Calculation of the Leggett–Takagi relaxation in the entire temperature range for ^{3}He-*B*

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(Submitted 6 June 1995; resubmitted 21 September 1995)
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An expression is presented for Leggett–Takagi relaxation with arbitrary $\omega_L \tau$, which makes it possible to include the entire temperature range for ^{3}He-*B*.

In the present paper we shall examine the following problem. It is known that the relaxation of the Brinkman–Smith mode in the B phase of ^{3}He in the hydrodynamic region is weak to the extent that τ (quasiparticle relaxation time) is small, and that in the collisionless region it is also weak to the extent that τ^{-1} is small. In the hydrodynamic region there exists an expression for the relaxation time of the Brinkman–Smith mode that is valid in the region[1] $\omega_L \tau \ll 1$. This condition corresponds to the temperature range near the superconducting transition temperature. At the same time, an expression has been derived for the Brinkman–Smith relaxation mode in the collisionless region, in which the condition $\omega_L \tau \gg 1$ is satisfied.[2] This is the applicability condition in the temperature region near zero Kelvin. In the more interesting intermediate temperature region, in which the condition $\omega_L \tau \sim 1$ is satisfied, no formulas have been derived for the damping of the precession of nonlinear NMR. In this letter our goal is to bridge this gap.

In general, the problem of calculating the relaxation time of nonlinear NMR on the basis of the nonlinear kinetic equation is extremely complicated. In our case, however, there is a substantial simplification: For tilt angles $\beta < \beta_L$ (Leggett's angle) Larmor precession occurs in ^{3}He-*B* without a shift of the precession frequency. The spin of the quasiparticles rotates coherently together with the spin of the condensate, which is not affected in this case by the additional dipole forces. In the region $\beta < \beta_L$ the solution of the kinetic equation will therefore be the equilibrium distribution function. Dipole forces which drive the condensate spin away from the quasiparticle spin appear at angles β that exceed β_L by a small amount, but these forces are weak to the extent that $\Delta\beta = \beta - \beta_L$ is small. For this reason, the nonequilibrium correction to the distribution function is also small to the extent that $\Delta\beta = \beta - \beta_L$ is small. To find this nonequilibrium correction and obtain an expression for the effective relaxation of the Brinkman–Smith mode, we start from a combined system of three equations:[3,4] the hydrodynamic equation for the total spin

$$\dot{\mathbf{S}} = \gamma \mathbf{S} \times \mathbf{H} + \mathbf{R}_d, \tag{1}$$

the equation of motion of the order parameter

$$\dot{\mathbf{d}}(\mathbf{p})=\left[\mathbf{d}(\mathbf{p}),\ \left\{\gamma\mathbf{H}-\gamma^2\left(\frac{Z_0}{4\chi_{n0}}\mathbf{S}+\frac{1}{\chi_{p0}}\mathbf{S}_p\right)\right\}\right], \qquad (2)$$

and the nonlinear kinetic equation

$$\delta\dot{\vec{\nu}}_k(\mathbf{r},t)+\left(\nabla_k E_k\frac{\partial}{\partial\mathbf{r}}\right)\delta\vec{\mu}_k(r,t)+\delta\vec{\mu}_k(\mathbf{r},t)\times\delta\mathbf{E}_k(\mathbf{r},t)=\mathbf{I}(\delta\vec{\nu}_k), \qquad (3)$$

$$\delta\vec{\mu}_k(\mathbf{r},t)=\delta\vec{\nu}_k(\mathbf{r},t)-\frac{\partial\varphi}{\partial E_k}\delta\mathbf{E}, \quad \varphi_k=-\ \tan\frac{1}{2}\beta E_k,$$

$$\delta\mathbf{E}_k=k_i\mathbf{A}_i+\frac{\xi_k}{E_k}\mathbf{X}-\left(1-\frac{\xi_k}{E_k}\right)\mathbf{d}_k[\mathbf{d}_k(k_i\mathbf{A}_i-\mathbf{X})], \quad \mathbf{X}=\mathbf{V}-\vec{\Omega}-f_0^a\mathbf{S}.$$

