Diffraction of light by a cosmic string

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The light curve of a star due to the passage of a cosmic string between the star and the observer is obtained. In contrast to the microlensing by a point-like object, the light curve of a star due to a cosmic string depends on the wavelength. The mechanism proposed for the increase in the luminosity of a star could be responsible for some recently observed microlensing events. © *1995 American Institute of Physics.*

1. INTRODUCTION

The increase in the luminosity of stars as a result of gravitational lensing by compact objects was first calculated by Einstein[1] and Tikhov.[2] Interest in gravitational lensing revived, however, only after the publication of studies by Liebes[3] and Refsdal.[4] Gravitational lensing by different massive cosmic formations later attracted a great deal of attention[5–11] (see the review[10] and the monograph[11]). Two possibilities were mainly discussed: 1) lensing by individual stars (point-like massive objects) and 2) change in the luminosity of a star by compact objects with a distributed mass such as a galaxy. Chang and Refsdal,[6] and also Gott[7] examined the possibility of explaining fluctuations in the luminosity of images of the quasar 0957+561 by microlensing by stars lying close to the line of sight. Gott[7] also underscored the fact that microlensing by point objects provides a unique possibility for detecting low-mass objects in galactic halos. However, this possibility attracted the attention of astronomers only after the publication of the paper by Paczynski.[9] Paczynski published plots of the theoretical geometric-optics curves of the increase in the luminosity of stars by lensing by compact bodies. In the beginning of the 1990s, several experimental groups[12–15] started the search for so-called "microlensing events." The results of an analysis of 16 events of this type which were discovered by three experimental groups — MASHO,[12,16] EROS,[13] and OGLE[14] — have now been published.

We know of no papers, however, in which a different possibility for the change in the observed luminosity of celestial bodies is studied in detail — doubling of the images by a cosmic string. The cosmologists who introduced the concept of a cosmic string only mentioned this possibility.[17–19] A string is a form of dark matter, but its manifestations can be observed through several effects.[20–23]

It should be noted that the geometric-optics approximation was employed in most papers on gravitational lensing (for the wave approach see Refs. 8 and 10). In many cases this approach is sufficient for compact lensing objects. Wave optics must be used to study gravitational lensing by a cosmic string, especially when a second geodesic arises.

2. COSMIC STRING

A cosmic string is a one-dimensional region with nonzero components of the energy-momentum tensor. It is characterized by two local parameters: the linear mass density μ and the radius ρ_S of the transverse cross section.[21] Cosmic strings arise as topological defects during the evolution of the early universe, and are therefore stable formations.[17–19] The radius ρ_S of the cross section depends on the linear mass density μ and can be several orders of magnitude greater than the Planck length[21] $l_{Pl}=(\hbar G/c^3)^{1/2}\sim 10^{-33}$ cm. In the present paper the thickness of the section can therefore be ignored. Strings can be either closed or infinitly long. The latter possibility is unlikely in the present-day universe. We designate by R the radius of a closed string.

To describe the space in the neighborhood of a string, it is convenient to employ a dimensionless linear mass density $\mu^*=(\mu G/c^2)$. To describe the space around a string at distances much smaller than R, we introduce cylindrical coordinates such that the z axis is oriented along with the string. The metric of the space outside the string is the metric of a flat space, but the azimuthal angular coordinate φ assumes values in the range $2\pi(1-4\mu^*)$. This space is said to be conical with an angular defect $\delta=8\pi\mu^*$. There is no gravitational attraction near a string. However, the global structure of the conical space is different from the structure of a flat space. This is the reason for a number of nonlocal effects, with the help of a string which can be detected.[21]

3. MONOCHROMATIC LIGHT CURVE

It is easy to see that when a string lies near the line of sight between an observer and a star, according to geometric optics the observer can see two images of the star (Fig. 1). The angle δ_O between two geodesics at the point O is

$$\delta_O=\delta\left(\frac{D_{LS}}{D_{LS}+D_{OS}}\right)\sin\varphi, \tag{1}$$

where φ is the angle between the string and the line of sight. In the present letter, however, we shall restrict the analysis to the case in which the angular distance between two images is so small that an observer cannot resolve the two images. In this case, the observer again sees one star but with a variable luminosity.

