Nonlinear stage of the ionization-field instability in a rf discharge

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The dynamics of a rf discharge at the nonlinear stage of the ionization-field instability, which results in the spatial modulation of the parameters in the direction of the electric-field vector, was investigated. It was shown that as a result of the development of the instability, pulsating small-scale structures (regular or stochastic), which are characterized by high (supercritical) values of the plasma density, are formed in the discharge. © *1995 American Institute of Physics.*

Discharges produced in gases by alternating electromagnetic fields of different frequencies (including rf, microwave, and optical) are subjected to so-called ionization-field instabilities which arise as a result of the mutual amplification of the perturbations of the field and density of the plasma.[1–6] The dynamic structures formed as a result of the development of these instabilities substantially determine the general character of the radiation with the plasma which it produces and the possibilities of using this plasma in different applications. Specifically, they make it difficult to use such discharges in laser technology and plasma chemistry (in the cases where the working medium must be uniform). At the same time, since they give rise to structures with high electron density and temperature in the discharge, the instabilities can play a positive role in the proposals for using electromagnetic radiation to produce radio-reflecting or "ozone-restoring" regions of artificial ionization in the atmosphere.[7,8]

Our objective in the present work is to investigate the dynamics of a discharge with a developed aperiodic "plasma-resonance" instability which predominates over other types of instabilities in a wide range of gas pressures, in which it is not suppressed by diffusion and the electron collision frequency ν is lower than the frequency ω of the field.[1,5] This instability, which leads to spatial modulation of the plasma density in the direction of the electric field vector, can be regarded as a unique ionization analog of the well-known modulation instability of a collisionless plasma with a focusing nonlinearity, but in contrast to the latter, it can develop (demonstrating, as we shall see, a substantially different nonlinear dynamics) not only in a narrow resonance region near the critical-density surface, but also in virtually the entire transmission region of the plasma.

Although the initial (linear) stage of the instability was analyzed and some forms of the discharge, which apparently appear as a result of its development, were observed experimentally quite a long time ago,[1,4,5] the space-time evolution of the discharge that corresponds to the nonlinear stage of the discharge has still not been investigated theoretically. We performed a computer simulation of the space-time evolution on the basis of solving a one-dimensional integrodifferential diffusion equation for the plasma density that describes the development of the instability in the longitudinal (parallel to the density gradient) rf electric field. The one-dimensional model ("flat capacitor" model) employed is suitable for describing small-scale fragmentation of discharges in both quasistatic and wave fields under the condition that the characteristic scale of the instability is small compared with the wavelength λ.

Still using the approximations which are usually employed for the theory of a non-equilibrium rf discharge in a cold gas[9–11] (local and instantaneous average heating of the electrons and local polarization response of the plasma at the frequency ω of the field) and assuming that the electron balance is controlled mainly by ionization as a result of electron impact, attachment to neutral molecules (assuming fast recombination of the negative and positive ions which are produced), and diffusion (free or ambipolar), we write the initial equations for the field and plasma density in the form

$$\frac{\partial(\varepsilon E)}{\partial x}=0, \quad \frac{\partial N}{\partial t}=D\frac{\partial^2 N}{\partial x^2}+(\nu_i-\nu_a)N. \tag{1}$$

Here $E(x,t)$ is the slow (on the scale of the period $2\pi/\omega$) complex envelope of the electric field, which is represented in the form $\mathbf{E}=\frac{1}{2}\mathbf{x}_0 E\exp(-i\omega t)+$ c.c., $N(x,t)$ is the electron concentration, $\varepsilon=1-(N/N_c)(1-i\nu/\omega)$ is the complex permittivity of the plasma, $N_c=m(\omega^2+\nu^2)/4\pi e^2$ is the critical concentration, and e and m are the electron charge and mass. The frequency ν_i of ionizing electron collisions is assumed to be a fixed function (rapidly increasing) of the amplitude of the field, for which the power-law approximation $\nu_i\sim|E|^\beta$, where β depends on the type of gas [for air $\beta\simeq 4-5$ (Ref. 12)], can be used in a wide range of values of $|E|$. For simplicity, the effective electron collision frequency ν, the attachment frequency ν_a, and the diffusion coefficient D, which depend on $|E|$ much more weakly, are assumed to be constants. It is convenient to represent the difference of the ionization and attachment frequencies which appears in the equation for N as

