Anomalies in the spectra of quantum oscillations for dilute two-dimensional electronic systems

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Magnetotransport characteristics of a two-dimensional hole gas located at a Si/SiGe heterojunction are measured for two carrier concentrations. A notable discrepancy is observed between values of the magnetic level filling factor determined from the periodicity of the Shubnikov–de Haas oscillations and the values of the Hall conductivity expected at these filling factors. In particular, the sample of lower concentration exhibits a re-entrant transition between the quantum Hall effect state and insulating state. It is shown that all the observed effects can be described by a model [S. I. Dorozhkin, JETP Lett. **60**, 595 (1994)] which takes into account the long-range potential modulation in the sample, with amplitude comparable to the Fermi energy. © *1995 American Institute of Physics.*

The normal picture of the quantum oscillations for a two-dimensional (2D) electronic system in strong magnetic fields ($\omega_c \tau > 1$) consists of a sequence of integer quantum Hall effect states (IQHESs). Here ω_c is the cyclotron frequency and τ is the relaxation time. In each IQHES, the diagonal magnetoconductivity σ_{xx} is nearly zero, while the Hall conductivity σ_{xy} is quantized as $\sigma_{xy} = ie^2/h$, where i is an integer which decreases with increasing magnetic field. It is generally accepted that i is equal to the number of bands of delocalized states below the Fermi level, each one of which is located at a particular magnetic level center.

The first reported deviation from this picture was for a dilute two-dimensional electron gas in a Si-MOSFET,[1,2] where the standard sequence of quantum Hall effect states was interrupted by a re-entrant insulating state, with both magnetoconductivities σ_{xx} and σ_{xy} close to zero. A similar observation was reported recently[3,4] for p channels in Si/SiGe heterostructures. This anomaly is characterized by the nonmonotonic dependence of the Hall conductivity σ_{xy} on magnetic field. The somewhat similar phenomenon of magnetic-field-induced delocalization was also observed recently for a dilute two-dimensional electron gas in a GaAs/AlGaAs heterostructure.[5] Another type of anomaly, which was pointed out in Ref. 3, is an inconsistency between the number of occupied magnetic levels N and the measured quantized value of $\sigma_{xy}=ie^2/h$, with $N\neq i$ instead of the expected $N=i$. A recent model[6] proposed by one of the authors can account for such anomalies in terms of percolation phenomena. In the present paper we report the evolution as a function of carrier concentration of quantum transport anomalies in p channels of Si/SiGe heterostructures and show that the observed effects can be described in terms of this model.[6]

We have made measurements on three Hall bar samples, fabricated from two different wafers, each one consisting of a Si/SiGe heterostructure epilayer grown on a Si substrate by solid-source molecular beam epitaxy. A detailed discussion of the growth method has been given elsewhere.[7] In brief, both samples contain a buried $Si_{0.8}Ge_{0.2}$ layer, which is a quantum well for holes in this system, and each is doped remotely by positioning a Si:B region on the surface of the structure, with some thickness L_s of nominally undoped Si separating the well and doping layer. The two-dimensional hole gas resides in an approximately triangular potential well, sited on the edge of the alloy layer nearest to the dopant atoms. The 2D carrier concentrations in the two wafers differ by a factor of ~ 2 (see below), which is due only to the particular values of L_s (all other structural and doping properties were the same for the two heterostructures). The corresponding low-field Hall mobilities were approximately the same: ~ 2000 cm^2/V·s. No conduction paths in parallel with the 2D hole gas were present in the samples at liquid-helium temperatures. Hall bar samples used for transport measurements were of length 4.70 mm and width 1.00 mm, with a distance of 1.72 mm between the potential probes used for the determination of R_{xx}. Experiments were performed in a ^{3}He cryostat down to 0.5 K and in a ^{3}He/^{4}He dilution refrigerator down to 30 mK. The sample current was set low enough to avoid any non-Ohmic effects.

