,v}\cos(k_yd), \quad (8)$$

and the eigenfunctions are

$$\psi_{c,v\nu}(m,n)=\exp(ik_y dn)J_{m-\nu}(1/f_{c,v}). \tag{9}$$

For what follows it is important that for $edE \gg t_{c,v}$ the splitting between the levels in the Wannier–Stark ladder is greater than the width of the initial miniband. In this case a magnetic field of any strength is a small perturbation. The magnetic field appears in Eqs. (2) and (4) via the phase factors in the hopping integrals, so that the energy scale of the perturbation from the magnetic field does not exceed the width of the initial allowed band $(t_{c,v})$, i.e., the magnetic field modifies the spectrum within the initial miniband $(2t_{c,v})$. In the Wannier–Stark ladder regime a magnetic field can therefore be taken into account by means of perturbation theory (the magnitude of the perturbation is much smaller than the distance between the levels). We obtain the following expression for the matrix element of the perturbation between the states ν and λ (8):

$$V_{c,v\nu\lambda}=4\sum_{m,n} t_{c,v}\psi_{c,v\nu}(m,n)\psi_{c,v\lambda}(m,n)\cos(2\pi\alpha m-k_y d) \tag{10}$$

$$=4t_{c,v}J_{\nu-\lambda}\{2\sin(\pi a)\cos[2\pi\alpha\nu+(\nu-\lambda)\sin^{-1}(\cos(\pi\alpha))-k_y d]\}.$$

Here we used the formula for the summation of Bessel functions.[18]

In a strong electric field the complex structure of the spectrum is therefore destroyed because of the presence of the magnetic field (Hofstadter butterfly[15]) and the spectrum degenerates into an equidistant ladder of levels which is modulated as a function of the number; here the amplitude of the modulation is small compared to the splitting between neighboring levels in the ladder without a magnetic field. We obtain

$$\varepsilon_{c,v\nu}=\varepsilon_{c,v}^{(0)}+edE\nu+2t_{c,v}J_0(2\sin(\pi\alpha))\cos(2\pi\alpha\nu-k_y d). \tag{11}$$

For weak fields $(\alpha \ll 1)$ the correction to the spectrum is linear in the magnetic field

$$\delta\varepsilon_{c,v\nu}=2t_{c,v}\sin(k_y d)\frac{ed^2H}{c}\nu, \quad \alpha\nu\ll 1. \tag{12}$$

The correction to the wave function of the νth state is

$$\delta\psi_{c,v}(m,n)=\exp(ikdn)\sum_{\lambda\neq\nu}\frac{V_{\nu\lambda}}{edE(\nu-\lambda)}J_{m-\lambda}\left(\frac{1}{f_{c,v}}\right). \tag{13}$$

It is a small term of order $t_{c,v}/edE \ll 1$.

The light absorption coefficient can be represented in the form [the correction $\delta\psi$ (13) can be ignored with respect to the parameter $1/f_{c,v} \ll 1$]

$$I(\omega)=d_0^2\sum_{\lambda,\mu}\int dk_y J_{\lambda-\mu}^2\left(\frac{1}{f_{c,v}}\right)\delta[\omega-\varepsilon_g-edE(\lambda-\mu)-2J_0(2\sin(\pi\alpha))$$

$$\times(t_c\cos(2\pi\alpha\lambda-k_y d)-t_v\cos(2\pi\alpha\mu-k_y d))]. \tag{14}$$

Here d_0 is the interband dipole matrix element with the atomic orbitals of the tight-binding basis, and $\varepsilon_g=\varepsilon_c^{(0)}-\varepsilon_v^{(0)}$ is the splitting between the initial quantum-size levels in isolated wells (ignoring the hops between the wells) in the conduction and valence bands (it should not be confused with the band gap taking into account hops in $\varepsilon_g^{(0)}$).

In the absence of a magnetic field the absorption coefficient consists of a series of steps corresponding to transitions between levels in the Wannier–Stark ladder in the valence band and the conduction band

$$I(\omega)=\frac{d_0^2}{d}\sum_{\lambda,\mu} J_{\lambda-\mu}^2\left(\frac{1}{f_{cv}}\right)\frac{\theta[\omega-\varepsilon_g-edE(\lambda-\mu)]\theta\{t_{cv}^2-[\omega-\varepsilon_g-edE(\lambda-\mu)]^2\}}{\sqrt{t_{cv}^2-[\omega-\varepsilon_g-edE(\lambda-\mu)]^2}}, \tag{15}$$

where $1/f_{cv}=1/f_c-1/f_v$, $t_{cv}=2(t_c-t_v)$, and $\theta(x)$ is the unit step function.

