# Coherent precession of magnetization in $^3$He-$A$ in the collisionless region

V. P. Mineev and Yu. G. Makhlin
*L. D. Landau Institute of Theoretical Physics, 117334 Moscow, Russia*

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Magnetization precession in the $A$ phase of $^3$He near the transition temperature in the collisionless region was investigated. The solution describing a coherently precessing two-domain structure in a nonuniform external field was found. In contrast to the $B$ phase, in the limit of a uniform external field this solution transforms into uniform spin precession. The behavior of the $A$ and $B$ phases of $^3$He in different regimes (hydrodynamic and collisionless) is compared. 

## INTRODUCTION

The dynamics of nonuniform magnetization distributions which precess in liquid $^3$He in an external magnetic field is now being vigorously investigated. The discovery of a structure that precesses uniformly in the normal liquid in a nonuniform field[1,2] has attracted interest in the temperature range slightly below $T_c$. Long-lived coherently precessing structures have also been discovered in this region.[3]

The equations of the collisionless ($\omega_L \tau \gg 1$) nonuniform spin dynamics of the $B$ phase of superfluid $^3$He near the transition temperature, where the main contribution to the spin current is associated with the nondissipative spin diffusion characteristic of a Fermi liquid, were derived in Ref. 4 . The solution found in Ref. 4 for these equations represents a two-domain structure reminiscent of the structure in a normal liquid[2] but with the thickness and shape of the domain wall determined by the dipole forces, rather than by the spatial dependence of the magnetic field.

In the phase diagram the superfluid $B$ phase in a magnetic field is separated from the normal liquid by a temperature interval (narrow at low pressures and wide at high pressures) in which the $A$ phase exists. In the present letter we will examine the uniformly precessing states in this region. We will show that in the collisionless regime a coherently precessing, two-domain structure exists in the $A$ phase. The form and thickness of the transitional layer between the domains is determined by the balance of the dipole forces and the nonuniformity of the magnetic field.

## EQUATIONS OF THE SPIN DYNAMICS

The equations of the nondissipative collisionless spin dynamics near the transition into the $A$ phase are the same as in the $B$ phase:[4]

$$\frac{\partial \mathbf{S}}{\partial t} = \mathbf{S} \times \vec{\omega}_L - \nabla \mathbf{J} + \mathbf{R}_D, \tag{1}$$

$$\frac{\partial \mathbf{J}}{\partial t}=\mathbf{J}\times(\vec{\omega}_L+\kappa\mathbf{S})-\frac{w^2}{3}\nabla(\mathbf{S}-\vec{\omega}_L), \tag{2}$$

$$\frac{\partial \mathbf{d}(\mathbf{k})}{\partial t}=\mathbf{d}(\mathbf{k})\times(\vec{\omega}_L-\mathbf{S}), \tag{3}$$

where $\mathbf{S}$ and $\mathbf{J}$ are the spin density and the spin-current density, $\mathbf{d}(\mathbf{k})$ is the order parameter, $\vec{\omega}_L$ is the Larmor frequency of the external field, and $\mathbf{R}_D$ is the dipole moment determined by the instantaneous value of the order parameter. The Fermi-liquid effects are described by the parameter $\kappa=-F_0^a/(1+F_0^a)$, where $F_0^a$ is the zeroth harmonic of the spin-spin part of the Fermi-liquid interaction (we ignore higher-order harmonics), and $w^2=v_F^2(1+F_0^a)$. All quantities are assumed to depend only on the coordinate $z$ so that we drop the spatial index in the derivatives and the spin current. We employ units in which the magnetization $\chi$ of the liquid and the gyromagnetic ratio $\gamma$ for the $^3$He atoms are related by the relation $\chi=\gamma^2$. As shown by Leggett,[5] it is not important to take into account in the equations of the spin dynamics the anisotropy of the magnetic susceptibility of the $A$ phase, since the direction of the order-parameter vector $\mathbf{d}(\mathbf{k})$ does not depend on the direction of the wave vector $\mathbf{k}$ on the Fermi sphere.