The temperature evolution of the magnetoresistance per square R_{xx} and of the Hall resistance R_{xy} for sample 1 is shown in Figs. 1a and 1b, respectively. The minima of R_{xx} at high temperature ($T=1.23$ K) are periodic in the inverse magnetic field. The filling factors ν defined from this periodicity are shown at the minima, and the corresponding carrier concentration is $n_{s1}=5.3\cdot 10^{11}$ cm^{-2}. Decreasing the temperature leads to a more complicated picture, where the R_{xx} minimum at $\nu=3$ is replaced by a maximum, while the minima at higher filling factors maintain their position. The Hall resistances at different temperatures are shown in Fig. 1b. The positions of integer filling factors in this figure correspond to those in Fig. 1a. The magnetic field dependence of the off-diagonal conductivity σ_{xy} calculated from the data of Figs. 1a,b is shown in Fig. 1c for the lowest temperature $T=30$ mK. Note the following anomalies of σ_{xy}, which can be observed in this figure. (i) σ_{xy} is close to $2e^2/h$ in the magnetic field range corresponding to filling factors between approximately 3.5 and 2. (ii) In the vicinity of $\nu=2$, σ_{xy} drops from approximately $2e^2/h$ to e^2/h.

The off-diagonal conductivity for sample 2 is shown in Fig. 2 for different angles θ between the magnetic field and the direction normal to the plane of the two-dimensional system. The carrier concentration determined from the oscillation periodicity in this sample is $n_{s2}=2.5\cdot 10^{11}$ cm^{-2}. The main anomaly in Fig. 2 is the appearance of the insulating state ($\sigma_{xx}\to 0$, $\sigma_{xy}\to 0$) between filling factors 3 and 1, which exists alongside

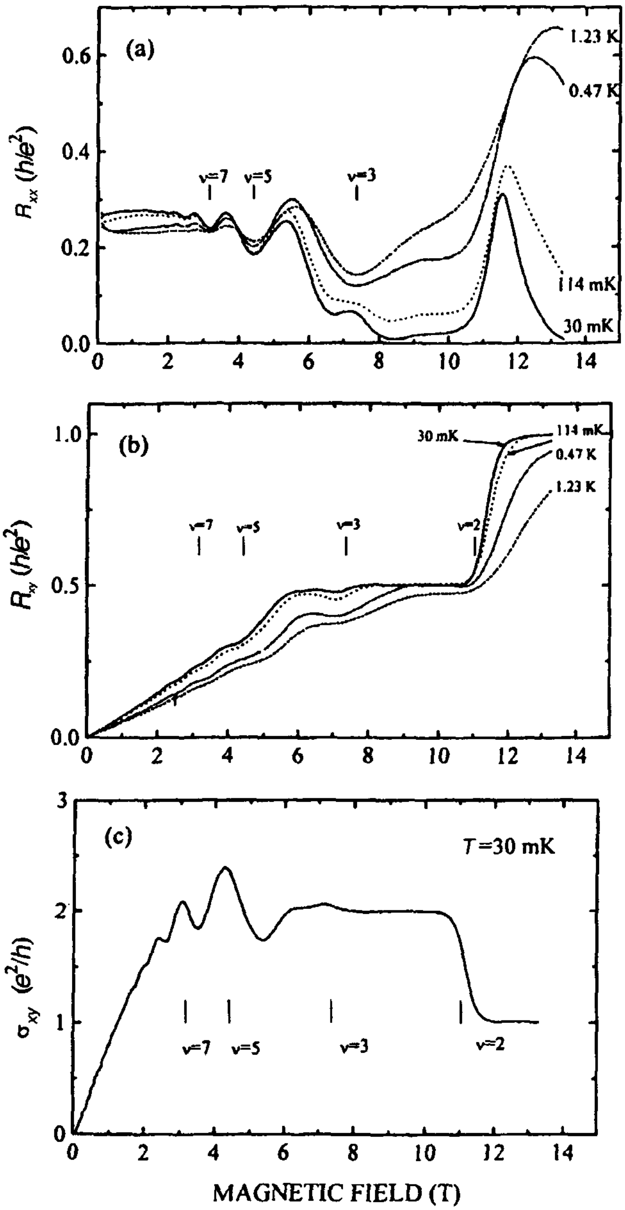

FIG. 1. a: Magnetoresistance per square R_{xx} for sample 1 (in units of $h/e^2=25.813$ kΩ) versus magnetic field normal to the sample plane at various temperatures (shown by the curves). The filling factors ν are defined from the R_{xx} minima positions at $T=1.23$ K. (b): Hall resistance R_{xy} versus magnetic field for sample 1. (c): Off-diagonal conductivity σ_{xy} (in units of e^2/h) for sample 1 at $T=30$ mK. The curve is calculated from the data of Figs. 1(a),(b) .

