Current fluctuation in an ideally conducting ring

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It is shown that in the noninteracting-electron approximation an ideally conducting ring manifests nonergodicity. In equilibrium with zero average current the current-current correlation function does not decay with time. When the interaction with the reservoir is taken into account explicitly, ergodicity is restored and the decay time of the correlation function is determined by the broadening of the discrete levels. © *1995 American Institute of Physics.*

At low temperatures an ideal ring manifests an unusual property for normal conductors — in the presence of a finite magnetic flux Φ which passes through the ring, a nondecaying current (so-called persistent current) flows through it.[1-5] It turns out that the current fluctuations in such a ring are very unusual. In the present paper we shall study these fluctuations for an ideal quasi-one-dimensional conductor in equilibrium and with a zero flux.

The main feature of an ideal connected conductor is the existence of discrete current-carrying levels. Each level is doubly degenerate (in addition to the spin degeneracy) with respect to the direction of the current. In a quasi-one-dimensional ideal conductor the wave functions (WFs) in polar coordinates are

$$\Psi=\frac{1}{\sqrt{2\pi}}e^{i\varphi M}\chi(\rho,z),$$

where M is the angular momentum, and χ is the normalized wave function in the cross section of the ring.

The current operator does not mix states which carry currents of opposite signs if $|M'|=|M|$ but it mixes the states slightly if the condition $|M'-M|\ll M$ is satisfied. The total current through the cross section of the ring can therefore be written approximately as follows:

$$\hat{I}(t)=I_c\sum_{\sigma=\pm1;M',M>0}[a^{\dagger}_{M'\sigma}a_{M\sigma}-b^{\dagger}_{M'\sigma}b_{M\sigma}]\exp[-i(E_M-E_{M'})t/\hbar]. \tag{1}$$

Here $I_c=e\hbar M_F/2\pi R^2m$, $a^{\dagger}_M$ are the creation operators for states with angular momentum M, $b^{\dagger}_{-M}\equiv a^{\dagger}_M$, $E_M=\hbar^2M^2/2mR^2$, and R is the radius of the ring.

The spectral density of the current fluctuations consists of a set of delta functions:

$$S(\omega)=\sum_n f_n\delta(\omega-n\omega_0), \tag{2}$$

where $\omega_0 = \hbar M_F / mR^2$.

Fluctuations with frequencies $\omega < \omega_0$ are of special interest. To describe zero frequencies, we shall consider the time-independent contribution to the current operator

$$\hat{I} = \sum_{\sigma=\pm 1; M>0} [a^{\dagger}_{M\sigma} a_{M\sigma} - b^{\dagger}_{M\sigma} b_{M\sigma}] I_c . \tag{3}$$

For a system with a variable number of particles and a density matrix

$$\hat{\rho} = \exp\left[-\beta \sum_{M>0} [\hat{a}^{\dagger}_M \hat{a}_M - \hat{b}^{\dagger}_M \hat{b}_M](E_M - \mu) \right], \quad \beta = 1/k_B T, \tag{4}$$

the average current is

$$\langle \hat{I} \rangle = \mathrm{Tr}\{\hat{\rho}\hat{I}\} = 0. \tag{5}$$

At the same time, the current correlation function in such a system is finite:

$$\langle \hat{I}^2 \rangle = \lim_{t\to\infty} \langle \hat{I}(0)\hat{I}(t) \rangle = 4 I_c^2 \sum_{M>0} n(E_M)[1 - n(E_M)], \tag{6}$$

where $n(x-\mu)$ is the Fermi function.

Comparison of expressions (5) and (6) shows that in the present approximation the system is nonergodic. This occurs because, in contrast to systems with a continuous spectrum, the one-particle electron dynamics alone does not guarantee that a system with a discrete spectrum will relax from one current state into another. The Fermi distribution gives the probabilities of certain current states in the ring so that the average (with respect to the distribution function) current is equal to zero, but the time-average of the current is not zero because of the absence of relaxation. The case in which a state with a zero total current was prepared initially is an exception. Taking into account a weak but finite interaction that mixes states with opposite angular momenta leads to temporal evolution of the system in the entire phase volume accessible to it with "visiting frequency" again given by the distribution (4). In this case the correlation function $\langle \hat{I}(0)\hat{I}(t) \rangle$ in the limit $t \to +\infty$ is

$$\langle \hat{I}(0)\hat{I}(t) \rangle = \langle \hat{I}^2 \rangle \exp[-t/\tau_M], \tag{7}$$

where $\langle \hat{I}^2 \rangle$ is given by expression (6), and τ_M is the characteristic "mixing" time of the states with $M = \pm |M|$. The delta functions in the spectral density (2) are broadened on a scale $\sim \tau_M^{-1}$, and the amplitude of the peaks $\propto \tau_M$; specifically,

$$S(0) = 2\tau_M \langle \hat{I}^2 \rangle. \tag{8}$$

As we can see, the intensity of the interaction, even (and especially) if it is weak, largely determines the long-time dynamics of the fluctuations. The fact that in the limit $k_B T \ll \hbar\omega_0$ the correlation function $\langle \hat{I}^2 \rangle$ does not depend on the temperature is also worth noting. This correlation function is

$$\langle \hat{I}^2 \rangle = I_c^2. \tag{9}$$

Correspondingly, the temperature dependence in expressions (7) and (8) lies entirely in the function $\tau_M(T)$. At the same time, when spectral density is averaged over the interval $\Omega \gg \omega_0$ with $\Omega \ll k_B T$, we obtain the more-familiar expression which is determined entirely by the one-particle dynamics:

$$\bar{S}(0) \simeq \frac{8\pi k_B T I_c^2}{\hbar \omega_0^2} = 2k_B T \frac{2e^2}{h}. \tag{10}$$

