Superfield approach to the Batalin–Vilkovisky quantization method for gauge theories

P. M. Lavrov,[1)] P. Yu. Moshin, and A. A. Reshetnyak

Tomsk State Pedagogical University, 634041 Tomsk, Russia; Tomsk State University, 634050 Tomsk, Russia

(Submitted 27 July 1995; resubmitted 4 September 1995)
Pis'ma Zh. Éksp. Teor. Fiz. **62**, No. 10, 760–763 (25 November 1995)

Rules are given for Lagrangian quantization of general gauge theories on the basis of the superfield implementation of the standard BRST symmetry. It is proved that the S-matrix is gauge-independent. The superfield form of the Ward identities is obtained. © *1995 American Institute of Physics.*

1. The modern methods[1–3] of explicitly covariant quantization of gauge theories are based on the principle of invariance under BRST (Becchi–Rouet–Stora–Tyutin) transformations.[4,5] This principle has been implemented in its most general form in the Batalin–Vilkovisky (BV) quantization scheme.[2,3] The complete set of variables in the method of Refs. 2 and 3 includes the fields ϕ^A (consisting of the initial classical fields, the ghost and antighost fields, and the Lagrange multipliers), the antifields ϕ^*_A of opposite statistics, the standard sources J_A for the fields ϕ^A, and the auxiliary fields λ^A which introduce the gauge.

The BRST-symmetry transformations were formulated for Yang–Mills theories in terms of geometry in the form of translations in hyperspace.[6–9] However, a closed superfield form of the BV-quantization method for arbitrary gauge fields has still not been found. In the present paper we propose a superfield approach to the BV-quantization scheme that reveals the geometric significance of the BRST symmetry incorporated into this method. We employ the condensed notation of Ref. 10 and the conventions adopted in Refs. 2 and 3 .

2. Let (x^μ,θ) be a hyperspace, where x^μ are the space–time coordinates, $\mu=(0, 1, ..., D-1)$, and θ is a scalar Grassman coordinate, and let $\Phi^A(\theta)$ and $\Phi^*_A(\theta)$ be the set of superfields and the corresponding superantifields, respectively, given in it:

$$\epsilon(\Phi^A)\equiv\epsilon_A, \quad \epsilon(\Phi^*_A)=\epsilon_A+1.$$

The components of the superfields and the superantifields are determined by the decomposition with respect to θ:

$$\Phi^A(\theta)=\phi^A+\lambda^A\theta, \quad \Phi^*_A(\theta)=\phi^*_A-\theta J_A, \quad \epsilon(J_A)=\epsilon(\phi^A),\epsilon(\lambda^A)=\epsilon(\phi^*_A) \tag{1}$$

and they are identical to the set of variables in the BV-quantization scheme [convenience dictates the choice of signs in the decomposition (1)].

We now determine the generating functional $Z[\Phi^*]$ of the Green's functions as a functional of the superantifields in the form of the following functional integral:

$$Z[\Phi^*]=\int d\Phi' d\Phi^{*\prime}\rho[\Phi^{*\prime}]\exp\left\{\frac{i}{\hbar}(S[\Phi',\Phi^{*\prime}]-U'\Psi[\Phi']+(\Phi^{*\prime}-\Phi^*)\Phi')\right\}. \tag{2}$$

Here $S=S[\Phi,\Phi^*]$ is the quantum action, which satisfies the generating equation

$$\frac{1}{2}(S,S)+VS=i\hbar\Delta S, \tag{3}$$

where $(\,,\,)$ is an antibracket defined for arbitrary functionals $F=F[\Phi,\Phi^*]$ and $G=G[\Phi,\Phi^*]$ according to the rule (we present at the same time the component representation of the objects introduced in the superfield description)

$$(F,G)=\int d\theta\left\{\frac{\delta F}{\delta\Phi^A(\theta)}\frac{\partial}{\partial\theta}\frac{\delta G}{\delta\Phi^*_A(\theta)}(-1)^{\epsilon_A+1}-(-1)^{(\epsilon(F)+1)(\epsilon(G)+1)}(F\leftrightarrow G)\right\}$$

$$=\frac{\delta F}{\delta\phi^A}\frac{\delta G}{\delta\phi^*_A}-(-1)^{(\epsilon(F)+1)(\epsilon(G)+1)}(F\leftrightarrow G), \tag{4}$$

and Δ is an operator of the form

$$\Delta=-\int d\theta(-1)^{\epsilon_A}\frac{\delta_l}{\delta\Phi^A(\theta)}\frac{\partial}{\partial\theta}\frac{\delta}{\delta\Phi^*_A(\theta)}=(-1)^{\epsilon_A}\frac{\delta_l}{\delta\phi^A}\frac{\delta}{\delta\phi^*_A}. \tag{5}$$

