A note on radiative corrections to μ and τ decays

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The radiative corrections of order $(\alpha/2\pi)(m_e^2/m_\mu^2)$ to μ decay and order $(\alpha/2\pi)(m_\mu^2/m_\tau^2)$ to τ decay are calculated. The decay width is enhanced by a factor of $4.48\cdot 10^{-3}$ $(\alpha/2\pi)$ in the muon case and 0.283 $(\alpha/2\pi)$ for the $\tau\to\mu\nu_\tau\bar{\nu}_\mu(\gamma)$ decay. The influence of these corrections on the electroweak data is discussed.

The present-day level of accuracy in the electroweak experiments requires a complete account of the $O(\alpha)$ radiative corrections.[1] Any deviation of the experimental data from the corresponding predictions can be considered as an indication of the presence of some new physics beyond the Standard Model. Therefore, the careful determination of these corrections is of great importance.

In this letter we reconsider the one-loop electromagnetic corrections to the muon and τ decays.[2] We calculate the contributions of order $(\alpha/2\pi)(m_e^2/m_\mu^2)$ to the total μ decay rate and of order $(\alpha/2\pi)(m_\mu^2/m_\tau^2)$ to the muon decays of τ. The order of magnitude of this correction has been discussed in Ref. 3 (see also Ref. 4) and has been estimated to be of order 10^{-7}–10^{-8}. We integrate numerically the well-known one-loop radiatively corrected electron spectrum[2] in muon decay and extract the first term of the expansion in m_e^2/m_μ^2. It turns out that the numerical factor in front of this term enhances this correction by two orders of magnitude. With this correction taken into account, the total decay rate of muon is equal to

$$\Gamma(\mu\to\text{all})=\frac{G_F^2 m_\mu^5}{192\pi^3}\left[f\left(\frac{m_e^2}{m_\mu^2}\right)+\frac{3}{5}\frac{m_\mu^2}{m_W^2}+\frac{\alpha(m_\mu)}{2\pi}g\left(\frac{m_e^2}{m_\mu^2}\right)\right], \tag{1}$$

$$f(x)=1-8x+8x^3-x^4-12x^2\log x, \tag{2}$$

$$g(x)=\frac{25}{4}-\pi^2-24x\left(\log x+\frac{17}{6}\right)+o(x), \tag{3}$$

$$\alpha^{-1}(m_\mu)=\alpha^{-1}-\frac{1}{3\pi}\log\frac{m_\mu^2}{m_e^2}+\frac{1}{6\pi}\approx 136.1 \tag{4}$$

with

$$m_\mu=105.658389\pm0.000034\ \text{MeV}, \tag{5}$$

$$m_e=0.51099906\pm0.00000015\ \text{MeV}, \tag{6}$$

$$m_W=80.22\pm0.26\ \text{GeV}. \tag{7}$$

Thus, the corrections are:

$$\frac{3}{5}\frac{m_\mu^2}{m_W^2}\approx1.04\cdot10^{-6}, \tag{8a}$$

$$\left(g\left(\frac{m_e^2}{m_\mu^2}\right)-g(0)\right)\frac{\alpha}{2\pi}\approx4.48\cdot10^{-3}\frac{\alpha}{2\pi}\approx5.2\cdot10^{-6}, \tag{8b}$$

$$\frac{\alpha^2}{3\pi}\log\frac{m_\mu^2}{m_e^2}\left(\pi^2-\frac{25}{4}\right)\approx3.5\cdot10^{-5}. \tag{8c}$$

One can see that the correction calculated here is five times larger than the correction due to the finite W mass. Nevertheless, this correction is irrelevant in respect to the concrete value of the weak constant $G_F=(1.16639\pm0.00002)\cdot10^{-5}\ \text{GeV}^{-2}$,[1] since it is 7 times smaller than (8c), which is quoted as the theoretical uncertainty for the weak constant.[1]