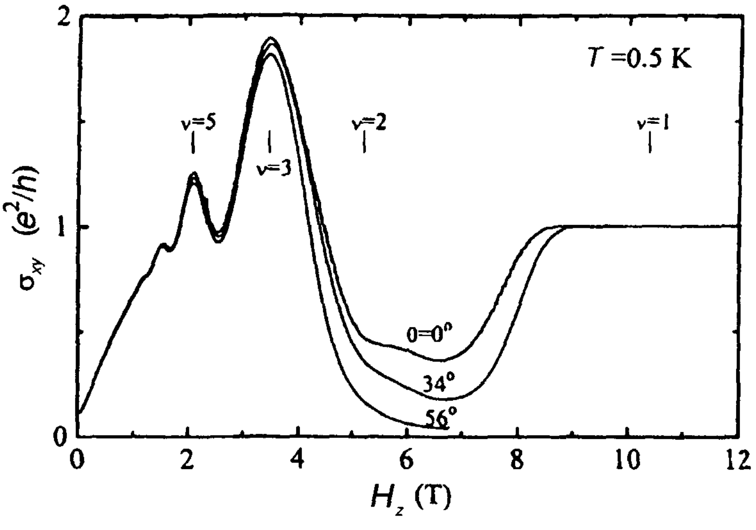

FIG. 2. Off-diagonal conductivity σ_{xy} versus normal magnetic field H_z at different angles θ between the magnetic field and the direction normal to the two-dimensional system for sample 2. $T=0.5$ K. Values of the filling factors were defined from the original R_{xx} curves used to calculate σ_{xy}.

the quantum Hall effect state with $\sigma_{xy}=e^2/h$. The degree of insulating behavior is enhanced by magnetic field tilting, a feature which is described and discussed in more detail elsewhere.[4]

We now show that the anomalies in σ_{xy} mentioned above can be described formally in terms of a simple model first introduced in Ref. 6. The applicability of this model to real samples will be addressed below. We choose a model periodic potential in the plane of the two-dimensional system, with the values $V=0$ and $V=2V_0$ on the white and black squares, respectively, of a chessboard (see the inset in Fig. 3a). The transition region width δ is much less than the square side length a but large in comparison with the magnetic length $l=(\hbar c/eH)^{1/2}$:$l\ll\delta\ll a$. In this paper we make one adjustment to the model as it was introduced originally,[6] which is to use an asymmetric form for the potential in the transition regions. For example, such asymmetry appears if we assume a quadratic variation of the potential $V(\mathbf{r})$ within the transition regions:

$$V(\mathbf{r})=\begin{cases} 0 & 0\leqslant x\leqslant a/2-\delta/2 \\ 2V_0(x-a/2+\delta/2)^2/\delta^2 & a/2-\delta/2\leqslant x\leqslant a/2+\delta/2. \\ 2V_0 & a/2+\delta/2\leqslant x\leqslant a \end{cases}$$

This set of equations is valid for the region in the (x,y) plane $0\leqslant x\leqslant a$, $0\leqslant y\leqslant \min\{x,a-x\}$, which is bounded by the dashed lines and the x axis in the inset of Fig. 3a, but it is possible to map out the potential over the $x-y$ plane by a series of reflections about the boundary lines of this elemental region. It is straightforward to determine that the percolation threshold for this potential occurs at an energy $E_p^0=eV_0/2$. In such a potential, the magnetic levels $E_n^{\pm}(\mathbf{r})=(n+1/2)\hbar\omega_c\pm E_z/2+eV(\mathbf{r})$ are inhomogeneously