In Eqs. (2) and (3) we have introduced the operators

$$U=-\int d\theta\frac{\partial\Phi^A(\theta)}{\partial\theta}\frac{\delta_l}{\delta\Phi^A(\theta)}=-(-1)^{\epsilon_A}\lambda^A\frac{\delta_l}{\delta\phi^A}, \tag{6}$$

$$V=-\int d\theta\frac{\partial\Phi^*_A(\theta)}{\partial\theta}\frac{\delta}{\delta\Phi^*_A(\theta)}=-J_A\frac{\delta}{\delta\phi^*_A} \tag{7}$$

(the derivatives with respect to θ are everywhere understood to be left-derivatives), the functionals

$$\rho[\Phi^*]=\delta\left(\int d\theta\Phi^*(\theta)\right)=\delta(J),$$

$$\Phi^*\Phi=\int d\theta\Phi^*_A(\theta)\Phi^A(\theta)=\phi^*_A\lambda^A-J_A\phi^A, \tag{8}$$

and the fermion functional $\Psi=\Psi[\Phi]$, which fixes the gauge.

We note that the component representations of the antibracket [Eq. (4)] and the operator Δ [Eq. (5)] are identical to the definitions of the corresponding objects in the BV-quantization formalism, whose algebraic properties are well known.

It is easy to verify that the operators U, V, and Δ have the properties

$$U^2=0,\quad V^2=0,UV+VU=0,\quad \Delta U+U\Delta=0,\quad \Delta V+V\Delta=0, \tag{9}$$

and the action of V and U on an antibracket is given by the rules

$$V(F,G)=(VF,G)-(-1)^{\epsilon(F)}(F,V,G),$$

$$U(F,G)=(UF,G)-(-1)^{\epsilon(F)}(F,U,G). \tag{10}$$

An important property of the integrand in Eq. (2) with $\Phi^*=0$ is its invariance under the following transformations of the global supersymmetry with Grassman parameter μ:

$$\delta\Phi^A(\theta)=\mu\frac{\partial}{\partial\theta}\Phi^A(\theta)=\mu U\Phi^A(\theta),$$

$$\delta\Phi^*_A(\theta)=\mu\frac{\partial}{\partial\theta}\Phi^*_A(\theta)+\mu\frac{\partial}{\partial\theta}\frac{\delta S}{\delta\Phi^A(\theta)}=\mu(V\Phi^*_A(\theta)+(S,\Phi^*_A(\theta))). \tag{11}$$

These transformations make it possible to prove gauge-independence of the vacuum functional $Z_\Psi\equiv Z[0]$. Indeed, let us change the gauge fermion according to the rule $\Psi\rightarrow\Psi+\delta\Psi$ and make in the functional integral for $Z_{\Psi+\delta\Psi}$ a change of variables of the form (11) with the following choice of the parameter μ:

$$\mu=-\frac{i}{\hbar}\delta\Psi. \tag{12}$$

Then, using Eq. (3), we obtain $Z_\Psi=Z_{\Psi+\delta\Psi}$, and we arrive at the result that the S-matrix is gauge-independent.

Another consequence of the invariance property of Z are the Ward identities for the generating functional of the Green's functions. We make in the functional integral (2) the change of variables (11), using an equation for the boson functional $S=S[\Phi,\Phi^*]$. Simple transformations yield the Ward identities for $Z[\Phi^*]$ in the form

$$-\int d\theta\frac{\partial\Phi^*_A(\theta)}{\partial\theta}\frac{\delta}{\delta\Phi^*_A(\theta)}Z[\Phi^*]=VZ[\Phi^*]=0. \tag{13}$$

We now consider the component form of the relations (11) and (13). The transformations (11) assume the form

$$\delta\phi^A=\lambda^A\mu,\quad \delta\lambda^A=0,\quad \delta\phi^*_A=\mu\left(\frac{\delta S}{\delta\phi^A}-J_A\right),\quad \delta J_A=0 \tag{14}$$

and for $J=0$ they are identical in form to the BRST transformations in the BV-quantization scheme. In this connection the transformations (11) can be interpreted as a superfield implementation of the standard BRST-symmetry transformations. The component representation of the Ward identity (13)

$$J_A \frac{\delta}{\delta\phi_A^*} Z(J,\phi^*) = 0 \tag{15}$$

is formally identical to the familiar Ward identities for gauge theories.