Since the space around a string is locally flat, we can use the well-known principles of wave optics.[24] To calculate the amplitude of an electromagnetic wave at the point of the observer within the framework of Fresnel's diffraction problem, it is necessary to know the distribution of the amplitude and phase on some surface. In the present calculations this surface consists of two half-planes, whose edges lie on the string.

Adopting as the unit of measurement the luminosity of the star when the string is located far away from the line of sight, we obtain the following expression for the curve of the change in the observed luminosity of the star at the point O (for $\alpha>0$):

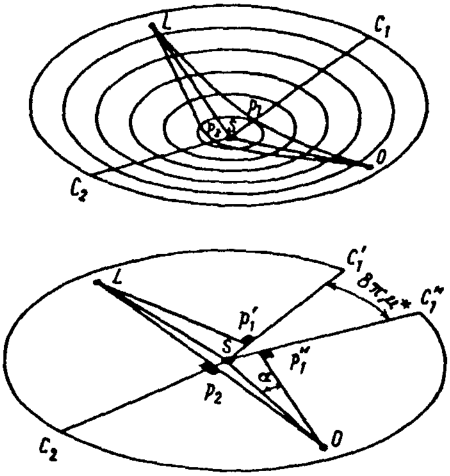

FIG. 1. Embedding of a two-dimensional projection of the space around a string — surface perpendicular to the string — in a three-dimensional flat space together with a planar unfolding of this projection. The section for the unfolding is made along the straight line SC perpendicular to the straight line (geodesic) $L_{P1}O$. When the string (S) lies near the line of sight, the star (L) and the observer (O) are connected by two minimal geodesics: $L_{P1}O$ and $L_{P2}O$. These geodesics give two images of the lensed star which are formed by electromagnetic waves and are separated from one another by an angular distance δ_O.

$$I(\alpha)=\left|\begin{array}{l}\frac{1}{2}\left(\exp\left(-\frac{i\pi}{4}\right)+\frac{1}{\sqrt{2}}\mathrm{sign}(\delta_O-\alpha)\int_0^{t_1}\exp\left(-\frac{i\pi t}{2}\right)\frac{dt}{\sqrt{t}}\right)\exp\left(\frac{i\pi t_1}{2}\right)\\ \qquad +\frac{1}{2}\left(\exp\left(-\frac{i\pi}{4}\right)+\frac{1}{\sqrt{2}}\int_0^{t_2}\exp\left(-\frac{i\pi t}{2}\right)\frac{dt}{\sqrt{2}}\right)\exp\left(\frac{i\pi t_2}{2}\right)\end{array}\right|^2, \qquad (2)$$

where $t_1=2(\delta_O-\alpha)^2\Lambda$, and $t_2=2\alpha^2\Lambda$. Here we have introduced the concept of the "wave distance"

$$\Lambda=\frac{D_{OS}}{D_{LS}}\left(\frac{D_{OS}+D_{LS}}{\lambda}\right), \qquad (3)$$

where λ is the wavelength of the observed radiation from the star. Expression (2) holds for $\alpha\ll1$. As α increases (for $\alpha>\delta_O$), however, the value of $I(\alpha)$ differs negligibly from one, even for $\alpha\ll1$. Multiplying Λ by δ_O^2, we obtain the only parameter that determines the shape of the light curve

$$N=\frac{1}{2}\delta^2\frac{D_{OS}D_{LS}}{D_{OS}+D_{LS}}\frac{1}{\lambda}\sin^2\varphi. \qquad (4)$$

We call this parameter the "excess zone number."

Expression (2) was made valid for all $\alpha>0$ by introducing $\mathrm{sign}(\delta_O-\alpha)$ for the change in the sign of one of the Fresnel integrals for $\alpha>\delta_O$, when the string is "too far" from the line of sight in order for the observer and the star to be connected by two minimal geodesics. For negative values of α in Eq. (2) α can be replaced by $\delta_O - \alpha$.