For strong fields E ($1/f_{cv}\ll 1$) the asymptotic expansion of the Bessel functions[16] gives

$$J_\nu\left(\frac{1}{f}\right)\propto\frac{1}{\Gamma(|\nu|+1)}\exp(-|\nu|\ln(|f|)).$$

Transitions occur between the levels originating from the nearest–neighbor quantum wells in the superlattice. The square-root singularity arises because of the one-dimensional character of the spectrum in a strong electric field. For transitions which are vertical in the spatial coordinate ($\lambda=\mu$) the singularity occurs at the edges of the allowed bands. Absorption is absent if the frequency is greater than the width of the allowed bands plus the initial splitting between the quantum-size levels in the conduction and valence bands ($\omega>2t_{c,v}+\varepsilon_g$).

The following expression is obtained for the absorption coefficient in a magnetic field:

$$I(\omega)=\frac{d_0^2}{d}\sum_{\lambda,\mu} J_{\lambda-\mu}^2\left(\frac{1}{f_{cv}}\right)\frac{\theta[\omega-\varepsilon_g-edE(\lambda-\mu)]\theta\{\bar{t}_{cv}^2-[\omega-\varepsilon_g-edE(\lambda-\mu)]^2\}}{\sqrt{\bar{t}_{cv}^2-[\omega-\varepsilon_g-edE(\lambda-\mu)]^2}}, \tag{16}$$

$$\bar{t}_{cv\lambda\mu}^2=4J_0^2(2\sin(\pi\alpha))\{[t_c\cos(2\pi\alpha\lambda)-t_v\cos(2\pi\alpha\mu)]^2 +[t_c\sin(2\pi\alpha\lambda)-t_v\sin(2\pi\alpha\mu)]^2\}.$$

A magnetic field leads to an effective change in the width of the allowed band. In weak magnetic fields the frequency shift for transitions between a pair of levels λ and μ in the valence and conduction bands is linear in the magnetic field and is proportional to $\propto(t_c\lambda-t_v\mu)ed^2H/c$.

In summary, the absorption coefficient in strong electric fields ($eEd/t_{c,v}\alpha\gg 1$) does not have any special smallness, in contrast to the weak-field case, in which the effective-mass approximation is valid. The predicted features of the electronic spectrum and absorption in strong, crossed, electric and magnetic fields can be directly checked experimentally on semiconductor superlattices.

Here we studied a two-dimensional lattice. For a three-dimensional lattice the qualitative results will remain the same, since motion along the z axis is free. The absorption coefficient in this case can be expressed in terms of elliptic integrals and the square-root singularity is replaced by a logarithmic singularity.

I thank S. V. Iordanskiĭ and S. S. Nazin and the participants of the theoretical seminar at the Institute of Solid State Physics of the Russian Academy of Sciences for helpful discussions. This work was supported by the Russian Fund for Fundamental Research (Project 95-02-06108).

[1] A. G. Aronov, Fiz. Tverd. Tela (Leningrad) **5**, 552 (1963) [Sov. Phys. Solid State **5**, 402 (1963)].
[2] L. V. Keldysh, Zh. Éksp. Teor. Fiz. **34**, 1138 (1958) [Sov. Phys. JETP **34**, 788 (1958)].
[3] W. Franz, Z. Naturforsch. A **13**, 484 (1958).
[4] A. G. Aronov and G. E. Pikus, Zh. Éksp. Teor. Fiz. **51**, 505 (1966) [Sov. Phys. JETP **24**, 339 (1967)].
[5] F. Bloch, Z. Phys. **52**, 555 (1928).
[6] H. M. James, Phys. Rev. **76**, 1611 (1949).
[7] G. Wannier, Phys. Rev. **117**, 432 (1960).
[8] E. E. Mendez in *20th International Conference on the Physics of Semiconductors*, 1990, Vol. 2, p. 1206.
[9] R. E. Peierls, Z. Phys. **80**, 763 (1933).
[10] J. M. Luttinger, Phys. Rev. **84**, 814 (1951).
[11] W. Kohn, Phys. Rev. **115**, 1460 (1959).
[12] G. H. Wannier, Rev. Mod. Phys. **34**, 645 (1962).
[13] E. L. Blount, Phys. Rev. **126**, 1636 (1962).
[14] M. Ya. Azbel', Zh. Éksp. Teor. Fiz. **46**, 927 (1964) [Sov. Phys. JETP **19**, 634 (1964)].
[15] D. R. Hofstadter, Phys. Rev. B **14**, 2239 (1976).
[16] Janke-Emde-Lösch, Tafeln, *Höherer Funktionen Seschste Auflage Neubearbeitet* von F. Lösch, B. G. Teubner Verlagsgesellschaft, Stuttgart, 1960.
[17] H. Fukuyama, R. A. Bari, and H. C. Fogedby, Phys. Rev. B **8**, 5579 (1973).
[18] A. P. Prudnikov, Yu. A. Brychkov, and O. I. Marychev, *Integrals and Series (Special Functions)* [in Russian], Nauka, Moscow, 1983.

Translated by M. E. Alferieff