The interaction of dyons in the SU(2) gauge theory

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Dyonic classical solutions of SU(2) gluodynamics are considered, and their interaction and contribution to the Wilson loop are investigated both analytically and numerically © *1995 American Institute of Physics.*

1. INTRODUCTION

The QCD vacuum is known to possess properties of confinement and chiral symmetry breaking (CSB). The first is characterized, for example, by the area law for the Wilson loop,[1] while the second is associated with nonzero values of chiral quark condensate. Both properties have been found in lattice calculations [2] and are due to nonperturbative fluctuations of the gluonic field in the vacuum.

At present there is no model of the QCD vacuum with properties of confinement and CSB, based directly on the QCD Lagrangian. The most elaborated model is the instanton gas or liquid model[3] which ensures CSB but lacks confinement.[4] Thus there is an urgent need to look for a more realistic model of QCD vacuum which possesses both basic properties. Confinement is associated widely with monopole-like degrees of freedom,[5] which may be of a purely quantum or quasiclassical character. In the latter case one should look for classical solutions of the Yang–Mills equations of monopole-like form. These solutions have been known for a long time.[6,7] They have both color-electric and color-magnetic fields and we shall therefore call them dyons.[8]

The dilute dyonic gas has been suggested some time ago as a model of QCD vacuum,[9] and some simple estimates of the Wilson loop have been done for dyons of finite time extent and exhibiting nonzero string tension.

Recently, interest in dyons has revived. In particular, lattice studies of classical and quantum fields of a dyon have been done, and a qualitative quasi-Abelian picture of confinement due to dyons has been suggested.[10]

The purpose of this letter is to make a first step in a more general consideration of dyonic gas. This step is connected to the problem of dyon interaction.

This letter is organized as follows. In Sec. 2 we define a single dyon solution in different gauges. In Sec. 3 we calculate Wilson loop for a single dyon and dyon–antidyon pair and demonstrate the phenomenon of non-Abelian screening in the latter case. In Secs. 4 and 5 we consider a dyonic gas model and the problem of dyon interaction. Section 6 contains concluding remarks and prospects for future investigations.

2. DYONIC SOLUTION

One way to present the (anti)dyonic solution is to consider the case when an infinite number of (anti)-instantons (in the 't Hooft ansatz[11]) are equally spaced along a straight line (time axis):

$$A_i^a=\epsilon_{iab}\partial_b \ln \rho \mp \delta_{ia}\partial_0 \ln \rho, \quad A_0^a=\pm\partial_a \ln \rho,$$

$$\rho(r,t)=\sum_{n=-\infty}^{\infty} \frac{1}{r^2+(t-2\pi n)^2}=\frac{1}{2}\frac{\sinh r}{r}\frac{1}{\cosh r-\cos t} \tag{1}$$

(we are using units in which $\gamma=2\pi/b=1$, where b is the distance between instantons). These solutions are (anti)self-dual. The potentials and fields look like

$$A_i^a=-\epsilon_{iab}n_b f(r,t)\pm\delta_i^a g(r,t), \quad A_0^a=\mp n^a f(r,t),$$

$$f(r,t)=\frac{1}{r}+\frac{\sinh r}{\cosh r-\cos t}-\frac{\cosh r}{\sinh r}, \quad g(r,t)=\frac{\sin t}{\cosh r-\cos t}, \tag{2}$$

$$\pm E_i^a=H_i^a=n_i n_a\left(f'-\frac{f}{r}+f^2\right)-\delta_i^a\left(f'+\frac{f}{r}-g^2\right)\mp\epsilon_{iab}n_b(g'+fg).$$

For large $r (r\gg 1)$ the potentials A_i^a, A_0^a fall off as $1/r$ and the fields E_i^a, H_i^a fall off as $1/r^2$. The total action of a dyon is proportional to its time extent:

$$S=\frac{1}{2g^2}\int d^3r\int_0^T dt[(E_i^a)^2+(H_i^a)^2]=\frac{4\pi}{g^2}T. \tag{3}$$

In the future we will refer to this dyonic solution as the "dyon in the 't Hooft gauge". In another gauge ("dyon in the Rossi gauge") the dyonic solution can be made static.[7] Namely, making the rotation

$$U=\exp\left(-i\frac{\tau_i}{2}n_i\Theta\right), \quad \Theta=\tan^{-1}\left(\frac{\sin t \sinh r}{\cosh r \cos t-1}\right),$$

we find that potentials and fields are

$$A_i^a=\epsilon_{aib}n_b\tilde{f}, \ \tilde{f}=\left(\frac{1}{r}-\frac{1}{\sinh r}\right), \quad A_0^a=\mp n^a\tilde{g}, \ \tilde{g}=\left(\frac{1}{r}-\frac{\cosh r}{\sinh r}\right), \tag{4}$$

$$\pm E_i^a=H_i^a=n_i n_a\left(\tilde{f}'-\frac{\tilde{f}}{r}+\tilde{f}^2\right)-\delta_{ia}\left(\tilde{f}'+\frac{\tilde{f}'}{r}\right).$$