The equations were derived under the assumptions that the characteristic size of the spatial nonuniformity is large compared to $v_F/\omega_L$ and the coherence length, and that the characteristic frequency $\omega_L$ is small compared to the order parameter: $\omega_L \ll \Delta$. For the magnetic fields used in the experiments the latter assumption excludes from the analysis only a very narrow range of temperatures near the transition where the order parameter is still very small. Equation (1) without the gradient term and Eq. (3) are the Leggett equations for a uniform spin dynamics.

In the $A$ phase the order parameter has the form

$$\mathbf{d}(\mathbf{k})=\Delta\cdot\mathbf{V}(\mathbf{\Delta}'+i\mathbf{\Delta}'',\hat{\mathbf{k}}), \tag{4}$$

where $\mathbf{V}$ is a unit vector in spin space, and $\mathbf{\Delta}'$ and $\mathbf{\Delta}''$ are a pair of orthonormal orbital vectors. Because the so-called orbital viscosity is high, the orbital dynamics (i.e., the dynamics of the vectors $\mathbf{\Delta}'$ and $\mathbf{\Delta}''$, as well as the vector of the orbital angular momentum $\mathbf{l}=\mathbf{\Delta}'\times\mathbf{\Delta}''$ which forms together with them a right-handed reference frame) is frozen at not too low temperatures. The dipole moment in the $A$ phase is

$$\mathbf{R}_D=\Omega_A^2\mathbf{V}\times\mathbf{l}(\mathbf{V}\cdot\mathbf{l}), \tag{5}$$

where $\Omega_A$ is the frequency of the longitudinal resonance.

We are interested in the precessing solutions of the system of equations (1)–(3) in a nonuniform external field.

## PRECESSING SOLUTIONS

The existence of exact precessing solutions of Eqs. (1)–(3) in the $B$ phase is associated with the fact that during the precessional motion of the magnetization the dipole moment appearing in Eq. (1) moves together with the magnetization. In the $A$ phase the dipole moment (5) depends on the orientation of the order parameter relative to the direction of the orbital-anisotropy vector $\mathbf{l}$. There are therefore no exact precessing solutions in the $A$ phase. However, in sufficiently strong magnetic fields, such that $\omega_L \gg \Omega_A$, there exist almost precessing solutions, when motions with amplitudes which are small in the parameter $(\Omega_A/\omega_P)^2$ are superimposed on the main precessing motion of the magnetization with the frequency $\omega_P$ close to $\omega_L$.

The frequency shift $\omega_P - \omega_L$ found by Brinkman and Smith[6] in the spatially uniform case is determined by the magnitude of the dipole moment $\mathbf{R}_D$ averaged over the fast precessing motion.

In the spatially nonuniform case the magnitude of the terms connected with the nonuniformity $(\nabla \mathbf{J})$ is of the same order of magnitude as $\mathbf{R}_D$. Just as in the spatially uniform case, in solving Eqs. (1)–(3) the following system must therefore be solved as a first approximation:

$$\frac{\partial \mathbf{S}}{\partial t} = \mathbf{S} \times \vec{\omega}_P, \tag{6}$$

$$\frac{\partial \mathbf{V}}{\partial t} = \mathbf{V} \times (\vec{\omega}_P - \mathbf{S}). \tag{7}$$

Here $\vec{\omega}_P = \omega_P \hat{z}$ is the precession frequency. We shall examine the solutions with the equilibrium absolute magnetization $S = \omega_P$. The solution of Eqs. (6) and (7) in the laboratory coordinate system is described by the equations[7]

$$\mathbf{S} = \omega_P \hat{R}_z(\alpha - \omega_P t) \hat{R}_y(\beta) \hat{z}, \tag{8}$$

$$\mathbf{V} = \hat{R}_z(\alpha - \omega_P t) \hat{R}_y(\beta) \hat{R}_z(\Phi - \alpha + \omega_P t) \hat{x}. \tag{9}$$

Here $\hat{R}_y(\beta)$ is the matrix describing a rotation around the $\hat{y}$ axis by the angle $\beta$, the matrix $\hat{R}_z(\alpha - \omega_P t)$ is defined similarly, and so on. The family of zeroth-approximation solutions is degenerate with the respect to the angles $\alpha$, $\beta$, and $\Phi$, which can be arbitrary functions of the coordinates.

