Charged skyrmions in a system of 2D spin excitons in the Hartree–Fock approximation

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The existence of topological defects, known as skyrmions, within the spin excitons energy band of a 2D electron gas under a strong magnetic field at filling factor $\nu=1$ is investigated within the Hartree–Fock approximation. Using the linear momentum representation, it is shown that the inhomogeneity created in the system by a charged skyrmion can be described by a nonuniform rotation of the spin density operators in a condensate of spin excitons.

Chiral fields, namely fields which take on values in a nonlinear space, can have some nontrivial topological invariants.[1] The existence of such invariants in a physical system can lead to the creation of unusual topological defects. In particular, Belavin and Polyakov[2] have studied nonuniform metastable states of an isotropic 2D ferromagnet, i.e., a three-component order parameter in a 2D coordinate space, which is known as the nonlinear $O(3)$ model [see also Ref. 3]. To avoid any misunderstanding we shall use the term skyrmions (or antiskyrmions) to describe such states for any positive (or negative) degree of map.

The possibility of observing states of this sort experimentally in real magnetic systems has recently been raised in connection with a sensitive nuclear magnetic resonance experiment[4] in which the local spin polarization of a 2D electron system was directly measured. Theoretically such a system, at appropriate filling factors, can exhibit spin excitations with topological characteristics; at filling factor $\nu=1$, for example, where the ground state is completely spin polarized, they have been shown to be skyrmions.[5–8]

Fertig *et al.*[9] have developed a Hartree–Fock approach to study what they termed "charged spin-texture excitations"—the appropriate generalization of skyrmions for nonzero Zeeman splitting. They have found that their net spin is always considerably larger than 1/2. This important prediction seems to be confirmed by the experiment reported in Ref. 4.

In this paper we present a Hartree–Fock description of charged skyrmions in the absence of Zeeman splitting, which clearly shows the connection between these unusual point defects and the more common spin excitations (e.g., spin-excitons[10]).

We examine the properties of a system of interacting electrons confined in a 2D space under a strong magnetic field. In this system the states of a free electron on a given Landau level is characterized, in the Landau gauge for the vector potential, by a linear momentum $p_y(\equiv p)$ and a projection of the spin. The operators $\hat{a}_p(\hat{b}_p)$ annihilate electrons with momentum p and spin up (down).

In the absence of Zeeman splitting the Hamiltonian of the system is written as

$$\hat{H}=\frac{1}{2}\sum_{\mathbf{q},p_1,p_2}\tilde{V}(q)e^{iq_x(p_2'-p_1)}[\hat{a}^\dagger_{p_1}\hat{a}^\dagger_{p_2}\hat{a}_{p_2'},\hat{a}_{p_1'}+(\hat{a}\rightarrow\hat{b})+2\hat{a}^\dagger_{p_1}\hat{b}^\dagger_{p_2}\hat{b}_{p_2'},\hat{a}_{p_1'}], \tag{1}$$

where $p_1'=p_1-q_y, p_2'=p_2+q_y$, and the effective potential is $\hat{V}(q)=e^{-q^{2/2}}V(q)$ for electrons in the lowest Landau level. Here $V(q)$ is the Fourier component of the interaction potential. Note that all lengths are measured here in units of the magnetic length l_H.

In the Hartree–Fock approximation the mean value of the Hamiltonian (1) can be written as

$$\langle\hat{H}\rangle=\frac{1}{2}\sum_{\mathbf{q},p_1,p_2}e^{iq_x(p_2'-p_1)}\{[\tilde{V}(q)-2\pi E(q)][\langle\hat{a}^\dagger_{p_1}\hat{a}_{p_1'}\rangle\langle\hat{a}^\dagger_{p_2}\hat{a}_{p_2'}\rangle+(\hat{a}\rightarrow\hat{b})]+\tilde{V}(q)$$
$$\times\langle\hat{a}^\dagger_{p_1}\hat{a}_{p_1'}\rangle\langle\hat{b}^\dagger_{p_2}\hat{b}_{p_2'}\rangle-2\pi E(q)\langle\hat{a}^\dagger_{p_1}\hat{b}_{p_1'}\rangle\langle\hat{b}^\dagger_{p_2}\hat{a}_{p_2'}\rangle. \tag{2}$$

In Eq. (2) the energy

$$E(q)\equiv\int\frac{d^2p}{(2\pi)^2}\tilde{V}(p)e^{i\vec{p}\cdot\vec{q}}$$

describes the dispersion law for spin excitons in the system.