Equations (1) and (3) are, respectively, the hydrodynamic equation for the total spin and the kinetic equation for the quasiparticles. They are always valid. Equation (2) was derived in Ref.4 for the motion of the order parameter under certain assumptions. One of the assumptions for which Eq. (2) is valid is to study almost-periodic solutions, when the effective relaxation time of these solutions is larger than the reciprocal of the frequency. In this case Eq. (5.34) in Ref. 4 can be substituted into Eq. (5.33) which yields Eq. (5.35) or, in our case, Eq. (2). In our case, as follows from Eq. (15), $\delta\dot{\beta}/\omega_L\ll 1$, and the case which was considered by us prevails.

Equation (3) was written in a rotating coordinate system. In this system all gradients and derivatives with respect to the time and the external magnetic field are as follows:

$$A_{\alpha\beta}=-\frac{i}{m}U^{-1}(\alpha,\beta,\gamma)\nabla U(\alpha,\beta,\gamma), \quad V_{\alpha\beta}=-iU^{-1}(\alpha,\beta,\gamma)\frac{\partial}{\partial t}U(\alpha,\beta,\gamma),$$

$$\Omega_{\alpha\beta}=U^{-1}(\alpha,\beta,\gamma)\frac{1}{2}\omega_L\sigma U(\alpha,\beta,\gamma), \quad A_{\alpha\beta}=\frac{1}{2}A_\lambda\sigma^\lambda_{\alpha\beta}, \quad V_{\alpha\beta}=\frac{1}{2}V_\lambda\sigma^\lambda_{\alpha\beta}, \qquad (4)$$

$$\Omega_{\alpha\beta}=\frac{1}{2}\Omega_\lambda\sigma^\lambda_{\alpha\beta} \quad (\lambda=x,y,z).$$

We seek the solution of the combined system of kinetic and hydrodynamic equations in the form

$$\delta\vec{\nu}_k=\frac{\partial\varphi}{\partial E_k}\delta\mathbf{E}+\delta\vec{\mu}_k,$$

where $\delta\vec{\mu}_k$ is the nonequilibrium correction to the distribution function. As will be seen from the solution, it is proportional to the first power of the deviation from Leggett's angle. We can seek the solution of the system of hydrodynamic equations as a functional of $\delta\beta$ (deviation from Leggett's angle) and of $|\delta\vec{\mu}|$ (nonequilibrium correction to the distribution function). The system (1), (2) will then be independent of Eq. (3) and we obtain by the method described in Refs. 1 and 2 the following expression for the precession frequency $\dot{\alpha}$:

$$\dot{\alpha}=\omega_L+\frac{16}{15}\frac{\Omega^2}{\omega_L}\left(\frac{1}{4}+\cos\beta\right)+O(|\delta\vec{\mu}|\delta\beta). \tag{5}$$

The second term in Eq. (5) originates from the equilibrium quasiparticle distribution function and contains a term linear in $\delta\beta$. The last term contains the product of two small quantities $|\delta\mu|\delta\beta$, which, as will be seen from the solution, is a quadratic function of $\delta\beta$.

We now switch to the kinetic equation (3). We discard all terms which depend on gradients, since we are interested in the uniform precession. We seek the solution of the kinetic equation (3) in the form

$$\delta\vec{\nu}_k=\frac{\partial\varphi}{\partial E_k}\delta\mathbf{E}+\delta\vec{\mu}_k,\quad \omega=\omega_L+\delta\omega(|\delta\vec{\mu}|),$$

where $\delta\omega(|\delta\vec{\mu}|)$ is the correction, which depends on $|\delta\vec{\mu}|$, to the Larmor precession frequency. We know the exact nonlinear solution of the kinetic equation

$$\delta\vec{\nu}_k=\frac{\partial\varphi}{\partial E_k}\delta\mathbf{E},\quad \omega=\omega_L, \tag{6}$$

which is simply the Larmor precession. Moreover, as shown in Ref. 4, there exists a simple relation between the derivatives of the order parameter and the magnetization of the system:

