Spiral instability of a longitudinal magnetic vortex in a thin, current-carrying superconducting film

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The left-handed spiral instability of a linear vortex in a planar superconductor in a magnetic field oriented parallel to the transport current is investigated on the basis of an exact solution for a spiral Abrikosov magnetic vortex in a plate of a type-II superconductor. It is shown that the critical instability current in thin films is comparable to the experimentally observed values and can determine the resistive behavior in a wide range of fields. © *1995 American Institute of Physics.*

The spiral instability of a flux vortex of a magnetic field oriented parallel to the transport current was first studied by Clem.[1] This instability can be summarized by saying that the Lorentz force exerted on the vortex by the transport current tends to increase the radius of a left-handed spiral vortex, while the external magnetic field and the linear tension force of the vortex line tend to increase the radius. As a result of this competition, there arises a critical instability current $j_{in}(H)$, above which a linear vortex which is oriented parallel to the current becomes unstable against certain left-handed helical fluctuations, expands, and exits from the sample. This effect is similar to the "corkscrew" instability of the longitudinal flow of a charged liquid in magnetohydrodynamics.[1] Clem's result was later extended by Brandt to the case of a longitudinal vortex lattice.[2]

This instability is extremely important, since it makes it possible to construct a consistent picture of the resistive behavior of a superconductor (SC) in a magnetic field oriented parallel to the current without invoking exotic dissipation mechanisms[3] and without constructing special "current-carrying" vortices.[4] It also makes it possible to explain the experimentally observed oscillations of the voltage and longitudinal magnetic moment.[5]

Since the transport current and hence the Lorentz force are exponentially small in the volume of a bulk superconductor, the critical instability current of an isolated vortex is found to be exponentially large and consequently unphysical (in contrast to the instability of a vortex lattice[2]). In samples with a low dimension — thin filaments and films — the transport current is not small anywhere and therefore an isolated vortex can be unstable in a wide range of fields. In the latter case, however, the interaction of a vortex with the surface of the samples must be taken into account, which was not done in the early works.

In Refs. 6 and 7 I derived in the London approximation the exact solution for a spiral vortex in a superconducting cylinder with an arbitrary radius and constructed the current-versus-field diagram of the resistive state. In the case of a thin superconductor

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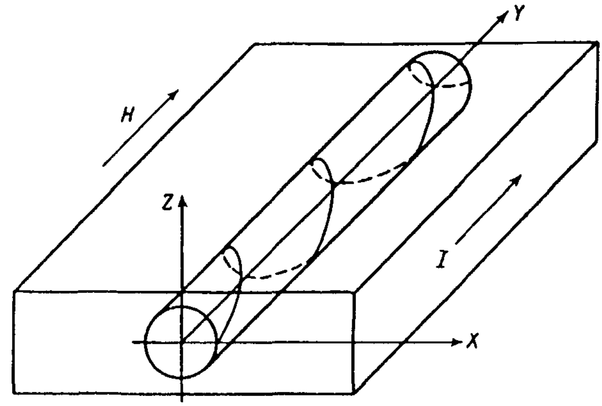

FIG. 1. Spiral magnetic vortex, wound on an imaginary cylinder of radius r, in a superconducting plate carrying a transport current I in an external magnetic field $\mathbf{H}$ oriented parallel to the current.

this diagram is indeed determined by the instability. In the present study I examined a spiral vortex in a current-carrying superconducting plate in a magnetic field oriented parallel to the current and determined the critical instability current of an isolated vortex. It turned out that this current is comparable to the critical pinning current and the critical current for the penetration of vortices through an edge (geometric) barrier in thin films.[8] This effect is not a well-defined effect in the same sense as the geometric barrier effect is not well defined,[8] i.e., it does not depend on the particular features of the structure of the superconducting film or on the quality of the edges and the surface of the film.