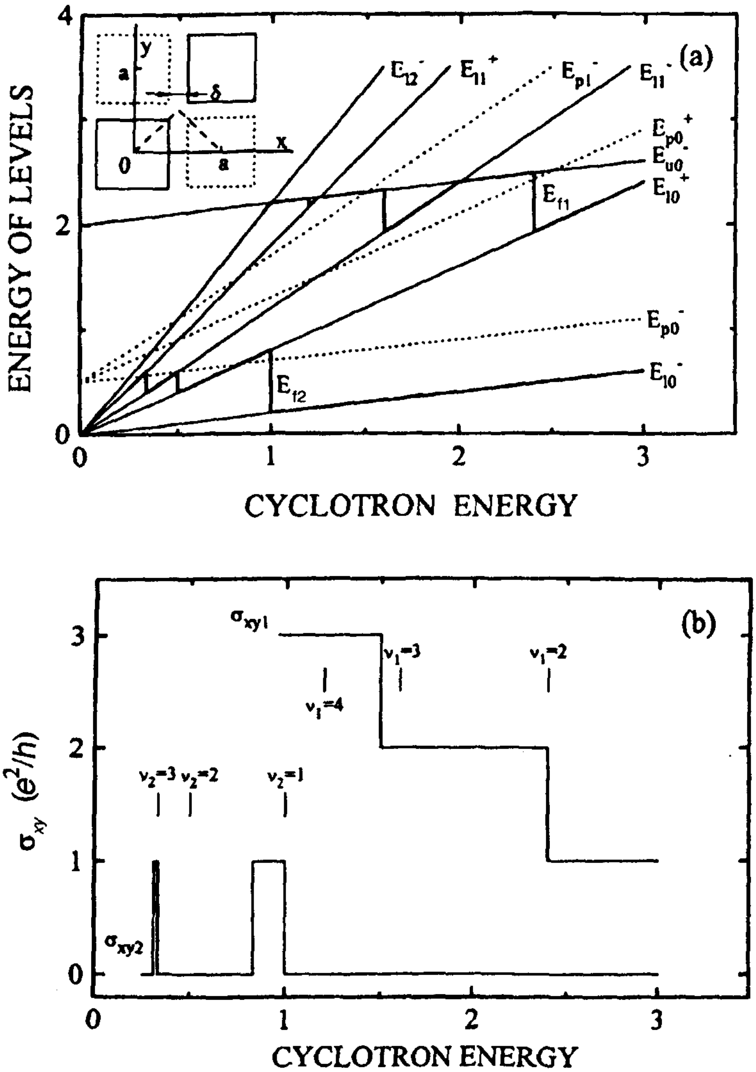

FIG. 3. a: Fermi energies E_{f1} and E_{f2} (wide solid lines) versus the cyclotron energy for two different values of the carrier concentration, characterized by the zero-magnetic-field Fermi energy E_f^0: $E_{f1}^0=2.2eV_0$, $E_{f2}^0=0.5eV_0$. The Zeeman splitting is assumed to be proportional to the cyclotron energy: $E_z=0.6\hbar\omega_c$. $T=0$ K. Magnetic sublevels are shown by narrow solid lines and marked in accordance with the notation used in the text. The extended states of the magnetic levels are shown by dotted lines with the corresponding notation. All the energies are given in units of eV_0. Inset: Definition of parameters describing the model potential. The squares of the chessboard are shown by solid and dotted lines for white and black squares, respectively. b: Off-diagonal conductivity σ_{xy}, calculated as described in the model, versus the cyclotron energy for the choice of parameters used in Fig. 3(a).

broadened, with the extended states located at energies $E_{pn}^{\pm}=(n+1/2)\hbar\omega_c\pm E_z/2+eV_0/2$. Here n is an integer, $\omega_c=eH_z/m^*c$ is the cyclotron frequency (m^* is the effective mass of a carrier), and the Zeeman splitting energy is represented by E_z. Given that the total area of the transition region is relatively small ($\delta \ll a$), the density of states for a given magnetic level may be considered to consist of two peaks, situated at $E_{ln}^{\pm}=(n+1/2)\hbar\omega_c\pm E_z/2$ and $E_{un}^{\pm}=E_{ln}^{\pm}+2eV_0$, and since the corresponding areas of the sample are equal we can assign to each such magnetic sublevel the same degeneracy factor $eH_z/2hc$.