$$\nu_i-\nu_a=\nu_a[(|E|/E_c)^\beta-1], \tag{2}$$

where E_c is the critical amplitude, determined from the condition of uniform breakdown $\nu_i=\nu_a$. As follows from Eqs. (1) and (2), the dynamics of the spatially periodic perturbations of the amplitude of the field and density, which are characterized by a fixed period L along the coordinate x $[N(x,t)=N(x+L,t),\ E(x,t)=E(x+L,t)]$ and fixed (time-independent) value of the field amplitude averaged over a period L

$$E_0=L^{-1}\int_0^L E(x)dx=\text{const},$$

is described in the dimensionless variables $t\to\nu_a t$, $x\to x/L_a$, $E\to E/E_c$, $n=N/N_c$ ($L_a=\sqrt{D/\nu_a}$ is the electron-attachment diffusion length) by the equations

$$\frac{\partial n}{\partial t}=\frac{\partial^2 n}{\partial x^2}+F\{n\}, \quad F\{n\}=(|E|^\beta-1)n, \tag{3}$$

$$E=\frac{E_0\varepsilon_{\text{eff}}}{\varepsilon}, \quad \varepsilon=1-n(1-i\delta), \quad \varepsilon_{\text{eff}}^{-1}=L^{-1}\int_0^L\varepsilon^{-1}dx, \tag{4}$$

where $\delta=\nu/\omega$, $\varepsilon_{\rm eff}$ is the effective permittivity, which determines the average polarizability of the plasma along the x axis, and E_0 and L are, here and below, dimensionless quantities ($E_0 \to E_0/E_c$, $L \to L/L_a$). An important feature of the nonlinear diffusion equation obtained for n is that it contains nonlinearities of two types: a "local" nonlinearity determined by the form of the resonance curve $|E|^\beta(n) \sim [(1-n)^2+(n\delta)^2]^{-\beta/2}$ and an "integral" nonlinearity determined by $\varepsilon_{\rm eff}$ which is a functional of the density $n(x)$ along the entire interval $(0, L)$.

Linearizing Eqs. (3) and (4) against the background of the uniform (in general, nonstationary) state $n=n_a(t)$, $\varepsilon=\varepsilon_a=1-n_a(1-i\delta)$, $E=E_0=$ const; i.e., setting $n=n_a(t)+n_v(t)\cos kx$, $k=2\pi/L$, $n_v \ll n_a$, and taking into account the fact that in this approximation $\varepsilon_{\rm eff}=\varepsilon_a$, we find the rate of change of the uniform and nonuniform components of the density:

$$\frac{dn_{a,v}}{dt}=\gamma_{a,v}n_{a,v}, \quad \gamma_a=E_0^\beta-1, \quad \gamma_v=\beta E_0^\beta|\varepsilon_a|^{-2}n_a(1-n_a-\delta^2 n_a)+E_0^\beta-1-k^2. \tag{5}$$

For any sign of the difference E_0-1, i.e., both in the process of ignition ($\gamma_a>0$) and in the process of extinction ($\gamma_a<0$) of the discharge, there exists a range of values of the average density n_a, where the initially small nonuniform perturbation n_v grows exponentially with time, and the time constant γ_v of this increase (instability increase rate) can be much larger than the corresponding constant $|\gamma_a|$ for n_a. Specifically, in the case $|E_0^\beta-1| \ll 1$, $k^2 \ll 1$, $\delta^2<1$ the condition $\gamma_v>|\gamma_a|$ holds in a wide range $n_{a1}<n_a<n_{a2}$, where $n_{a1}=(2|1-E_0^\beta|+k^2)\beta^{-1} \ll 1$ and $n_{a2}=1/(1+\delta^2) \sim 1$.

