Breaking of electroweak symmetry in the minimal supersymmetric standard model

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It is shown that, up to a gauge transformation, the electrically neutral vacuum of the minimal supersymmetric standard model is the only nontrivial minimum of the Higgs potential. © *1995 American Institute of Physics.*

The minimal supersymmetric standard model (MSSM), which is the minimum possible supersymmetric extension of the standard model, has been widely investigated during the last 15 years.[1–6] Interest in this model remains undiminished.[7] The reasons for this interest are that the model is relatively simple and that it makes precise predictions concerning the physics directly beyond the standard model in the energy range accessible to modern and future accelerators.[2,3,6,7] Many review articles and original studies have dealt with the MSSM.[8,9] In the present paper we call attention to an important and previously unnoticed feature of the breaking of the electroweak symmetry in this model. To explain what we have in mind, we briefly recall the basic features of this phenomenon in the MSSM.

In the MSSM, just as in the standard model, the electroweak symmetry is broken via the Higgs mechanism.[10] A distinction, nonetheless, can be made from the standard model. Because of the supersymmetry, we must have at least two Higgs doublets,[11] since superfields of the same chirality must be present in the superpotential. Elimination of the auxiliary nondynamical components of the gauge and Higgs superfields gives the following scalar Higgs potential:[3,4,11]

$$V=m_1^2|H_1|^2+m_2^2|H_2|^2+\mu(H_1\epsilon H_2+H_1^*\epsilon H_2^*)+\frac{g^2+g'^2}{8}(|H_1|^2-|H_2|^2)^2 + \frac{g^2}{2}|H_1^\dagger H_2|^2, \tag{1}$$

where H_1 and H_2 are scalar Higgs doublets:[9]

$$H_1=\begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix}, \quad H_2=\begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}.$$

For conciseness, we imploy the following notation:

$$|H_1|^2=|H_1^0|^2+|H_1^-|^2, \quad |H_2|^2=|H_2^+|^2+|H_2^0|^2, \quad H_1\epsilon H_2=H_1^\alpha\epsilon_{\alpha\beta}H_2^\beta,$$

where α and β are $SU(2)$ indices.[2)]

For the Higgs fields to acquire nonzero vacuum averages this potential must have a nontrivial minimum. In the preceding papers dealing with works on the MSSM it was assumed at first that only the neutral components of the Higgs doublets acquire nonzero vacuum averages, and then a minimum of the Higgs potential along these neutral components was sought.[3,4,9,12,13] This procedure is justified from the physical standpoint, since the presence of a charged vacuum average would result in nonconservation of electric charge.[13] Nonetheless, the following question arises: Is this neutral minimum the only minimum of the potential (1) or can other minima, which are gauge-nonequivalent to it, also exist? This is not clear *a priori*. We shall show below that, up to a gauge transformation, all minimal field configurations are equivalent to the neutral configurations

$$H_1=\begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \quad H_2=\begin{pmatrix} 0 \\ v_2 \end{pmatrix}. \tag{2}$$

To solve this problem, we first rewrite the potential (1) in a more convenient form. It is easy to show that any two $SU(2)$ doublets satisfy the useful relation

$$|H_1|^2|H_2|^2=|H_1^\dagger H_2|^2+|H_1\epsilon H_2|^2.$$

Using this identity, we can introduce an angle α:

$$|H_1^\dagger H_2|=|H_1||H_2|\sin\alpha, \quad |H_1\epsilon H_2|=|H_1||H_2|\cos\alpha,$$

$$H_1\epsilon H_2=|H_1\epsilon H_2|e^{i\beta}=|H_1||H_2|\cos\alpha\ e^{i\beta},$$

and write the potential (1) as follows:

$$V=m_1^2|H_1|^2+m_2^2|H_2|^2+2\mu|H_1||H_2|\cos\beta\cos\alpha+\frac{g^2+g'^2}{8}(|H_1|^2-|H_2|^2)^2$$
$$+\frac{g^2}{2}|H_1|^2|H_2|^2\ \sin^2\alpha. \tag{3}$$

When the Higgs potential (1) is written in this form, the condition for stability in strong fields in the direction of vanishing of the fourth-order terms ($|H_1|=|H_2|$, $\sin\alpha=0$) is obvious:

$$m_1^2+m_2^2>2|\mu|.$$

Here some remarks are in order. The last term in the potential is ordinarily dropped because it vanishes on the gauge-equivalent field configurations (2).[5,6,9] If we wish to find the minimum exactly, then this term must be taken into account.