Let us consider a superconducting plate which fills the space $|z|\leq d$. Let a transport current flow in the direction of the positive y axis along the plate and let an external field $\mathbf{H}$ be applied parallel to the surface in the same direction (see Fig. 1). The total magnetic field in the plate consists of the Meissner term, which satisfies the boundary condition $\mathbf{H}_M=\mathbf{H}$ on the boundaries of the plate $z=\pm d$, and the vortex field $\mathbf{h}$. The latter field satisfies the equations

$$\mathbf{h}+\lambda^2\nabla\times\nabla\times\mathbf{h}=\Phi,\quad |z|\leq d,$$

$$\nabla\times\mathbf{h}=0,\quad |z|\geq d,\quad \nabla\cdot\mathbf{h}=0 \tag{1}$$

with the boundary conditions that the field is continuuous at the boundaries of the plate $z=\pm d$ and $\mathbf{h}\to 0$ in the limit $z\to\infty$ or $x\to\infty$. In the first equation (the London equation) in (1) λ is the penetration depth of the magnetic field in the superconductor. The special right-hand side Φ in the London equation

$$\Phi=\Phi_0\frac{\mathbf{e}_l}{\mathbf{e}_l\mathbf{e}_\varphi}\delta(\rho-r)\delta(y-L\varphi) \tag{2}$$

describes a vortex spiral wound on an imaginary cylinder of radius r, whose axis is oriented along the y axis. It is convenient to define the quantity Φ in the cylindrical coordinates (ρ,φ,y), where $\rho^2=x^2+z^2$, $\tan\varphi=z/x$, $\mathbf{e}_l$ is a unit vector which is tangent to the nucleus of the vortex, and $\mathbf{e}_\varphi$ is the unit azimuthal vector. The constant slope angle

α of the spiral with respect to the xz plane is determined by the relation $\mathbf{e}_1\mathbf{e}_\varphi = \cos\alpha$, where $\tan\alpha = L/r$ and $2\pi L$ is the height of a loop of the spiral. As $r\to 0$, the spiral degenerates into a linear vortex lying on the y axis.

The periodicity of the problem makes it convenient to switch from (1) to the equations for the Fourier components of the field, which can be determined from the relations

$$h^i(x,y,z) = \frac{1}{2\pi}\sum_k \exp(iky/L)\int dq\, h_k^i(q,z)\exp iqx. \tag{3}$$

Equations (1) then acquire the simple form

$$\left(\frac{\partial^2}{\partial z^2} - p^2\right) h_k^i(q,z) = 0, \quad |z| \geqslant d,$$

$$\left(\frac{\partial^2}{\partial z^2} - Q^2\right) h_k^i(q,z) = f_k^i(q,z), \quad |z| \leqslant d, \tag{4}$$

where $p^2 = q^2 + k^2/L^2$, $Q^2 = q^2 + k^2/L^2 + \lambda^{-2}$, and $f_k^i(q,z) = -\lambda^{-2}\Phi_k^i(q,z)$.

The solution of Eqs. (4) outside the plate is trivial and is determined by exponential functions which decay at infinity:

$$h_k^i(q,z) = h_k^i(q,\pm d)\exp[p(d\mp z)], \quad z \geqslant d \ (z \leqslant -d). \tag{5}$$

We find the solutions of Eqs. (4) inside the plate by the general method for solving inhomogeneous second-order equations:[9]

$$h_k^i = C_+^i \exp Qz + C_-^i \exp(-Qz) - \frac{\Phi_0\theta(z-r)}{4\pi Q\lambda^2}[I_-^i(\pi/2)\exp Qz - I_+^i(\pi/2)$$

$$\times\exp(-Qz)] - \frac{\Phi_0\theta(r-|z|)}{4\pi Q\lambda^2}[I_-^i(T)\exp Qz - I_+^i(T)\exp(-Qz)], \tag{6}$$

where

$$I_\pm^i(T) = \int_{-\pi-T}^{T} dt\,\phi_i(t)\exp(-ikt - iqr\cos t \pm Qr\sin t), \tag{7}$$

and $\phi_x = -(r/L)\sin t$, $\phi_y = 1$, $\phi_z = (r/L)\cos t$, and $T = \sin^{-1} z/r$.