It is easy to establish the relation between the proposed approach and the BV-quantization rules. For this, we confine our attention to the special solution of Eq. (3) in the form of the functional $S[\Phi,\Phi^*]$ which does not depend on the variables λ^A

$$\frac{\delta\bar{S}}{\delta\lambda^A} = \int d\theta\theta \frac{\delta\bar{S}}{\delta\Phi^A(\theta)} = 0 \tag{16}$$

and is linear in J_A

$$\bar{S}[\Phi,\Phi^*] = S(\phi,\phi^*) + J_A\phi^A, \tag{17}$$

where $S(\phi,\phi^*)$ satisfies the standard master equation of Refs. 2 and 3 :

$$\frac{1}{2}(S,S) = i\hbar\Delta S. \tag{18}$$

We choose the boundary condition for Eq. (18) in the form

$$\bar{S}\Big|_{\Phi^*=\hbar=0} = S, \tag{19}$$

where S is the initial gauge-invariant classical action (we note that the condition (19) is identical to the generating equation (3)). Then, taking into consideration the definitions (6) and (8) and the component composition Φ^A, Φ_A^* and confining our attention to the class of gauges which depend only on the fields ϕ^A, we arrive at the following representation for the generating functional $Z = Z(J)$ of the Green's functions for the fields ϕ^A:

$$Z(J) = Z[\Phi^*]\Big|_{\phi^*=0} = \int d\phi d\phi^* d\lambda \exp\left\{ \frac{i}{\hbar}\left[S(\phi,\phi^*) + \left(\phi_A^* - \frac{\delta\Psi}{\delta\phi^A}\right)\lambda^A + J_A\phi^A\right]\right\}. \tag{20}$$

The latter relation, together with Eqs. (17)–(19), determine the generating functional of the Green's functions within the BV-quantization formalism.

3. In the present letter rules were proposed for Lagrangian quantization of gauge theories of general form on the basis of a superfield implementation of the standard BRST symmetry. The relation of this formulation to the BV-quantization rules was indicated. It was shown that the S-matrix is gauge-independent. The geometric significance of the Ward identities in gauge theories is that the generating functional $Z[\Phi^*]$ of the Green's functions is invariant under translations in hyperspace (x^μ,θ) along the Grassman coordinate θ.

We thank the referee for critical remarks which improved the content of this paper.

This work was supported by the Russian Fund for Fundamental Research (Project 94-02-03234) and the International Science Foundation (Grant RI 1300).

[1)]e-mail: lavrov@tspi.tomsk.su

[1]B. DeWitt and J. W. van Holten, Phys. Lett. B **79**, 389 (1978)
[2]I. A. Batalin and G. A. Vilkovisky, Phys. Lett. B **102**, 27 (1981).
[3]I. A. Batalin and G. A. Vilkovisky, Phys. Rev. D **28**, 2567 (1983).
[4]C. Becchi, A. Rouet, and R. Stora, Commun. Math. Phys. **42**, 127 (1975).
[5]I. V. Tyutin, Preprint No. 39, Physics Institute of the USSR Academy of Sciences, Moscow, 1975.
[6]L. Bonora and M. Tonin, Phys. Lett. B **98**, 48 (1981).
[7]L. Bonora, P. Pasti, and M. Tonin, J. Math. Phys. **23**, 839 (1982).
[8]C. M. Hull, B. Spence, and J. L. Vasquez-Bello, Nucl. Phys. B **348**, 108 (1990).
[9]L. Baulieu, Phys. Rep. **129**, 1 (1985).
[10]B. S. DeWitt, *Dynamical Theory of Groups and Fields*, Gordon and Breach Science Publishers, N. Y., 1965 [Russian translation, Mir, Moscow, 1987].

Translated by M. E. Alferieff