Expression (10) is identical to the spectral density of equilibrium fluctuations in a quasi-one-dimensional ideal conductor that is connected to reservoirs.

We now consider a more complicated situation in which the number of particles in the ring is fixed. In this case the density matrix is again diagonal in the noninteracting-electron approximation, and its diagonal elements are

$$\rho = Z^{-1} \exp\left[-\beta \sum_{n=1}^{N} E_{M(n)} \right], \quad Z = \mathrm{Tr}\left\{ \exp\left[-\beta \sum_{n=1}^{N} E_{M(n)} \right] \right\}. \tag{11}$$

Since the probabilities now depend on the total energy, additional correlations appear between the electrons. Let us consider the case $\hbar\omega_0 \gg k_B T$. Each one-electron level is quadruply degenerate, and for $k_B T \ll \hbar\omega_0$ the magnitude of the fluctuations depends strongly on the number of electrons l in the last filled level at $k_B T = 0$. If $l = 4$ (the level is completely filled), then excitations with the minimum energy $\Delta = \hbar\omega_0$ are simply an electron at the level with energy $E_F + \Delta$ in one of four states and the absence of an electron in one of the states with energy E_F. There are 16 such excitations. The probability of finding the system in one of the states α is

$$\rho_{\alpha\alpha} \approx \frac{\exp[-\beta\Delta]}{1 + 16\exp[-\beta\Delta]}. \tag{12}$$

Calculating the square of the current from the formula

$$\langle \hat{I}^2 \rangle = \sum_{\alpha} \rho_{\alpha\alpha} \langle \alpha | \hat{I} | \alpha \rangle \langle \alpha | \hat{I} | \alpha \rangle, \tag{13}$$

we obtain for $k_B T \ll \Delta$

$$\langle \hat{I}^2 \rangle = 8\exp[-\beta\Delta](2I_c)^2. \tag{14}$$

For $l < 4$ the analysis is similar, but there is a specific feature associated with the degeneracy of the ground state of the system. We give as an example the case with $l = 1$. The order of the degeneracy of the ground state is equal to 4. Since in the limit of low but finite temperature (and hence a coupling with a reservoir) all variants of the ground state are equally probable, the "correct" limit for the ground state as $k_B T \to 0$ must be the limit where an electron with maximum energy is in a mixed state with zero projections of the spin and zero current. The main contribution to the current fluctuations is now provided not by the excited states but rather by the current-carrying states with minimum energy. They are all realized with equal probability $\approx 1/4$. For the squared current we thus obtain

$$\langle \hat{I}^2 \rangle = I_c^2. \tag{15}$$

In calculating the current correlation function with allowance for the interaction it should be kept in mind that the different states can have different lifetimes, for example, states with different total spins. The long-lived states will then account for the largest contribution to the correlation function $\langle I(0)I(t)\rangle$ in the limit $t \to \infty$. For $l=1$, $T \to 0$

$$\langle I(0)I(t)\rangle \propto I_c^2 \exp[-t/\tau_{\max}]. \tag{16}$$

For $l=4$, $T \to 0$

$$\langle I(0)I(t)\rangle \propto \exp[-\beta\Delta] I_c^2 \exp[-t/\tau_{\max}]. \tag{17}$$

We call attention to the fact that for $l=4$ the fluctuations vanish in the limit $k_B T \to 0$, while for $l=1$, 2, and 3 in the limit $k_B T \to 0$ there is a finite spectral noise density $S(0) \sim I_c^2 \tau_M$, which originates from the degeneracy of the ground state. It is obvious that the result (9), obtained for the case with a variable number of particles, has the same origin. This dependence of the fluctuations on the degeneracy of the ground state is similar to the situation with an average current and a finite flux Φ that passes through the ring — the average current ("persistent current") abruptly acquires a finite value for arbitrarily small flux if the ground state is degenerate.

Although the elastic scattering removes the degeneracy with respect to the angular momentum, and although for $\Phi=0$ all states do not carry current, for $\Phi \neq 0$ the characteristic states carry a finite current[2] and the situation becomes similar to an ideal ring with $\Phi=0$. In addition, as shown in Ref. 6 , in a system with an odd number of electrons and spin-orbit interaction the characteristic states can carry a finite current even in the presence of elastic scattering and $\Phi=0$. The phenomenon of "pseudononergodicity," which was described above, should be observed in this case.

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