This means that all gauge-invariant quantities are not simply time-periodic but also static. The remarkable feature of the dyonic solution in the Rossi gauge is that it looks like the t'Hooft–Polyakov monopole in the Bogomol'nyi–Prasad–Sommerfield limit if the zero component of the dyonic potential is replaced by the scalar field of the monopole. Thus, the (anti)dyon, like the monopole, carries magnetic charge. Unlike the monopole, it carries electric charge, its electric charge being equal to (minus) plus the magnetic charge.

3. WILSON LOOP FOR ISOLATED DYON

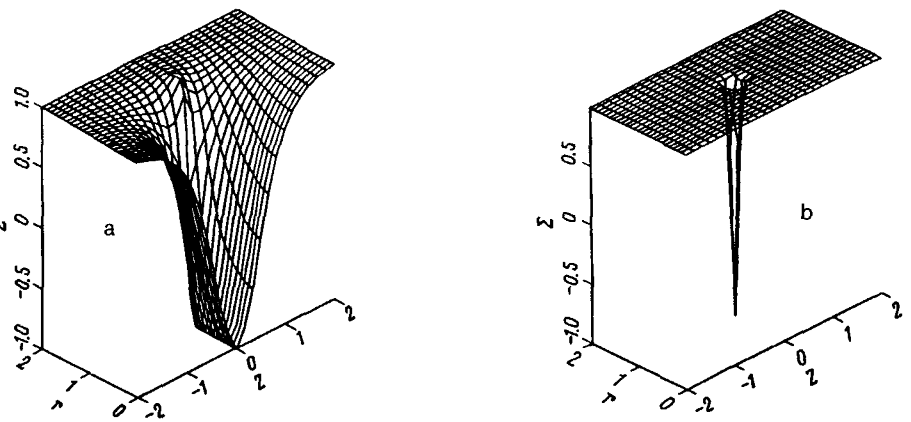

FIG. 1. The Wilson loop integral for an isolated dyon (a) and for a pair of dyons (b). The loop lies in XY plane and has radius equal to 10. The result is presented as a function of the cylindric coordinates (r,z) of the dyon (a) or pair of dyons (b), measured in units of the radius of the loop.

We will consider a spatial loop of circular form. When the distances from the dyon to all points of the loop are much larger than the size of the dyon (the size of the dyon equals $1/\gamma=1$ in our notation), the calculations are very simple. At large distances from the dyon the fields of the dyon can be made purely Abelian by an appropriate gauge rotation. The integral $i\oint A_i dx_i$ is then a measure of the flux of magnetic field through the loop and is equal to Ω (Ω is the solid angle subtended by the loop from the point where the dyon is located). We thus have the answer

$$W=\frac{1}{2}\mathrm{Tr}\; P\; \exp\left(i\oint A_i^a dx_i \frac{\tau_a}{2}\right)=\frac{1}{2}\mathrm{Tr}\;\exp\left(i\Omega\frac{\tau_3}{2}\right). \tag{5}$$

When the distance from the dyon to some points of the loop is smaller than the size of the dyon, the finite-size effects are important and the result (5) is smoothly modified. Figure 1(a) shows the results of numerical calculations for a loop of radius equal to 10.

An interesting feature of non-Abelian screening can be seen for two dyons or dyon–antidyon pair of zero separation. In the Rossi gauge, for example, the spatial potential is then doubled. The magnetic field of such a potential has no Coulomb part and falls off exponentially. The Wilson loop is trivial anywhere except for the boundary of the loop [see Fig. 1(b)].