Strictly speaking, expression (2) is not exact, since the distances D_{OS}^i between the observer and the string along the geodesics (and also the distances D_{LS}^i from the star to the string along the geodesics) are different for t_1 and t_2 and are related by a simple relation $D_{OS}^1 \cos\alpha = D_{OS}^2 \cos(\delta_O - \alpha)$. However, for small angular defects δ (which are unlikely to exceed several angular seconds[21]) the difference between the exact expression and expression (2) is negligible.

Expression (2) obtained for the light curve is valid for large string radii R, when the string can be regarded as being linear on a scale of several hundred Fresnel zones

$$R \gg \sqrt{\frac{D_{LS} \cdot D_{OS}}{(D_{LS}+D_{OS})} \lambda \frac{1}{\sin^2 \varphi}}. \tag{5}$$

In deriving expression (2) we assumed that the distances D_{OS} and D_{LS} are small compared to the radius R of the string. However, the result is also valid for $D \gg R$, since at distances of the order of R the gravitational potential of a closed string is too weak to affect the result.

4. EXPERIMENTAL LIGHT CURVE

In a real situation the observer, the string, and the star move along individual orbits. On the basis of the approach adopted in the present letter, it is sufficient to study the motion of the observer with respect to a line passing through the star and a point on the string closest to the minimal geodesic connecting the star and the observer. It is obvious that in most cases the angle α can be treated as a time argument.

In astronomy optical measurements are performed in several spectral intervals. For example, the main measurements in an experiment performed by the OGLE group were performed in the standard photometric band I.[14] To obtain the measured light curve from the monochromatic light curve, the latter must be integrated over the spectral interval. As an illustration, some curves obtained by integrating over a spectral interval with the relative width of the spectral band I are shown in Fig. 2a. These curves are more regular than the monochromatic curves. There is some additional smoothing because of the finite angular size of the light source. Light curves for a light source of angular size Δ are shown in Fig. 2c. Figure 3 gives an example of the light curves for one event observed in four standard spectrometric bands (see next section).

It is obvious from the examples presented above that the main difference between the events induced by the passage of a string and the events associated with lensing by a compact object lies in the wavelength dependence of the shape of the light curve. The presence of side peaks with small excess zone numbers can also be important for identifying the string nature of a microlensing event. The only certain manifestation of lensing by a compact object is a maximum amplification of the luminosity greater than a factor of 6, since the maximum possible amplification of the luminosity by lensing by a string is slightly greater than a factor of 5. It should be noted that the light curves due to lensing by a compact object can be very diverse and can exhibit wavelength dependence when the lensing object is a double star.[25]

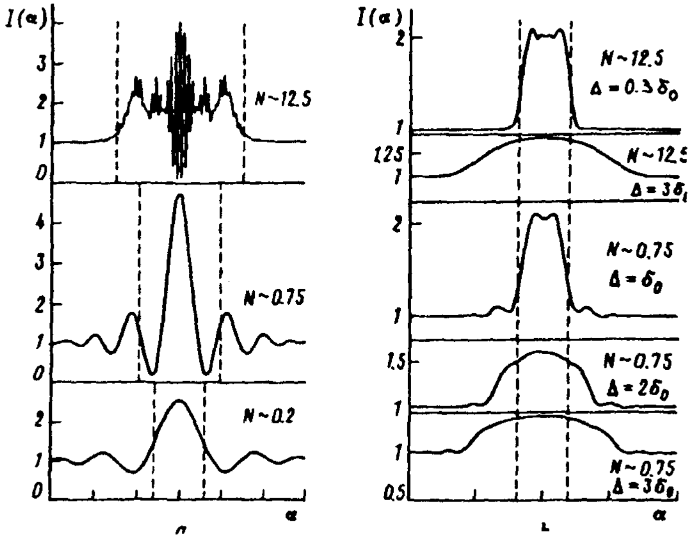

FIG. 2. Light curves averaged (a) over a spectral interval with a 30% width and (b) over a spectral interval with a 30% width and over a finite angular size of the stellar disk. Δ — Angular diameter of the star in units of δ_0. The dashed lines delimit the range of angles with two geodesics.