There are six boundary conditions, expressing the continuity of the field components at $z = \pm d$, that can be used to find the twelve unknown constants $C_\pm^i$ and $h_k^i(q,\pm d)$. The missing equations for determining the solution can be found by using the second and third equations in (1) inside and outside the plate and the fact that the solutions $\exp(\pm Qz)$ are linearly independent. Finally, the following expressions can be obtained for the constants in (6):

$$C_\pm^i = \frac{\Phi_0}{4\pi Q\lambda^2}[I_-^i(\pi/2)\exp Qz - I_+^i(\pi/2)\exp(-Qz)] + g_\pm^i$$

$$-\frac{\Phi_0 r}{4\pi\lambda^2 L}\,\frac{[Q\cos h2Qd + p\sin \mathrm{h}2Qd]I_+^z(\pi/2) + QI_-^z(\pi/2)}{[(Q^2+p^2)\sin \mathrm{h}2Qd + 2Qp\cos \mathrm{h}2Qd]\sin \mathrm{h}2Qd}f_\pm^i, \tag{8}$$

where $g^x_\pm = g^y_\pm = 0$, $g^z_\pm = (\Phi_0 r/4\pi Q\lambda^2 L) I^z_\pm(\pi/2)/\sin \mathrm{h}2Qd$, $f^x_\pm = \pm iq/p$, $f^y_\pm = \pm ik/pL$, and $f^z_\pm = p/Q$. Expressions (5)–(8) give a complete description of the field generated by a spiral vortex in space.

To study the stability of a linear vortex lying on the y axis against helical distortions, it is necessary to calculate the Gibbs free energy of the system taking into account the work performed when the radius of the system is changed by the sources of the constant magnetic field and the transport current. In the spirit of Ref. 10, this energy can be represented in the form

$$G = F - \Delta W_H \pm \Delta W_I, \tag{9}$$

where F is the characteristic energy of the vortex, ΔW_H is the work performed by the source of the field, and ΔW_I is the work performed by the current source. In the last term the upper sign corresponds to a right-handed spiral vortex and the bottom sign corresponds to a left-handed spiral vortex.

The exact calculation of the energy F (defined in the standard manner[11]) is problematic, but to analyze the stability of a linear vortex, it is sufficient to study small deviations from the position of equilibrium at $r=0$. In the expansion in small r it is sufficient to retain in the expression for the energy terms up to powers r^2 inclusively, which gives

$$F = \left(\frac{\Phi_0}{4\pi\lambda}\right)^2 \left[\ln \frac{d}{\xi} + \frac{r^2}{2L^2} \ln \frac{\lambda}{\xi} + \frac{r^2}{\lambda^2} - \frac{r^2}{2d^2}\right]. \tag{10}$$

The work (per unit length of the plate along the y axis) performed by the source of the field in the simple geometry of a plate is given by

$$\Delta W_H = \frac{1}{2\pi L}\frac{1}{4\pi}\int dV \mathbf{h}\cdot\mathbf{H} = \frac{H\Phi_y(r)}{4\pi}, \tag{11}$$

where Φ_y is the flux through the vortex in the direction of the y axis. In Eq. (11) only the contribution to the energy which depends on the position of the vortex is taken into account. The Meissner term was therefore dropped. The flux Φ_y can be found from expressions (5) and (6) for the field as an integral of the y component of the field over any plane y-const. It is easy to show that the result does not depend on the value of y. The result is

$$\Phi_y(r) = \Phi_0 \left[1 - \frac{I_0(r/\lambda)}{\cos hd/\lambda}\right], \tag{12}$$

where $I_0(x)$ is a zeroth-order modified Bessel function.[12] In the limit $r \to 0$, this result is identical to the result obtained by Shmidt.[13]

The work ΔW_I performed by the source of the transport current can be found by a direct calculation of the work performed by the Lorentz force exerted on the vortex by the transport current $f_L = \mathbf{J}\times\mathbf{\Phi}/c$. The transport-current density $\mathbf{j}$ has only one component: the component along the y axis. Since the angle between the current $\mathbf{j}$ and the local direction $\mathbf{e}_l$ of an element of length of the flux line remains constant, this force is equal in modulus to $f_L = (j\Phi_0/c)\cos\alpha$ (per unit length) and is directed toward the symmetry axis of the plate $x=z=0$. Then the force acting on an element of length dl of the vortex is $f_L dl=(j\Phi_0/c)dl\cos\alpha=(j\Phi_0/c)rd\varphi$. Therefore, the work required to stretch a single loop of the spiral up to the radius r is

$$\Delta G_I=\int_0^r d\rho\rho\int_0^{2\pi} d\varphi\frac{j\Phi_0}{c}=\frac{I(r)\Phi_0}{c}, \tag{13}$$

where $I(r)$ is the current flowing through the section πr^2 inside the vortex spiral.