4. DYONIC GAS MODEL OF THE QCD VACUUM

We propose a superposition ansatz for the dyonic gas, similar to that for the instantonic gas,[3] namely

$$A_\mu(x)=\sum_{i=1}^{N} A_\mu^{(i)}(x), \tag{6}$$

where the individual (anti)dyon field $A_\mu^{(i)}$ depends on the position $R^{(i)}$, $O(4)$ orientation $\omega^{(i)}$, and SU(2) (color) orientation $\Omega^{(i)}$:

$$A_{\mu}^{(i)}(x)=\Omega^{(i)+}A_{\mu}(x,R^{(i)},\omega^{(i)})\Omega^{(i)}. \qquad (7)$$

The resulting potential (6) is no longer the solution of the Yang–Mills equations; it can only be close to the solution for large dyon separations (gas approximation). Moreover, the resulting fields depend on the gauge we are using for the (anti)dyon solution (the sum of potentials in one gauge is not connected to that in another gauge by any gauge transformation). To keep the diluteness property of the dyonic system (6) one must therefore choose the gauge of individual solution $A_{\mu}^{(i)}$ in such a way as to get a minimal attraction. The interaction at large distances depends crucially on the asymptotic behavior of $A_{\mu}^{(i)}$, and the latter depends on the class of gauge, the different classes being connected by singular gauge transformations. In particular, in the Rossi gauge the topological charge comes from the point at infinity, while in the 't Hooft gauge is comes from the points $r=0$, $t=0$, 2π, ... The interaction within the given class depends also on the specific gauge chosen, but that dependence is largely taken into account by the orientation matrices $\Omega^{(i)}$ in (7).

In what follows a first, pilot study is reported of the dyon–dyon and dyon–antidyon interaction in two representative classes of gauges: the 't Hooft and the Rossi gauges.

5. DYON–DYON AND DYON–ANTIDYON INTERACTION

We will describe two dyons or a dyon–antidyon pair by the sum of their potentials. One should choose an appropriate gauge for the single (anti)dyon solution in such a way that the interaction in the gauge at large distances is small. The Rossi gauge[7] is not appropriate for this purpose: the interaction energy diverges in all cases except for the case of a dyon–antidyon pair in which the dyons are of equal size and have the same $O(4)$ orientation and the same SU(2) orientation. Even in this special case the interaction energy grows linearly with the separation of the dyon and antidyon. Another possibility is the 't Hooft ansatz (2). As we will see later, the interaction of dyons for large distances is repulsive in this ansatz, more repulsive than in other gauges (in the Abelian gauge, for example). We regard this fact as a self-consistent motivation for considering the dyons as a gas system. So far, this is the only reason why we have chosen the 't Hooft ansatz for the dyon solution. Here we have two peculiarities. Because the potentials of the dyons depend on time (they are periodic in time and the period is equal to 2π), the interaction energy is also time-dependent. Therefore, we will average the interaction energy over the period. The interaction energy also depends on the relative shift of the dyons in the time direction (relative phase). Thus, the interaction potential is a function of the separation r, relative phase φ, relative SU(2) orientation R, and relative velocity v. Here we will consider only the case when $R=I$ and $v=0$. The dependence on R and v will be the subject of a separate investigation. The functions $V_{dd}(r,\varphi)$ and $V_{d\bar{d}}(r,\varphi)$ were calculated numerically. They are plotted in Fig. 2a,b. For $r\rightarrow 0$, $\varphi\rightarrow 0$ the functions V_{dd} and $V_{d\bar{d}}$ are logarithmically divergent. For large $r(r\approx 10)V_{d\bar{d}}$ is approximately equal to $0.5/r$ and $V_{dd}\approx 20/r$, although the $1/r$ dependence is not clearly seen. We can compare this result for V_{dd} and $V_{d\bar{d}}$ with the results that we should expect in the Abelian gauge, where the dyon–antidyon interaction is zero (the magnetic charges have the same sign and the electric charges are of opposite sign) and the dyon–dyon interaction is $2/r$ (both electric and magnetic charges are of the same sign).

The absolute minimum of V_{dd} is zero ($r\rightarrow\infty$) and the absolute minimum of $V_{d\bar{d}}$ is