Indeed, the decay rates $\Gamma(\tau\to e\nu_\tau\bar{\nu}_e(\gamma))$ and $\Gamma(\tau\to\mu\nu_\tau\bar{\nu}_\mu(\gamma))$ can be obtained simply from (1) by the replacement $m_\mu\to m_\tau$ and $(m_e\to m_\mu;m_\mu\to m_\tau)$ respectively. The correction under consideration is relevant for $\Gamma(\tau\to\mu\nu_\tau\bar{\nu}_\mu(\gamma))$. Explicitly, one finds

$$\Gamma(\tau\to\mu\nu_\tau\bar{\nu}_\mu(\gamma))=\frac{G_F^2m_\tau^5}{192\pi^3}\left[f\left(\frac{m_\mu^2}{m_\tau^2}\right)+\frac{3}{5}\frac{m_\tau^2}{m_W^2}+\frac{\alpha(m_\tau)}{2\pi}g\left(\frac{m_\mu^2}{m_\tau^2}\right)\right] \tag{9}$$

with

$$\alpha^{-1}(m_\tau)\approx133.3, \tag{10}$$

$$m_\tau=1777.0\pm0.26\ \text{MeV (Ref.5)}. \tag{11}$$

Then, one gets

$$\frac{3}{5}\frac{m_\tau^2}{m_W^2}\approx2.89\cdot10^{-4}, \tag{12a}$$

$$\left(g\left(\frac{m_\mu^2}{m_\tau^2}\right)-g(0)\right)\frac{\alpha}{2\pi}\approx0.283\cdot\frac{\alpha}{2\pi}\approx3.25\cdot10^{-4}, \tag{12b}$$

$$\frac{\alpha^2}{3\pi}\log\frac{m_\tau^2}{m_e^2}\left(\pi^2-\frac{25}{4}\right)\approx5.3\cdot10^{-5}. \tag{12c}$$

We note that the first term in the expansion of $g(x)$ in (3) gives $g(m_\mu^2/m_\tau^2)-g(0)\approx0.23$.

In the case of $\tau\to\mu\nu_\tau\bar{\nu}_\mu(\gamma)$ decay, allowance for the nonzero muon mass decreases the one-loop correction by 8%. This correction slightly affects predictions for branching ratios of the τ leptonic decays:

$$\Gamma(\tau\to\mu\nu_\tau\bar{\nu}_\mu(\gamma))=0.9731\cdot\Gamma(\tau\to e\nu_\tau\bar{\nu}_e(\gamma)). \tag{13}$$

This correction is also relevant for testing of $e-\mu$ universality in τ leptonic decays. We rewrite (13) in the form

$$\Gamma(\tau\to\nu_\tau\, l\bar{\nu}_l(\gamma))=\frac{G_\tau G_l m_\tau^5}{192\pi^3}\left[f(x_l)+\frac{3}{5}\frac{m_\tau^2}{m_W^2}+\frac{\alpha(m_\tau)}{2\pi}\,g(x_l)\right], \tag{14}$$

where

$$G_l=\frac{g_l^2}{4\sqrt{2}M_W^2},\quad x_l=\frac{m_l^2}{m_\tau^2}.$$

The strength of each leptonic charged current is determined by g_l. Comparison of Γ_e and Γ_μ is a test of $e-\mu$ universality. Since $f(x_e)\approx1$, $g(x_e)\approx0$, one has:

$$\frac{\Gamma_\mu}{\Gamma_e}=f(x_\mu)\,\frac{g_\mu^2}{g_e^2} \tag{15}$$

with $f(x_\mu)\approx0.9731$.

Comparing with the experimental data[5] one finally gets

$$\frac{g_\mu}{g_e}=1.0005\pm0.0035\ . \tag{16}$$

To conclude, we have computed the correction of order $(\alpha/2\pi)(m_e^2/m_\mu^2)$ to the muon width and of order $(\alpha/2\pi)(m_\mu^2/m_\tau^2)$ to $\Gamma(\tau\to\mu\nu_\tau\bar{\nu}_\mu(\gamma))$. In the latter case this correction decreases the one-loop correction by 8%. Our results give precise predictions for the decay rates, which can actually be tested provided the experiments achieve higher accuracy and an explicit calculation of the two-loop correction for the total muon decay rate is done.

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