Our approach to the problem of skyrmions in a two-sublevel system in the absence of Zeeman splitting is closely related to the method of isospin operators used in the study of electrons in a silicon inversion layer, which have two degenerate valleys.[11] Following Refs. 11 and 12 we introduce isospin operators

$$\hat{S}_i(\mathbf{q})=\frac{1}{2}\sum_p e^{iq_x(p+q_y/2)}C^\dagger_p\sigma_iC_{p+q_y}, \tag{3}$$

where σ_i are the Pauli matrices, and $C_p\equiv(\hat{a}_p,\hat{b}_p)$.

An additional operator corresponds to a nonuniform density of particles

$$\hat{N}(q)\equiv\sum_p e^{iq_x(p+q_y/2)}(\hat{a}^\dagger_p\hat{a}_{p+q_y}+\hat{b}^\dagger_p\hat{b}_{p+q_y}). \tag{4}$$

In terms of the mean values of these operators, i.e.,

$$N(\mathbf{q})\equiv\langle\hat{N}(\mathbf{q})\rangle$$

and

$$\mathbf{S}(\mathbf{q})\equiv\langle\mathbf{S}(\mathbf{q})\rangle,$$

Eq. (2) takes the form

$$\langle\hat{H}\rangle=\sum_{\mathbf{q}}\left\{\frac{1}{2}[\tilde{V}(q)-\pi E(q)]N(\mathbf{q})\ N(-\mathbf{q})-2\pi E(q)\mathbf{S}(\mathbf{q})\cdot\mathbf{S}(-\mathbf{q})\right\}. \tag{5}$$

The fully polarized state of the system, which corresponds to a filling factor $\nu=1$, is $|\psi_0\rangle=\Pi_p\hat{a}_p^{\dagger}|0\rangle$.

The key element of our HF descriptions of skyrmions is the canonical transformation

$$\hat{A}_p=\sum_{p_1}(U_{p,p_1}\hat{a}_{p_1}+V_{p,p_1}\hat{b}_{p_1}) \tag{6}$$

with the unitarity conditions $\hat{U}\hat{U}^{\dagger}+\hat{V}\hat{V}^{\dagger}=1$, which converts the fully polarized state $|\psi_0\rangle$ into a new state $|\psi\rangle=\Pi_p\hat{A}_p^{\dagger}|0\rangle$.

In the new state: $\langle\psi|\hat{a}_p^{\dagger}\hat{a}_{p'}|\psi\rangle=(\hat{U}^{\dagger}\hat{U})_{p',p}$, $\langle\psi|\hat{b}_p^{\dagger}\hat{b}_{p'}|\psi\rangle=(\hat{V}^{\dagger}\hat{U})_{p',p}$, and $\langle\psi|\hat{a}_p^{\dagger}\hat{b}_{p'}|\psi\rangle=(\hat{V}^{\dagger}\hat{U})_{p',p}$.

To be explicit, for the skyrmion state under consideration here we define

$$\hat{U}=\cos\frac{\hat{\theta}}{2}e^{i\hat{\phi}/2},\quad \hat{V}=\sin\frac{\hat{\theta}}{2}e^{-i\hat{\phi}/2} \tag{7}$$

where the operators $\hat{\theta}$, and $\hat{\phi}$, which describe nonuniform rotation, should be projected on the lowest Landau level. Thus the corresponding matrix elements are:

$$(\hat{\theta})_{p,p'}=[\theta(\mathbf{r})]_{p,p'}=\sum_{\mathbf{q}}\theta(\mathbf{q})(e^{i\mathbf{q}\cdot\mathbf{r}})_{p,p'}\equiv\sum_{\mathbf{q}}\theta(\mathbf{q})(\hat{\rho}\mathbf{q})_{p,p'}, \tag{8}$$

where $\theta(\mathbf{q})$ is the Fourier transform of the angle $\theta(\mathbf{r})$ with respect to coordinates. A similar expression can be written for $(\hat{\phi})_{p,p'}$.