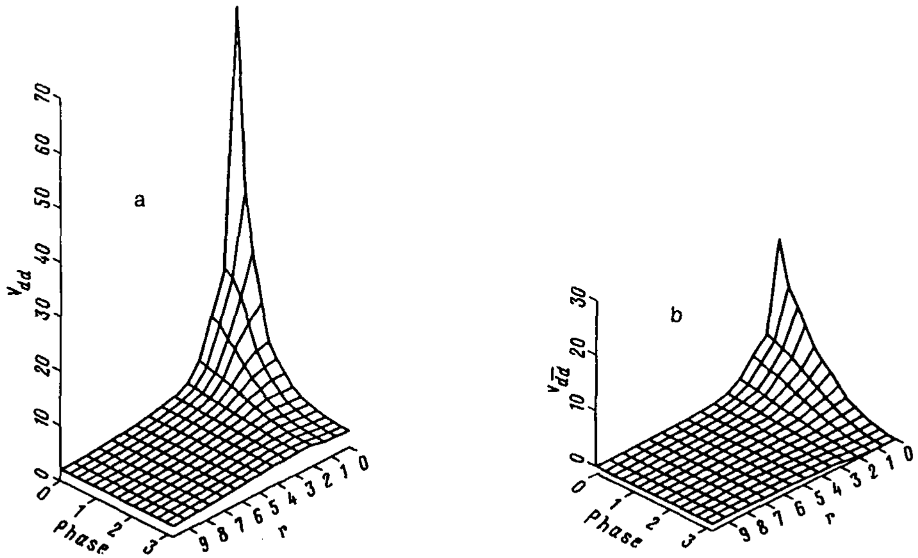

FIG. 2. The interaction energy for two dyons (a) and dyon–antidyon pair (b) as a function of their separation and relative phase. The interaction energy is measured in units of dyon mass $(4\pi/g^2)$.

-1.3 $(\varphi=\pi, r=0)$. So a dyon and antidyon attract strongly at small distances when the relative phase is π. The dyon–dyon interaction is also attractive in the region $\varphi=\pi$, $r=0$ but, in contrast to the dyon–antidyon case, the interaction energy here is positive.

6. CONCLUDING REMARKS

We have calculated the interaction energy for the dyon–dyon and dyon–antidyon pairs. In the 't Hooft ansatz for the dyonic solution the calculations show that this energy is minimal when the dyons have opposite phase. In this case, the dependence of the energy on the separation between dyons is different in the dd and $d\bar{d}$ systems. For large r one has $V_{dd} \gg V_{d\bar{d}}$ (a factor ≈ 40 for $r=10$). One should compare our results for V_{dd} with those obtained in the ansatz of Manton,[12] where V_{dd} vanishes. This latter property holds for an exact two-dyon solution of general form.[13]

As a next step we plan to study the dependence of V_{dd} and $V_{d\bar{d}}$ on the relative orientation in color space and relative motion of dyons (we use the word motion in analogy with the interaction of particles in Minkowski space). This will be the basis for considering a gas of dyons and calculating its contribution to the Wilson loop.

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[1] K. Wilson, Phys. Rev. D **10**, 2445 (1974).
[2] F. Karsch, in *QCD: 20 Years Later*, P. M. Zerwas and H. A. Kastrup, eds. (World Scientific, Singapore, 1993).
[3] C. G. Callan, R. Dashen, and D. J. Gross, Phys. Rev. D **17**, 2717 (1978); E. V. Shuryak, Nucl. Phys. B **203**, 93, 116, 140 (1982); D. I. Diakonov and V. Yu. Petrov, Nucl. Phys. B **245**, 259 (1984).
[4] C. G. Callan, R. Dashen, and D. J. Gross, Phys. Lett. B **66**, 375 (1977); D. I. Diakonov, V. Yu. Petrov, and P.

V. Pobylitsa, Phys. Lett. B **226**, 471 (1989); E. V. Shuryak, Nucl. Phys. B **328**, 85, 102 (1989).
[5]G. 't Hooft, in High Energy Physics, *Proc. EPS International Conference*, edited by A. Zichichi (Editrici Compositori, 1976); S. Mandelstam, Phys. Rev. **23**, 245 (1978).
[6]M. K. Prasad and C. M. Sommerfield, Phys. Rev. Lett. **35**, 760 (1975); R. Jackiw, Rev. Mod. Phys. **49**, 681 (1977); E. Corrigan and D. Fairlie, Phys. Lett. B **67**, 69 (1977); B. Harrington and H. Shepard, Phys. Rev. D **17**, 2122 (1978).
[7]P. Rossi, Phys. Rep. **86**, 317 (1982).
[8]B. Julia and A. Zee, Phys. Rev. B **3**, 3918 (1971).
[9]Yu. A. Simonov, Sov. J. Yad. Fiz. **43**, 557 (1985).
[10]J. Smit and A. van der Sijs, Nucl. Phys. B **355**, 603 (1991).
[11]R. Jackiw, C. Nohl, and C. Rebbi, Phys. Rev. D **15**, 1642 (1977).
[12]N. S. Manton, Nucl. Phys. B **126**, 525 (1977); *ibid.* **150**, 397 (1979).
[13]M. F. Atiyah, N. J. Hitchin, V. G. Drinfeld, and Yu. I. Manin, Phys. Lett. A **65**, 185 (1978).

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