Equations (3) and (4) were solved with the initial condition $n(x,0)=n_0+n_1\cos kx$ for large deviations of the density from the average value n_a. The calculations showed that the scenarios of space-time evolution of the discharge depend strongly on E_0. For $E_0=1$ the nonlinear stage of the instability is completed rapidly with the establishment of a uniform stationary state with density $n=n_a>n_{a2}$, which is stable with respect to nonuniform perturbations. In the case $E_0>1$, a uniform state with a monotonically increasing density is established (this increase stops only when the electron-ion recombination processes, which have been neglected here, are taken into account). The most interesting and diverse results were obtained for the region $E_0<1$. Specifically, for $1-E_0 \ll 1$, $k^2=(2\pi/L)^2 \ll 1$, $\delta<1$ [which are typical for the region behind the front of the breakdown wave, produced by the field of a converging microwave beam at gas pressures $p \simeq 1-30$ torr (Ref. 5)], a series of repeating fluctuations of the discharge, which are characterized by deep temporal and spatial modulation of the density and resonance amplification of the field in the rapidly moving regions of critical density $n=1$, is realized. The computational results for the parameters $E_0=0.95$, $\beta=4$, $\delta=\nu/\omega=0.5$, $L=10$, $n_0=0.8$, and $n_1=0.04$ are presented in Figs. 1 and 2. Two stages — fast and slow — can be separated in each repeating cycle of the space-time evolution of the plasma density (Fig. 1). At the first stage ($1.5<t<3$ and $7<t<8.5$) the development of the instability leads to a transition of the maximum of the density through the point $n=1$, after which a fast breakdown wave, which leads to establishment of an almost uniform state with $n \simeq n_a>n_{a2}$, is formed. At this stage, sharp temporal peaks of the real and imaginary parts of the effective permittivity $\varepsilon_{\rm eff}(t)$ of the plasma appear (Fig. 2).

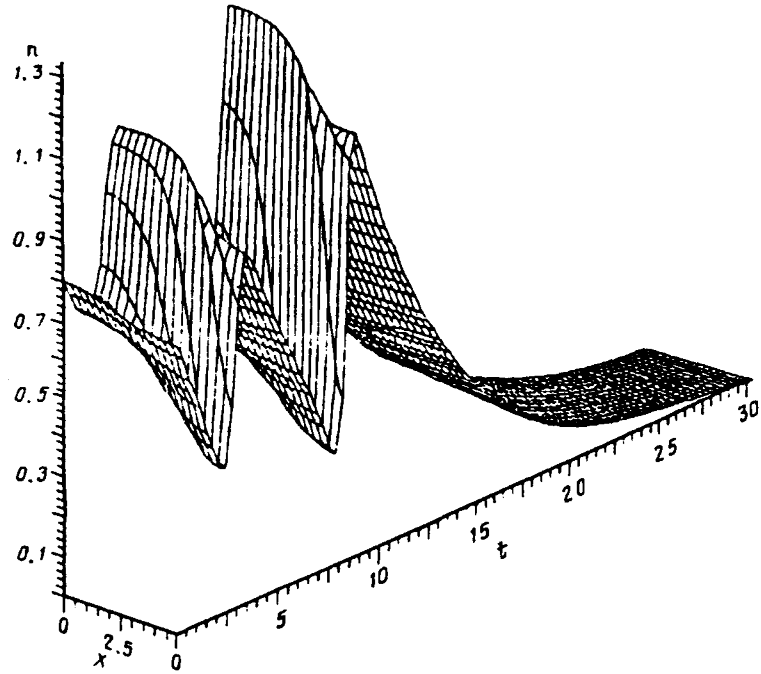

FIG. 1. Space-time evolution of the plasma density $n(x,t)$ in the absence of an external source of ionization.

These peaks indicate that sharp changes occur in the refractive and absorbing properties of the plasma as a whole, viewed as a macroscopic medium. At the second stage ($3 < t < 7$ and $t > 8.5$) the density $n \simeq n_a$ decreases slowly and finally reenters the zone of instability $n < n_{a2}$. If the nonuniform component of the perturbation $(n - n_a)$ has not relaxed too strongly by this time (as at the end of the last fluctuation in Fig. 1), it again