$$\mathbf{S}=\chi(-\mathbf{X}), \tag{7}$$

where χ is the susceptibility of ^{3}He-B. The vector $\delta\mathbf{E}$ can be expressed in terms of $\mathbf{X}$,[3,4] and the latter vector, in turn, can be expressed in terms of $\mathbf{S}$:

$$\delta\mathbf{E}=\frac{\xi_k}{E_k}\mathbf{X}+\left(1-\frac{\xi_k}{E_k}\right)\mathbf{d}(\mathbf{dX})=\frac{\xi_k}{E_k}\frac{\mathbf{S}}{\chi}+\left(1-\frac{\xi_k}{E_k}\right)\mathbf{d}\left(\mathbf{d}\frac{\mathbf{S}}{\chi}\right). \tag{8}$$

Linearizing Eq. (3) with respect to the solution (6), we obtain for small $\delta\vec{\mu}_k$ and $\delta\omega$ the equation

$$\left(i\omega+\frac{1}{\tau}\right)\delta\vec{\mu}_k+\delta\vec{\mu}_k\times\left\{\frac{\xi_k}{E_k}\frac{\mathbf{S}}{\chi}+\left(1-\frac{\xi_k}{E_k}\right)\mathbf{d}\left(\mathbf{d}\frac{\mathbf{S}}{\chi}\right)\right\} \tag{9}$$

$$+\delta\omega\varphi_k'\left\{\frac{\xi_k}{E_k}\frac{\mathbf{s}}{\chi}+\left(1-\frac{\xi_k}{E_k}\right)\mathbf{d}\left(\mathbf{d}\frac{\mathbf{s}}{\chi}\right)\right\}=0.$$

Here the vector $\mathbf{s}$ is equal in magnitude to the vector $\mathbf{S}$ and lies in a plane perpendicular to $\mathbf{S}$. Equation (9) expresses the obvious physical result that the nonequilibrium correction $\delta\vec{\mu}$ appears only when the precession frequency deviates from the Larmor frequency. Equation (9) can be written in both the Larmor coordinate system and the proper coordinate system, since the transformation from one to the other reduces to adding a term of order $\delta\omega|\delta\vec{\mu}|$, which can clearly be ignored. The proper coordinate system, where the vectors **S** and **s** are stationary, is now more useful to us. From the solution of the hydrodynamic equations (5) we have the following expression for $\delta\omega$ in our case:

$$\delta\omega=\frac{16}{15}\frac{\Omega^2}{\omega^2}\left(\frac{1}{4}+\cos\beta\right).$$

If we would take into account the last term in Eq. (5), we would obtain a term on the order of $(\delta\beta)^2$, which would exceed the accuracy of our analysis. After a $u-v$ transformation, the magnetization of the nonequilibrium part of the quasiparticles will be

$$\delta\mathbf{S}=\sum_k \delta\mathbf{S}_k=\sum_k \left\{\frac{\xi_k}{E_k}\delta\vec{\mu}_k+\left(1-\frac{\xi_k}{E_k}\right)\mathbf{d}(\mathbf{d}\delta\vec{\mu}_k)\right\}. \tag{10}$$

Substituting into this expression the solution of the kinetic equation (9), using Eq. (5), and taking into acount that $|\mathbf{S}|=|\mathbf{s}|=\omega_L$, we obtain the following expression for the modulus of the nonequilibrium magnetization:

$$\delta S_k=\frac{16}{15}\varphi_k'\tau\frac{\Omega^2}{\chi}\left(\frac{1}{4}+\cos\beta\right)$$
$$\times\frac{(i\omega\tau+1)^2\left[\frac{\xi_k^2}{E_k^2}+\frac{\Delta^2}{E_k^2}d_z^2\right]-\frac{\xi_k^2}{E_k^2}(i\omega\tau+1)\omega\tau+\frac{\Delta^4}{E_k^2}d_z^4(\omega\tau)^2}{(i\omega\tau+1)\left[(i\omega\tau+1)^2+\frac{\xi_k^2}{E_k^2}(\omega\tau)^2+\frac{\Delta^2}{E_k^2}d_z^2(\omega\tau)^2\right]}. \tag{11}$$