The equation fixing the spatial behavior of these functions is obtained as follows. We write Eqs. (1) and (2) and the zeroth-order solutions,[1)] Eqs. (8) and (9), in a coordinate system rotated around the vertical axis by the angle $\alpha - \omega_P t$, which depends on the coordinates. Substituting these solutions into Eq. (1), we average this equation over the time (over the precession period $2\pi/\omega_P$). Motions with frequencies different from $\omega_P$ and small amplitudes $[\sim (\Omega_A/\omega)^2]$ drop out of the equations, and we obtain

$$\mathbf{S} \times (\vec{\omega}_L - \vec{\omega}_P) - \nabla \mathbf{J} + \langle \mathbf{R}_D \rangle = 0. \tag{10}$$

Here $\mathbf{J}$ is the solution of Eq. (2) (also in a rotating system), which according to Ref. 4 , has the following form in the temperature and frequency ranges of interest to us:

$$\mathbf{J} \cong \frac{w^2}{3\kappa S^2} \mathbf{S} \times \nabla \mathbf{S}. \tag{11}$$

The calculation of the projections of the average dipole moment onto the axes of the rotating coordinate system $\tilde{x}$, $\tilde{y}$, $\tilde{z} = z$ gives the result

$$\langle R_{D\tilde{x}}\rangle = -\frac{1}{8}\Omega_A^2 \sin\beta(1+\cos\beta)\sin(2\Phi), \tag{12}$$

$$\langle R_{D\tilde{y}}\rangle = -\frac{1}{8}\Omega_A^2 \sin\beta[2\cos\beta+(1+\cos\beta)\cos(2\Phi)], \tag{13}$$

$$\langle R_{D\tilde{z}}\rangle = -\frac{1}{8}\Omega_A^2(1+\cos\beta)^2 \sin(2\Phi). \tag{14}$$

It is evident from expression (11) for the current that the divergence of the current is perpendicular to the magnetization. Making use of this property, we find from Eq. (10) that the component of the average dipole moment along the magnetization is equal to zero:

$$0=(\langle\mathbf{R}_D\rangle\mathbf{S}) = -\frac{1}{8}\Omega_A^2(1+\cos\beta)^2 \sin(2\Phi). \tag{15}$$

Since we are interested in the nonuniform distributions of the magnetization, we ignore the case $\cos\beta=-1$. We thus have

$$\sin(2\Phi)=0; \tag{16}$$

i.e., $\Phi=0,\ \pi/2,\ \pi,\ \ldots$ . The case of $\Phi$, which is a multiple of $\pi$, corresponds to a minimum of the dipole energy averaged with respect to the fast precession, and the case of angles $\Phi$, which are equal to half-integral multiples of $\pi$, corresponds to a maximum of the dipole energy.

It follows from Eqs. (12)–(14) and the restriction imposed on the angle $\Phi$ that the dipole moment, averaged in the rotating coordinate system, is directed along the $\tilde{y}$ axis.

The $z$ projection of Eq. (10) is $\nabla J_z=0$ . Since at the boundary of a closed (at least on one side) vessel there is no spin current, it suggests that $J_z=0$, i.e.,

$$(\mathbf{S}\times\nabla\mathbf{S})_z=0.$$

The magnetization gradient thus lies in the same vertical plane as the magnetization itself, and hence the position of this plane (fixed by the angle $\alpha$) does not depend on $z$:

$$\alpha(z)=\text{const.} \tag{17}$$

Finally, the $\tilde{y}$ projection of Eq. (10) leads to an equation for determining the spatial dependence of the angle of inclination $\beta$ of the magnetization from the vertical direction:

$$-\omega_P(\omega_L-\omega_P)\sin\beta-\frac{w^2}{3\kappa}\beta''-\frac{1}{8}\Omega_A^2 \sin\beta[2\cos\beta\pm(1+\cos\beta)]=0. \tag{18}$$

Here the $\pm$ sign is determined by the value of $\cos(2\Phi)$. The boundary condition for Eq. (18) is the condition $\beta'=0$ that there is no spin current. We are interested in the solution in a closed cell.

Before solving Eq. (18), we underscore again that in deriving this equation we made substantial use of the fact that the dipole energy is small: $\Omega_A \ll \omega_P$. In the $B$ phase a precessing solution exists for any ratio of the frequencies $\Omega_A$ and $\omega_P$.