The carrier concentration in each sample is fixed. As a result, the Fermi level E_f will oscillate with magnetic field due to the changing densities of states of the magnetic sublevels and, in some instances, will cross the energies of extended states (Fig. 3a). The zero-magnetic-field value E_f^0 is a function of carrier concentration. In Fig. 3a, oscillations of the Fermi level are shown for two values of the carrier concentration. Following the generally accepted point of view, we assume that the Hall conductivity σ_{xy} is equal to e^2/h times the number of delocalized bands below the Fermi level (see, for example, Ref. 8). The corresponding behavior of the conductivity σ_{xy} for the two values of carrier concentration is shown in Fig. 3b. The values of integer filling factor ν shown in Fig. 3b are determined from the positions of the Fermi energy discontinuities of Fig. 3a and are equal to the numbers of occupied sublevels. The results of this modeling, as displayed in Fig. 3b, give a good description of the apparently anomalous behavior of σ_{xy} seen in our experimental data, including the very pronounced effect of changing the carrier concentration, provided we assume that the minima of R_{xx} for the higher-temperature curves correspond to Fermi energy jumps between magnetic sublevels. We point out that the anomalies reported in Refs. 1–5 can also be ascribed to the same physical phenomena.

In proceeding to compare the model used in this work with real samples, we note first of all that a regular potential modulation is not the only effect which could give the results obtained here. The most important feature of this phenomenon is the existence of at least one peak[2)] in the density of states associated with a magnetic level which is separated from the delocalized state of the level by an energy splitting exceeding the magnetic level separation. Such a situation might arise due to nonlinear screening of the long-range potential fluctuations,[9] for example, but this case certainly needs more-detailed consideration. Some form of long-range potential modulation, regular or otherwise, is very likely to exist in our samples, but at the moment it isn't possible to identify a particular cause: possibly a departure from 2D growth, with 3D islanding occurring; or some kind of "self-organization" process affecting the distribution of dopant B or alloying Ge atoms. In addition, if the distribution of charged impurities in our samples is random, then long-range potential fluctuations should surely be present.[9] It is noted that experimental proof of the percolation nature of the metal–insulator transitions in sample 2 has been obtained[10] recently.

It should be made clear that we do not claim a complete quantitative explanation of our experimental results. For one thing, the form of the potential distribution in our samples is not known; and, furthermore, the ratio of the Zeeman and cyclotron energies is not known to great accuracy.[3] These are reasons why we have chosen to emphasize a description of the main effects, while not dwelling on minor differences between the experimental results and model calculations. Nevertheless, we consider that the success of our simple model in describing these nonstandard features of the Shubnikov–de Haas oscillations justifies our approach to the problem, and most certainly highlights an area which would benefit from more rigorous investigation, both experimental and theoretical in scope. In particular, a more accurate theoretical study of these effects should include (i) accurate calculations of both magnetoconductivities σ_{xy} and σ_{xx}, (ii) self-consistent variation of the real potential with magnetic field, (iii) homogeneous broadening of magnetic levels due to short-range potential fluctuations, and, specifically for our material, (iv) allowance for the complicated energy spectrum of two-dimensional holes, with a nonlinear dependence of the magnetic level energies on field. Such considerations might result in a somewhat different picture of the Shubnikov–de Haas oscillations. As to further experimental work, it would be of great interest to investigate the evolution of the magnetotransport characteristics described here in gated Si/SiGe samples with tunable carrier concentration: this is what we hope to do in the future.

In conclusion, we have observed various striking anomalies in the magnetotransport characteristics of two-dimensional hole systems existing in a Si/SiGe heterostructure. It is argued that oscillations of the Fermi energy relative to the extended states, which do not coincide with maxima of the density of states of magnetic levels because of the presence of a smooth, long-range potential modulation, can give rise to the apparently anomalous behavior observed. This effect is rather general and should be present in all two-dimensional electronic systems.

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[2)]Note that the existence of the upper sublevel may be of no importance, as, for example, in the case of lower carrier density (Fig. 3a), where this sublevel does not influence the Fermi energy oscillations. It is also rather obvious that the spatial periodicity of the potential modulation is not critical for the results.

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