With the help of Eqs (3), (4), and (7) we obtain the following results:

$$S_z(\mathbf{q})=\frac{1}{4}\sum_p e^{iq_x(p+q_y/2)}(e^{-i\hat{\phi}/2}\cos\hat{\theta}e^{i\hat{\phi}/2}+e^{i\hat{\phi}/2}\cos\hat{\theta}e^{-i\hat{\phi}/2})_{p+q_y,p}, \tag{9}$$

$$S_+(\mathbf{q})=\frac{1}{2}\sum_p e^{iq_x(p+q_y/2)}(e^{i\hat{\phi}/2}\sin\hat{\theta}e^{i\hat{\phi}/2})_{p+q_y,p} \tag{10}$$

and

$$N(\mathbf{q})=\frac{1}{2\pi}\delta(\mathbf{q})+\delta N(\mathbf{q}) \tag{11}$$

with

$$\delta N(\mathbf{q})=\frac{1}{2}\sum_p e^{iq_x(p+q_y/2)}(e^{-i\hat{\phi}/2}\cos\hat{\theta}e^{i\hat{\phi}/2}-e^{i\hat{\phi}/2}\cos\hat{\theta}e^{-i\hat{\phi}/2})_{p+q_y,p}. \tag{12}$$

We are interested here in the case when the spatial dependence of the angles $\theta(\mathbf{r})$ and $\phi(\mathbf{r})$ is sufficiently smooth that the characteristic length scale for the coordinate dependence is much longer than the magnetic length. Our main goal is to express the energy of the system in the HF approximation [Eq. (2)] as a functional of the spin and particle densities, which includes terms up to second order in a gradient expansion.

Let us consider the correction $\delta N(\mathbf{q})$ [Eq. (12)] to the density of particles due to our nonuniform rotation of the spin density. Taking advantage of the smooth spatial behavior of the rotation angles, which results in a small commutator $[\hat{\phi},\hat{\theta}]$, and the approximate identity

$$e^A e^B \approx (1+[A,B]) e^B e^A, \tag{13}$$

which is valid for any two operators A and B of this type, we get after very simple calculations

$$\delta N(\mathbf{q}) \approx \frac{i}{2}\sum_p e^{iq_x(p+q_y/2)}([\hat{\phi},\hat{\theta}]\sin\hat{\theta})_{p+q_y,p}. \tag{14}$$

A similar expression can be derived from the spin density, i.e.,

$$S_z(\mathbf{q}) \approx \frac{1}{2}\sum_p e^{iq_x(p+q_y/2)}(\cos\hat{\theta})_{p+q_y,p}, \tag{15}$$

while for $S_x(S_y)$ we should replace $\cos\hat{\theta}$ in Eq. (15) with $\sin\hat{\theta}\cos\hat{\phi}$ $(\sin\hat{\theta}\sin\hat{\phi})$.

For the small values of q considered in our case:[13]

$$[\hat{\rho}\mathbf{q}_1,\hat{\rho}\mathbf{q}_2] \approx -i([\mathbf{q}_1\times\mathbf{q}_2]\cdot\hat{z})\hat{\rho}\mathbf{q}_1+\mathbf{q}_2, \tag{16}$$

where $\hat{z}$ is a unit vector along the field direction.