If the plate has a macroscopic thickness, $d\gg\lambda$, then the current $I(r)$ will be exponentially small, just as in the case of a macroscopic cylinder. This will lead, in exactly the same way, to exponentially large values of the spiral-instability current. In what follows we will therefore consider a thin film with $d\ll\lambda$. In this case the transport current I is distributed uniformly along the thickness of the plate and the distribution of the current along the width of the plate W is given by the formula[14]

$$j(x)=\frac{I}{\pi d\sqrt{W^2-x^2}}. \tag{14}$$

For a spiral vortex with a small radius r at the center of the plate (in the most stable position) the work performed by the current source per unit length of the plate will then be

$$\Delta W_I=\frac{\Delta G_I}{2\pi L}=\frac{I\Phi_0 r^2}{2\pi c dWL}. \tag{15}$$

Finally, the Gibbs energy of the thin plate with a vortex is (per unit length)

$$G=\left(\frac{\Phi_0}{4\pi\lambda}\right)^2\left[\ln\frac{d}{\xi}+\frac{r^2}{2L^2}\ln\frac{\lambda}{\xi}+\frac{r^2}{\lambda^2}\ln\frac{d}{\lambda}-\frac{r^2}{2d^2}\right]-\frac{H\Phi_0}{4\pi}\left[\frac{d^2}{2\lambda^2}-\frac{r^2}{4\lambda^2}\right]-\frac{I\Phi_0 r^2}{2\pi c dWL}. \tag{16}$$

We now introduce, as in Ref. 1 , the force acting on a unit length of the vortex in the form $f=-\partial G/\partial r=rK(r,y)$, where $y=\lambda/L$. The function $K(0,y)$ has a maximum at $y_0=(\lambda/d)H_I/H_{c1}$, where $H_I=2I/W_c$ is the amplitude of the field generated above the center of the plate $x=0$, $z=d$ by the current, and H_{c1} is the lower critical field of the bulk superconducting material. These two quantities first become positive (and hence the vortex becomes unstable) for the current

$$I_{in}=\frac{cWdH_{c1}}{2\lambda}\sqrt{H-H_d/2H_{c1}}, \tag{17}$$

where $H_d=\Phi_0/2\pi d^2-(\Phi_0/\pi\lambda^2)\ln\lambda/d$ is the characteristic field, equal in order of magnitude to the lower critical field of the film.[13] The maximum current density in a real superconducting film is reached at the edge for $|W-x|\approx d$. Using the current distribution over the width of the film (14), we estimate the maximum current density to be

$$j_{in}\cong j_{c1}\sqrt{(W/d)(H-H_d)/2H_{c1}}, \tag{18}$$

where the characteristic current $j_{c1}=cH_{c1}/4\pi\lambda$ in thin films can be of the same order of magnitude $\sim 10^6$ A/cm^2 as the critical current $j_c=cH_{c1}/2\pi d$ for vortex entry through a geometric barrier,[8] in agreement with the experimental data.

In contrast with the macroscopic case, the instability current (18) is not exponentially large and is comparable to critical currents of a different nature — barrier and pinning currents which are observed in real films. Therefore, it determines, to the same degree, their resistive behavior. Specifically, in a wide supercritical region of fields and currents the longitudinal magnetic moment and the voltage can be expected to oscillate with a small number of characteristic frequencies as a result of vortex entry into the sample and subsequent exit of unstable vortex spirals from the sample.[1,7] Such oscillations have been observed in macroscopic samples of type-II superconductors.[5] It is of interest to make a frequency analysis of the current noise in a longitudinal magnetic field. Such an analysis can reveal the characteristic frequencies in the process of dissipation and thereby shed light on the mechanism of this process in a longitudinal field.

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