The potential (3) has a nontrivial minimum if the following conditions are satisfied:

$$\cos\beta=-\operatorname{sign}\mu, \quad \cos\alpha=1,$$

$$m_1^2|H_1|+\mu|H_2|+\frac{g^2+g'^2}{4}(|H_1|^2-|H_2|^2)|H_1|=0,$$

$$m_2^2|H_2|+\mu|H_1|-\frac{g^2+g'^2}{4}(|H_1|^2-|H_2|^2)|H_2|=0. \tag{4}$$

The first two equations in the system (4) are equivalent to the equations

$$H_1\epsilon H_2=-|H_1||H_2|\mathrm{sign}\mu, \tag{5}$$

$$H_1^\dagger H_2=0, \tag{6}$$

while the last two equations are the standard equations which have previously been examined in studies dealing with the MSSM,[7,8] but with $|H_1|$and $|H_2|$ instead of $|H_1^0|$ and $|H_2^0|$, respectively. These equations have a simple solution:

$$|H_1|^2=|H_1^0|^2+|H_1^-|^2=v_1^2,\quad |H_2|^2=|H_2^+|^2+|H_2^0|^2=v_2^2,$$

$$v_1^2=(m_1^2+m_2^2\mp\sqrt{(m_1^2+m_2^2)^2-4\mu^2})F_\pm(\mu^2),$$

$$v_2^2=(m_1^2+m_2^2\pm\sqrt{(m_1^2+m_2^2)^2-4\mu^2})F_\pm(\mu^2),$$

$$F_\pm(\mu^2)=\frac{1}{g^2+g'^2}\,\frac{\pm(m_1^2-m_2^2)-\sqrt{(m_1^2+m_2^2)^2-4\mu^2}}{\sqrt{(m_1^2+m_2^2)^2-4\mu^2}}.$$

Real and positive $|H_1|$ and $|H_2|$ are obtained if the following condition is satisfied: $m_1^2m_2^2\leqslant\mu^2$.

To analyze the conditions (5) and (6), we introduce the complex phases of the components of the Higgs doublets

$$H_1^0=|H_1^0|e^{i\gamma_1},\quad H_1^-=|H_1^-|e^{i\gamma_2},\quad H_2^+=|H_2^+|e^{i\delta_1},\quad H_2^0=|H_2^0|e^{i\delta_2},$$

and, assuming that $\mu<0$, we rewrite Eqs. (5) and (6), respectively, as follows:

$$|H_1^0||H_2^0|e^{i(\gamma_1+\delta_2)}\;-\;|H_1^-||H_2^+|e^{i(\gamma_2+\delta_1)}=v_1v_2,$$

$$|H_1^0||H_2^+|e^{i(\delta_1-\gamma_1)}\;+\;|H_1^-||H_2^0|e^{i(\delta_2-\gamma_2)}=0.$$

This immediately gives us the relationship between the phases and the moduli of the components of the Higgs doublets:

$$e^{i\delta_1}=-e^{-i\gamma_2}, \tag{7}$$

$$e^{i\delta_2}=e^{-i\gamma_1}, \tag{8}$$

$$|H_1^0||H_2^0|+|H_1^-||H_2^+|=v_1v_2, \tag{9}$$

$$|H_1^0||H_2^+|=|H_1^-||H_2^0|. \tag{10}$$

After parametrization,

$$|H_1^0|=v_1\cos\theta,\quad |H_1^-|=v_1\sin\theta,\quad |H_2^+|=v_2\cos\phi,\quad |H_2^0|=v_2\sin\phi,$$

where $0<\theta<\pi/2$ and $0<\phi<\pi/2$, we obtain easily from Eqs. (9) and (10) the relation

$$\theta+\phi=\frac{\pi}{2}.$$

Combined with Eqs. (7) and (8), this immediately enables us to write the final result for the minimum of the Higgs potential (1) in the form

$$H_1=v_1\begin{pmatrix} \cos\theta\ e^{i\gamma_1} \\ \sin\theta\ e^{i\gamma_2} \end{pmatrix}, \quad H_2=v_2\begin{pmatrix} -\sin\theta\ e^{-i\gamma_2} \\ \cos\theta\ e^{-i\gamma_1} \end{pmatrix}. \tag{11}$$

Expressions (11) can be written as follows:

$$H_1=U\begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \quad H_2=U\begin{pmatrix} 0 \\ v_2 \end{pmatrix},$$

where U is a $SU(2)$ matrix:

$$U=\begin{pmatrix} \cos\theta\ e^{i\gamma_1} & -\sin\theta\ e^{-i\gamma_2} \\ \sin\theta\ e^{i\gamma_2} & \cos\theta\ e^{-i\gamma_1} \end{pmatrix}.$$

It is therefore easy to see that, up to a gauge transformation, the field configurations (2) are the minimum of the potential (1). The main result of the analysis proposed in this paper is that the fields H_1 and H_2, which give to the potential (1) a minimum, can be transformed into the form (2) by means of only one $SU(2)$ matrix. In all other respects, the electroweak symmetry is broken in the MSSM completely analogously to the standard model. In conclusion, we note that if we had assumed $\mu>0$, then the matrix U would also have been unitary but its determinant would be equal to -1. For this reason, this variant should be dropped.

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