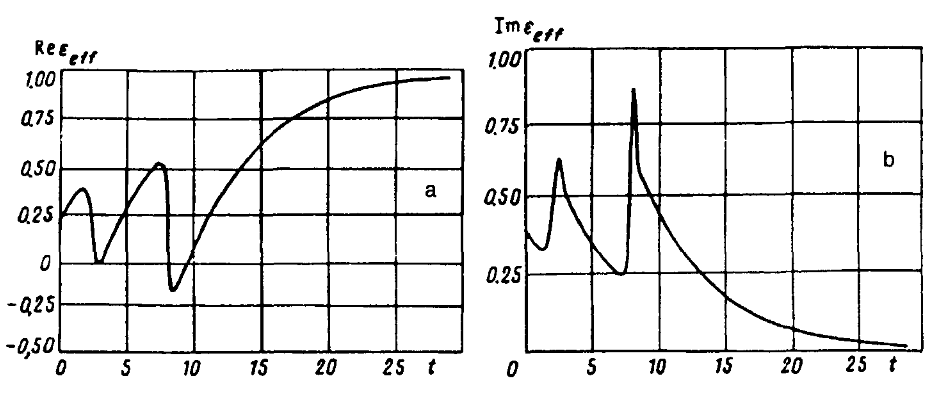

FIG. 2. Real (a) and imaginary (b) parts of the effective permittivity $\varepsilon_{\text{eff}}(t)$.

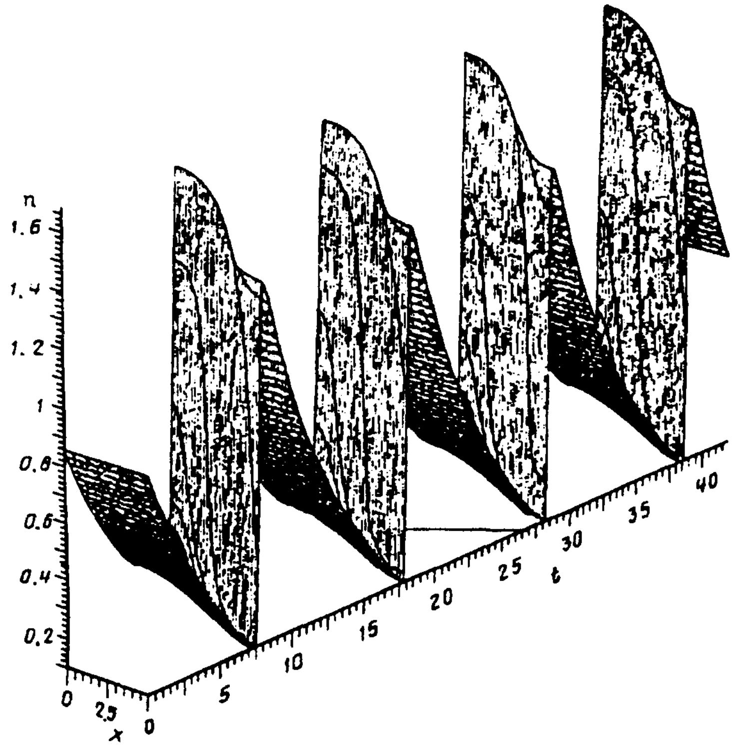

FIG. 3. Space-time evolution of the plasma density $n(x,t)$ in the presence of a weak external source.

increases substantially and the process is repeated. The last fluctuation in our model is completed at the relaxation stage after the formation of a state with a nonuniformity so weak that over the time that the density decreases from n_{a2} to n_{a1}, i.e., over the residence time of the plasma in the instability zone, there is not enough time for this instability to reach an appreciable magnitude and the discharge relaxes irreversibly to the state $n=0$. It is obvious that at this final stage of the evolution of the discharge weak external sources of ionization, which we ignored in Eq. (3) and which are capable of producing nonuniform seed perturbations of the density that are capable of driving the discharge into a regime of nonoverlapping fluctuations, can start to play a determining role. As an example of the realization of this regime, Fig. 3 shows the results of a numerical solution obtained (for the same values of the parameters E_0, β, δ, and L) with the addition to the right-hand side of Eq. (3) for n an external source of the form $I(x)=I_0(1+\cos kx)$ ($I_0=2\times10^{-4}$). Since the evolution of unstable disturbances in this regime actually starts again after each fluctuation, in the case of an external source of noise characterized by the random function $I(x,t)$, which is important for practical applications, the fluctuations should become stochastic, i.e., they should lead to the formation of structures with a random (uncorrelated with preceding fluctuations) arrangement of the maxima and minima of the density.

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