5. ESTIMATES AND SUGGESTIONS

According to the maximum luminosity amplification criterion formulated at the end of the preceding section, only a few of the microlensing events recorded by the OGLE, MACHO, and EROS are clearly unsuitable as candidates for lensing by a string. Some of the remaining events, together with the so-called "background" events,[16] could be caused by a string. To identify reliably the true reason for a microlensing event, it is necessary to have a synchronous series of observations in different spectrometric intervals. In addition, the typical time scale of a microlensing event caused by a string with angular defect

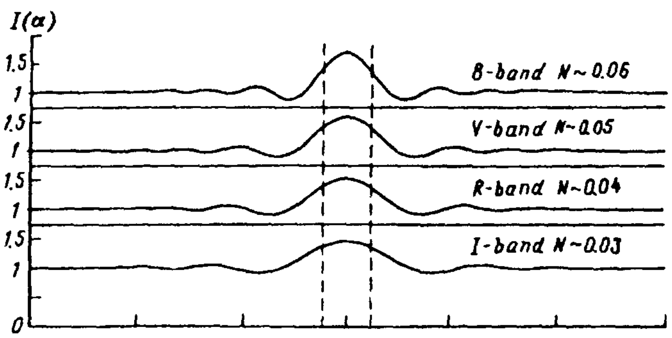

FIG. 3. Synchronous light curves for one event of lensing of a star by a string in four standard spectrometric bands. The dashed lines delimit the range of angles with two geodesics.

$\delta \sim 10^{-11}$(see below) is of the order of 10^4 s for stars at the center of the galaxy. (In the present letter we have answered that we are dealing with strings of only one type. In principle, the situation can be more complicated.) String events with a duration of about 1 month require a rare combination of motions of the string and the star. The events which can be observed by the existing "microlens" groups thus constitute only a small fraction of the string events that can be recorded on earth.

A preliminary qualitative analysis of the data presented by the "microlens" groups gives $\delta \sim 10^{-11}$. However, it could also turn out to be one or two orders of magnitude smaller. The corresponding linear mass density of a string is $\mu \sim 10^{16}$ g/cm. Let us estimate the minimum size of a string located several kiloparsecs from us, for which expression (2) is valid — $R \gg 10^9$ cm [see the condition (5)]. Another characteristic length associated with strings is determined by the decay of strings into gravitational waves. Strings with initial size exceeding

$$R \sim R_{rad} = \gamma \frac{G\mu t}{c}, \tag{6}$$

where t is the age of the universe and γ is a dimensionless factor of the order of 100, could have survived until now.[21,26] This length determines the lower limit of the size of strings of density μ that can be observed today. For $\mu \sim 10^{16}$ g/cm we obtain $R_{rad} \approx 0.1$ pc $\sim 10^{17}$ cm, which is much greater than the limit of applicability of expression (2). We can therefore ignore the limitation (5) for strings with this mass density μ. The mass of such a string with radius $R \sim R_{rad}$ is comparable to the mass of a star $M \sim 10^{34}$ g. Since the size of such a string is an order of magnitude smaller than the typical interstellar distances, the total mass of strings of this type in the galaxy can far exceed the total mass of ordinary matter.

In the present letter we have considered the diffraction of light by a cosmic string and we have constructed the light curve of a star for the case in which a cosmic string passes between the star and the observer. We found that for arbitrary excess zone number (6) the light curve can differ substantially from the light curve in the short-wavelength limit. For excess zone numbers $N \gg 1$, after integrating over the spectral interval and the finite angular size of the star, we obtain a stepped light curve which is consistent with the ray-optics approximation.

The string mechanism for the change in the luminosity of a star could be responsible for some recently recorded microlensing events. Note that for the same string event the light curve can have a different shape in different spectral intervals, which makes it possible to distinguish such an event from lensing by a compact object.

To determine the true nature of each microlensing event, the data must be approximated by theoretical curves, taking into account averaging over a spectral interval and integration over the stellar disk, with allowance for the distribution of the intensity over the disk.

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