Dynamics of the temperature of a recombining ensemble of fermions

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It is shown that the temperature of a recombining, statistically degenerate Fermi ensemble increases. © *1995 American Institute of Physics.*

Radiative recombination of particles is encountered in a number of problems. Examples of such processes are interband recombination of electrons and holes in semiconductors or annihilation in a system consisting of particles and antiparticles. In this connection, there arises the question of the behavior of the temperature of the recombining "annihilating" ensemble of particles. At first glance, one would think that the temperature of the ensemble should decrease as photons are emitted.[1)] However, it can be shown that the answer to this question is not so trivial and requires a more careful analysis: One must keep in mind the fact that in the process of recombination not only are photons emitted, but the density of the particles in the ensemble decreases.

In any sufficiently complicated system recombination is accompanied by other processes. In addition, recombination itself can be a complex process. However, since we desire to understand the role of the decrease in the particle density, we shall restrict the analysis to the use of a very simple model. We shall investigate an isolated ensemble of Fermi particles, from which particles "vanish" at a rate that depends on their energy. The energy of each particle, $E=E_g+\epsilon$, consists of the self-energy E_g and the energy ϵ of the thermal motion. We assume that the thermal energy of a particle is much smaller than its self-energy, so that in the calculations performed below we can employ a series expansion in powers of ϵ/E_g.

It is obvious that two equations are required to describe the process under consideration: one equation for the particle concentration and another equation for the energy of the ensemble. We shall employ the following equation for the particle concentration n:

$$\frac{dn(\mu,T)}{dt}=-\int_0^\infty w(E)\rho(\epsilon)f(\epsilon,\mu,T)d\epsilon;\ \ n(\mu,T)=\int_0^\infty \rho(\epsilon)f(\epsilon,\mu,T)d\epsilon, \qquad (1)$$

where $w(E)$ is the probability of spontaneous recombination of particles with energy E, $f(\epsilon,\mu,T)$ is the distribution function of the particles, T is the temperature, μ is the Fermi potential of the ensemble, and $\rho(\epsilon)$ is the density of the energy states. Introducing the temperature of the recombining ensemble, we assume that a thermal equilibrium is established in the ensemble much more quickly than the particles recombine.

The equation for the energy density $U(\mu,T)$ of the ensemble can be written from the law of conservation of energy. Each recombining particle carries off energy E. Therefore,

$$\frac{dU(\mu,T)}{dt}=-\int_0^\infty w(E)(E_g+\epsilon)\rho(\epsilon)f(\epsilon,\mu,T)d\epsilon;$$

$$U(\mu,T)=\int_0^\infty (E_g+\epsilon)\rho(\epsilon)f(\epsilon,T,\mu)d\epsilon. \tag{2}$$

Now we can write an equation for the change in the temperature of the ensemble; this equation will ultimately answer the question posed at the beginning of this paper. Using the fact that

$$\frac{dU(\mu,T)}{dt}=\frac{\partial U}{\partial n}\frac{dn}{dt}+\frac{\partial U}{\partial T}\frac{dT}{dt},$$

we can rewrite Eq. (2) in the form

$$\frac{dT}{dt}=\left(\frac{\partial U}{\partial T}\right)^{-1}\int_0^\infty w(E)\left(\frac{\partial U}{\partial n}-E_g-\epsilon\right)\rho(\epsilon)f(\epsilon,\mu,T)d\epsilon. \tag{3}$$

To expand Eq. (3), we must find an explicit expression for $U(\mu,T)$ and $n(\mu,T)$ in terms of the Fermi potential and the temperature, and we must assign the function $w(E)$. In the case where the energy ϵ is a quadratic function of the momentum we have $\rho(\epsilon)=\epsilon^{1/2}$. We assume that the state of the particles is statistically degenerate, and that the distribution function is the Fermi function. Then $U(\mu,T)$ and $n(\mu,T)$ can be calculated by the method recommended in Ref. 1. Up to second-order terms in the temperature we obtain

$$n(\mu,T)=\left(\frac{\mu-E_g}{\alpha}\right)^{3/2}\left[1+\frac{1}{2}\left(\frac{\pi kT}{2(\mu-E_g)}\right)^2\right],\quad \alpha=\left(\frac{6\pi^2}{g}\right)^{2/3}\frac{\hbar^2}{2m}, \tag{4}$$

$$U(\mu,T)=n\left\{E_g+\frac{3}{5}(\mu-E_g)\left[1+2\left(\frac{\pi kT}{2(\mu-E_g)}\right)^2\right]\right\}, \tag{5}$$

where g is the statistical weight, and m is the particle mass. It is obvious that in order for Eqs. (4) and (5) to be valid, the quantity $kT/(\mu-E_g)$ must be small.

Next, we assume that

$$w(E)=\frac{1}{\tau_s}\left(\frac{E}{E_g}\right)^q, \tag{6}$$

where τ_s does not depend on E. This assumption is based on the fact that the probability for spontaneous radiative recombination is proportional to the squared matrix element of the dipole moment and the cubed frequency of the emitted photon.[2] We therefore have $q=3$ if the matrix element of the dipole moment is essentially independent of the frequency. In general, $q\neq 3$.

Relations (4)–(6) make it possible to expand Eq. (3). In its general form this equation is complicated. We shall write it for a temperature close to zero, ignoring terms of order $(\mu-E_g)/E_g$ and higher:

$$\frac{d\theta^2}{dt}=\frac{1}{\tau_s}(\mu-E_g)^2;\quad \theta=\frac{\pi}{2}kT. \tag{7}$$

It follows from this equation that the time derivative of the temperature is strongly positive. A recombination thus leads to heating of a degenerate Fermi ensemble.

We now consider a nondegenerate (Boltzmann) ensemble, where

$$U=n\left(E_g+\frac{3}{2}kT\right). \tag{8}$$

In this case Eq. (3) (up to terms of higher order in kT/E_g) reduces to the equation

$$\frac{d}{dt}(kT)=-q\frac{(kT)^2}{E_g}. \tag{9}$$

We see that the change in temperature of a nondegenerate ensemble depends strongly on the sign of q. The temperature decreases if $q>0$, increases if $q<0$, and remains unchanged if $q=0$. This behavior has a simple interpretation. For $q>0$ the hotter particles are removed at a higher rate than the colder particles, so that the ensemble is cooled. For $q<0$ the situation is reversed. At $q=0$, the rate of removal of the particles does not depend on the energy of the particles and the temperature of the ensemble remains unchanged. As we can see, in the degenerate case a recombining ensemble of fermions is heated, irrespective of the value of q. The reason for this behavior of an ensemble is that a degenerate Fermi ensemble consists mainly of "cold" particles, whose energy is lower than the Fermi potential.

Equations (7) and (9) were derived in the limiting cases of degenerate and Boltzmann statistics. In general, the dynamical behavior of the temperature is expected to be complex. Only a numerical analysis, a subject for a special study elsewhere, can be carried out in this case.

When a semiconductor is exposed to a short (picosecond or femtosecond) radiation pulse, such that the energy of the photons exceeds the band gap, an ensemble of carriers whose temperature may be different from the lattice temperature is produced. Recombination heating (cooling) plays a large role in the process establishing a stationary temperature of the ensemble. It must therefore be taken into account when analyzing the nonequilibrium temperature dynamics in different semiconductor devices.[3,4] In particular, it is important to take this effect into account in order to understand the recovery of the gain in a semiconductor amplifier after a short radiation pulse has passed through it.[5]

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[1)]Our personal experience in scientific discussions shows that this is the most commonly held viewpoint.

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