Our final result for the correction to the density of particles in coordinate representation is thus

$$\delta N(\mathbf{r}) \approx -\frac{1}{4\pi}\{[\nabla\phi(\mathbf{r})\times\nabla\theta(\mathbf{r})]\cdot\hat{z}\}\sin\theta(\mathbf{r}) = \frac{1}{4\pi}\left(\mathbf{n}\cdot\left[\frac{\partial\mathbf{n}}{\partial x}\times\frac{\partial\mathbf{n}}{\partial y}\right]\right), \tag{17}$$

where $\mathbf{n}(\mathbf{r})$ is a unit vector field defined by the rotation angles $\theta(\mathbf{r})$ and $\phi(\mathbf{r})$ as

$$\mathbf{n}(\mathbf{r}) \equiv (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta).$$

This vector is proportional to the mean value of the spin density operator, i.e.,

$$\mathbf{n}(\mathbf{r}) = 4\pi S(\mathbf{r}),$$

which has nonzero transverse (i.e., x and y) components only when the number of spin excitons in the system is *macroscopic*. It is also easy to check that the part of the energy associated with the spin density in Eqs. (9) and (10) can be written as

$$\delta E\{\mathbf{S}\} = \frac{1}{32\pi}E(0)\int [\nabla\cdot\mathbf{n}(\mathbf{r})]^2 d^2r, \tag{18}$$

so that the total HF energy can be written as a functional of the unit vector field $\mathbf{n}(\mathbf{r})$:

$$E_{HF} \approx -\frac{1}{8\pi}E(0)\int d^2r\left(\mathbf{n}\cdot\left[\frac{\partial\mathbf{n}}{\partial x}\times\frac{\partial\mathbf{n}}{\partial y}\right]\right) + \frac{1}{32\pi}E(0)\int [\nabla\cdot\mathbf{n}(\mathbf{r})]^2 d^2r. \tag{19}$$

Note that since the spatial dependence of angles $\theta(\mathbf{r})$, $\phi(\mathbf{r})$ is yet unspecified the vector field $\mathbf{n}(\mathbf{r})$ is absolutely arbitrary, except for the fact that its norm is unity, and so the energy functional in Eq. (19) is only a variational form.

The integral

$$Q=\frac{1}{4\pi}\int d^2r\left(\mathbf{n}\cdot\left[\frac{\partial\mathbf{n}}{\partial x}\times\frac{\partial\mathbf{n}}{\partial y}\right]\right) \tag{20}$$

is equal, by virtue of Eq. (17), to the total number of particles added to or removed from the system in going from the fully polarized ground state $|\psi_0\rangle$ to the new state $|\psi\rangle$. It is known to be topologically invariant,[1-3] that is invariant under any smooth variation $\delta\mathbf{n}(\mathbf{r})$, and its values can be only integer numbers.

The new state $|\psi\rangle$ is therefore a topological defect characterized by the degree of map Q; it is positive for a skyrmion and negative for an antiskyrmion. According to well known results,[2,3] the minimal energy of a skyrmion for any given degree of map Q corresponds to the condition

$$\int(\nabla\mathbf{n})^2d^2r=2\left|\int d^2r\left(\mathbf{n}\cdot\left[\frac{\partial\mathbf{n}}{\partial x}\times\frac{\partial\mathbf{n}}{\partial y}\right]\right)\right|. \tag{21}$$

Thus for a given degree of map Q the minimal energy of a skyrmion is

$$E_{sk}=\frac{1}{4}\left(\frac{e^2}{\kappa l_H}\right)\sqrt{\frac{\pi}{2}}(|Q|-2Q). \tag{22}$$

In the case $|Q|=1$ this result coincides with the obtained numerically by Fertig *et al.*[9]

It is very important to stress here that the spin-rotation transformation (7) is unitary and does not change the total number of electrons. Thus by going to the new state $|\psi\rangle$ from the fully polarized ground state $|\psi_0\rangle$ the total topological charge does not change either. This can be done only by creating these topological defects in pairs of well separated skyrmions and the corresponding antiskyrmions with equal and opposite charges. The total energy of such skyrmion–antiskyrmion pair, with degree of map $Q=1$, is exactly equal to one-half of the total energy required to create a well separated electron–hole pair (large spin exciton).

Since Zeeman spin splitting is completely neglected in our model, its $O(3)$ symmetry is fully preserved, and there is no definite length scale for skyrmions. In this limiting case the skyrmion energy [Eq. (22)] is independent of the skyrmion size, provided, of course, that it is much larger than the magnetic length l_H.

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