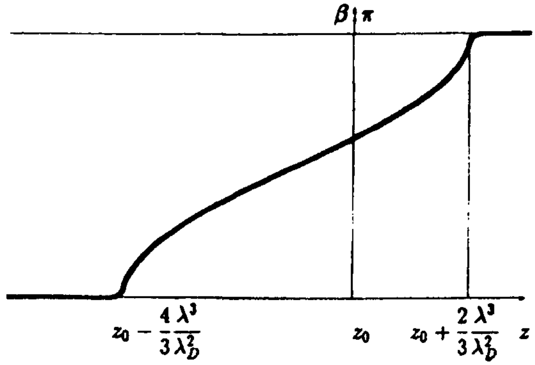


FIG. 1. Inclination angle of the magnetization as a function of the magnetic field for a two-domain structure in the $A$ phase.

## DOMAIN-WALL STRUCTURE

It is convenient to rewrite Eq. (18) in the form

$$\beta'' = -\frac{z-z_0}{\lambda_n^3}\sin\beta - \frac{1}{\lambda_D^2}\sin\beta\frac{[2\cos\beta\pm(1+\cos\beta)]}{3}. \tag{19}$$

Here we assume that the external field is a linear function of the coordinates: $\omega_L=\omega_P+\nabla\omega_L(z-z_0)$. We have introduced two characteristic lengths: the domain-wall thickness $\lambda_n=(w^2/3\omega_P\nabla\omega_L)^{1/3}$ in the normal liquid and the dipole length $\lambda_D=(8/9\kappa)^{1/2}w/\Omega_A$.

The solution of Eq. (19) for the minimum of the dipole energy (+ sign) is shown in Fig. 1. It describes a two-domain structure. Just as in the normal liquid, in one domain, which is located in the region of weaker fields, the magnetization has the equilibrium value and in the other domain the magnetization has the opposite value. While the thickness of the domain wall is equal to $\lambda_n$ in the normal liquid and $\lambda_D$ in the $B$ phase (where $\lambda_D \ll \lambda_n$), in the case of the $A$ phase the characteristic size of the transitional region is the length $\lambda_n^3/\lambda_D^2$, which corresponds to a frequency differential of the order of $\Omega_A^2/\omega_P$. In contrast to the $B$ phase, in the limit of a uniform field $\nabla\omega_L\to 0$, this length approaches infinity; i.e., the coherent precession of a two-domain structure transforms into uniform precession of the magnetization.

It can be shown that in the uniform case the solution corresponding to a maximum of the dipole energy with respect to $\Phi$ is unstable. This property apparently remains in the nonuniform case. The solution corresponding to a maximum with respect to $\Phi$ qualitatively looks just like the solution in Fig. 1.

## TWO-DOMAIN STATES FOR THE *A* AND *B* PHASES IN THE HYDRODYNAMIC AND COLLISIONLESS REGIONS

We wish to point out an interesting relationship between the two-domain solutions for different phases in different regimes.

As shown in Ref. 4 , the spin dynamics is determined by which contribution — equilibrium or nondissipative diffusion (Fermi liquid) — makes the greater contribution to the spin current. For $\omega_L\tau \gg 1$ the latter contribution is the determining contribution near the transition ($T>0.85T_c$). We call this region collisionless. At low temperatures, $T<0.85T_c$, or in weak fields, $\omega_L\tau \ll 1$, the equilibrium current plays the determining role and the equations of spin hydrodynamics operate.

In the hydrodynamic region (see the review in Ref. 8 ) the two-domain solution in the $A$ phase has a characteristic size $\lambda_D$ (with $w$ replaced by the velocity of the spin waves); i.e., it continues to exist in a uniform field. In the $B$ phase the characteristic size of the transitional region (from $\beta=0$ to $\beta=\pi$) is equal to $\lambda^3/\lambda_D^2$ (see, for example, Ref. 4 ), where $\lambda=(c_\parallel^2/\omega_P\nabla\omega_L)^{1/3}$ is the analog of $\lambda_n$ for the corresponding problem.

In the collisionless regime described above, the situation is reversed: The solution has a characteristic size $\lambda_n^3/\lambda_D^2$ in the $A$ phase and $\lambda_D$ in the $B$ phase.