The dissipative function is given by the formula[2,4]

$$W=\delta\mathbf{S}^2/\tau. \tag{12}$$

The total energy of the system is given by expression (4.14) from Ref. 4. The Larmor term ($\mathbf{S}\omega_L\sim\omega_L^2\delta\beta$) and all subsequent terms in expression (4.14) from Ref. 4 make a contribution linear in $\delta\beta$. However, their magnitudes are of the order of ($\mathbf{S}\delta\mathbf{S}\sim\omega_L(\Omega^2/\omega_L)\delta\beta\sim\Omega^2\delta\beta$), which is small compared to the Larmor frequency. We can therefore write

$$E=\omega_L S_\zeta\ \sin\ \beta=E_0+\sqrt{\frac{15}{16}}\omega_L^2\delta\beta. \tag{13}$$

Using the expression

$$(d/dt)E=-W, \tag{14}$$

we thus obtain the following expression for the relaxation rate:

$$\dot{\delta\beta}=\left(\frac{15}{16}\right)^{1/2}\frac{\tau}{\chi}\frac{\Omega^4}{\omega_L^2}F(\omega\tau), \tag{15}$$

where

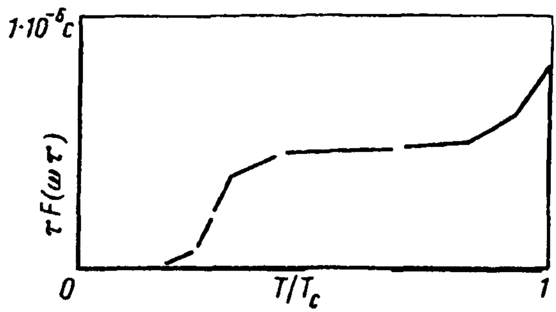

FIG. 1.

$$F(\omega,\tau)=\sum_k \mathrm{Re}^2\varphi_k' \frac{(i\omega\tau+1)^2\left[\frac{\xi_k^2}{E_k^2}+\frac{\Delta^2}{E_k^2}d_z^2\right]-\frac{\xi_k^2}{E_k^2}(i\omega\tau+1)\omega\tau+\frac{\Delta^4}{E_k^2}d_z^4(\omega\tau)^2}{(i\omega\tau+1)\left[(i\omega\tau+1)^2+\frac{\xi_k^2}{E_k^2}(\omega\tau)^2+\frac{\Delta^2}{E_k^2}d_z^2(\omega\tau)^2\right]},$$

$$\tau^{-1}=\frac{\pi^2}{6}\frac{\nu_0}{\hbar}f(T)\left[\frac{\hbar}{2\pi}\nu_0^2\langle W_S\rangle\right]=\tau_0^{-1}(T_c)\frac{f(T)}{f(T_c)},$$

where $f(T)$ is Yosida's function.[5] A plot of $\tau F(\omega\tau)$ is shown in Fig. 1. This is the main result of our study. We employed the values $\omega_L=2\pi\cdot 460$ kHz and $\tau_0(T_c)T_c^2=0.3$ μs·mK^2. As the temperature is lowered, in the hydrodynamic region first we see a plateau, which stems from the fact that $\tau\sim\exp(\Delta/T)$, $\chi_q\sim\exp(-\Delta/T)$, and the exponential in the numerator in Eq. (15) cancels out. Next, a further lowering of the temperature brings into play the function $F(\omega\tau)$, which leads to the dependence $F(\omega\tau)\sim(\omega_L\tau)^{-2}$, and the relaxation rate decreases exponentially. In agreement with the preceding studies,[1,2] in hydrodynamics we have for the Brinkman–Smith relaxation mode $\tau_{\mathrm{eff}}\sim(\omega_L^2/\tau\Omega^4)$ and in the collisionless region we have $\tau_{\mathrm{eff}}\sim(\tau\omega_L^4/\Omega^4)$.

It is a pleasure to thank A. F. Andreev and I. A. Fomin for helpful discussions, and F. S. Kopylov for assistance in the numerical calculations.

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Translated by M. E. Alferieff