Two-domain structures with different arrangement of the domains can exist in the hydrodynamic case in the $A$ phase and in the collisionless case in the $B$ phase, when the wall thickness $\lambda_D$ remains finite in a uniform field: A domain with equilibrium orientation can lie in both the region of a stronger field and the region of a weaker field. These configurations have similar energies, and complete degeneracy occurs in the uniform case.

The difference in the two-domain states in the given phase but in different regimes in helium-3 is a consequence of the fact that in the problem of the appearance and existence of precessing structures the spin current in the hydrodynamic (equilibrium) region is directed opposite to the spin current in the collisionless (Fermi liquid) region. Accordingly, the current either promotes or impedes the appearance of nonuniform structures.

The difference in the two-domain states in the same regime but in different phases of helium-3 is, however, a consequence of the difference in the convexities (upward or downward) of the effective dipole energy (averaged and minimized with respect to $\Phi$) as a function of $\cos\beta$. In the $B$ phase this energy is convex downward

$$U_{DB,\mathrm{eff}}=\frac{8}{15}\Omega_B^2\left(\cos\beta+\frac{1}{4}\right)^2\Theta(\beta-\theta_L), \tag{20}$$

where $\theta_L\approx 104°$, and in the $A$ phase the energy is convex upward

$$U_{DA,\mathrm{eff}}=-\frac{3}{16}\Omega_A^2\left(\cos\beta+\frac{1}{3}\right)^2. \tag{21}$$

## DISCUSSION

We have solved the equations of the spin dynamics of the $A$ phase near the transition temperature, which describe a two-domain, coherently precessing structure. In one domain the magnetization has the equilibrium orientation and in the other domain it has the opposite orientation. In the derivation of the equations we used the small value of the dipole energy.

The stability of this structure must be investigated. Just as in Ref. 4 , however, it can be assumed that the solution corresponding to the maximum (as a function of $\Phi$) of the dipole energy is unstable (relative to oscillations as a function of $\Phi$; compare with Ref. 8). The solution corresponding to a minimum (as a function of $\Phi$) is stable: with respect to $\Phi$ because this is a minimum and with respect to the other variables because a distribution that ensures a locally maximum value of the effective energy is realized: the sum of the spectroscopic energy

$$E_{sp}=-(\vec{\omega}_L-\vec{\omega}_P)\mathbf{S},$$

the dipole energy (21), and the "gradient" energy

$$E_{\nabla}=-\frac{w^2}{6\kappa S^2}(\nabla\mathbf{S})^2$$

(compare with Ref. 4 ). Of course, these arguments are not rigorous and an accurate investigation of the stability of the solutions is required.

A local minimum of the dipole energy is realized not only for the equilibrium absolute magnitude of the spin, but also for double or half the value.[9] Since the dependence of the effective dipole energy on $\beta$ under these conditions is rather complex, it is expected that a complex structure of the domain walls will arise for such distributions in the collisionless region.

It is entirely possible to observe experimentally the above-described structure in the $A$ phase in the pressure and magnetic field ranges where the condition $\lambda_D \ll \lambda_n$ is still achieved in the $A$ phase, i.e., quite close to $T_c$. As the temperature decreases from the normal phase, the characteristic length of the structure should change (i.e., the strength of the induction signal from it should change): It starts to increase at the transition into the $A$ phase (more accurately, when $\lambda_D = \lambda_n$) and it decreases sharply at the transition into the $B$ phase.

Energy dissipation in this region (in both the normal and superfluid liquids) is mainly determined by spin diffusion.[4] The dissipated power is inversely proportional to the thickness of the domain wall. Consequently, relaxation is much slower in the $A$ phase with $\lambda_D \ll \lambda_n$ than in the $B$ phase.

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[1)]The use of Eq. (9) presupposes that the vector **V** moves in a plane perpendicular to **S**. In the spatially uniform case $\mathbf{S}\cdot\mathbf{V}$ is an integral of motion, and the $\mathbf{S}\cdot\mathbf{V}=0$ corresponds to equilibrium initial conditions. In the nonuniform case $d(\mathbf{S}\cdot\mathbf{V})/dt=-(\mathbf{V}\cdot\nabla\mathbf{J})$. Averaging this equality over the fast precession, we find that $\mathbf{S}\cdot\mathbf{V}$ remains in the first approximation.

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