

Sign change of the flux-flow Hall effect in HTSC

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A novel mechanism for the sign change of the flux-flow Hall effect is proposed. The dependence of the superconducting thermodynamic potential Ω_{sc} on the chemical potential μ produces a topological contribution to the Hall conductivity $\delta\sigma_{xy} = -(\partial\Omega_{sc}/\partial\mu)ec/B$, which may lead to a double sign change of the Hall effect as a function of temperature. © 1995 American Institute of Physics.

The sign change of the Hall effect in the mixed state of high-temperature superconductors (HTSCs) is the most puzzling phenomenon in the physics of the magnetic properties of these materials.¹ In spite of the numerous attempts to explain this anomaly, even the origin of the sign reversal in the Hall resistivity remains unclear.¹ The sign change of the Hall resistivity has also been observed in conventional superconductors and thus is not a peculiarity of HTSCs.^{1,2} Comparing experimental data for different materials, Hagen *et al.*¹ have argued that the sign change is an intrinsic property of the vortex motion, and that the sign reversal occurs in the range of parameters where the transport mean free path l becomes of the order of the vortex core size ξ .

In this letter we propose an explanation for the sign reversal in the Hall effect. We present a coherent phenomenological derivation of the Hall conductivity σ_{xy} , consistently taking into account the scattering of normal excitations, σ_{xy}^s (Ref. 3), as well as a topological contribution σ_{xy}^t deriving from the dynamics of the order parameter:

$$\sigma_{xy} = \sigma_{xy}^s - \partial\Omega_{sc}/\partial\mu|_0^\infty ec/B. \quad (1)$$

Here $\Omega_{sc}(r)$ is the superconducting part of the thermodynamic potential density, which depends on the distance r from the vortex core, and μ is the chemical potential. The second term, σ_{xy}^t , turns out to reproduce the result obtained previously from the time-dependent Ginzburg–Landau equation (TDGL).^{4,5} It is this contribution which gives rise to a double sign change in the Hall effect.

This paper is organized as follows. First we study a model uncharged superconductor and find the additional topological contribution $\sigma_{xy}^I = -\delta n e c / B$, where δn is the difference between the carrier density at the center of the vortex core and that far outside. Then we include Coulomb screening and show that although it suppresses the density modulation, it does not affect the topological term in the Hall conductivity (1).

In order to describe the Hall effect one has to find the transverse force experienced by a vortex moving with velocity \mathbf{v}_L under the applied transport current \mathbf{j}_T . There are two contributions to the transverse force: The first one arises from the nondissipative momentum transfer from the moving vortex to infinity. This contribution consists of the Magnus and Iordanskii⁶ forces. The second contribution stems from the momentum transfer from the vortex to the normal excitations in the vortex core. The subsequent absorption of this transferred momentum by the thermal bath due to scattering of normal quasiparticles leads to dissipation and to the longitudinal Bardeen–Stephen friction force.

In order to understand both contributions let us derive the dynamic term in the adiabatic action for a moving vortex. The effective action for a superconductor depends on the phase of the order parameter χ only through gauge invariant combinations: $S = S(\nabla\chi - 2e\mathbf{A}/\hbar c, \partial\chi/\partial t + 2e\phi/\hbar)$, with \mathbf{A} and ϕ the vector and scalar potentials, respectively. Variation of the action with respect to the phase yields the current conservation law:

$$\frac{\delta S}{\delta\chi(\mathbf{r}, t)} = -\frac{\hbar}{2e} \nabla\mathbf{j} + \frac{\partial}{\partial t} \frac{\delta S}{\delta\partial\chi/\partial t} = 0, \quad (2)$$

with a current density $\mathbf{j} \equiv c \delta S / \delta\mathbf{A}(\mathbf{r}, t)$. The continuity equation $\nabla\mathbf{j}/e + \partial n / \partial t = 0$ for the particle current implies that the effective action has to contain the topological term (n is the particle density)

$$S_t = -\hbar \int dV dt \frac{n}{2} \frac{\partial\chi}{\partial t}. \quad (3)$$

The factor 1/2 is due to pairing and is absent in a superfluid Bose system. This topological term is irrelevant if $n = \text{const}$ and χ is single valued, but in the presence of a vortex this is the term in the action that governs its dynamics. For simplicity we consider the 2D case. Expressing the phase in the presence of a vortex as the sum of a singular contribution $\Theta(\mathbf{r} - \mathbf{R}_L(t)) = \arg(\mathbf{r} - \mathbf{R}_L(t))$ ($\mathbf{R}_L(t)$ is the vortex position) and a regular contribution $\chi = \Theta(\mathbf{r} - \mathbf{R}_L(t)) + \chi_r(\mathbf{r}, t)$, and taking only the singular contribution into account, we obtain

$$S_t = \frac{\hbar}{2} \int d^2r dt n \nabla\Theta \dot{\mathbf{R}}_L = \int dt \mathbf{a}(\mathbf{R}_L) \mathbf{v}_L. \quad (4)$$

The quantity $\mathbf{a}(\mathbf{R}_L) = (\hbar/2) \int d^2r n(\mathbf{r} - \mathbf{R}_L) \nabla\Theta(\mathbf{r} - \mathbf{R}_L)$ has the meaning of a ‘vector potential’ of a fictitious constant magnetic field. To see this, calculate

$$\nabla \times \mathbf{a} = \frac{\hbar}{2} \oint n \nabla\Theta d\mathbf{l} = \pi\hbar(n_\infty - n_0), \quad (5)$$

where we have replaced the surface integral by two contour integrals, at infinity and around $\mathbf{r}=\mathbf{R}_L$, with n_∞ and n_0 being the particle densities far outside the vortex core and on its axis, respectively. This term in the action produces the transverse force

$$\mathbf{F}_\perp = \pi\hbar(n_\infty - n_0)\mathbf{z} \times \mathbf{v}_L, \quad (6)$$

analogous to the Lorentz force experienced by a particle moving in a magnetic field⁷ (the unit vector \mathbf{z} is orthogonal to the plane). Note that this force is independent of charge and is not of electromagnetic origin.

Based on a similar Berry phase type argument, Ao and Thouless⁸ arrived at the conclusion that the relevant density in Eq. (6) is n_s , rather than n . This is correct only for the Galilean invariant case, where the density n and the superfluid density n_s , coincide, and the force (6) is just the Magnus force. In general there are two major differences between (6) and the Magnus force; first, n_∞ is the total density rather than the superfluid one, and thus this part is the sum of Magnus and Iordanskii⁶ forces. Second, and most important, there is an additional term proportional to the density at the vortex axis. In our derivation of Eq. (5) we have excluded the vortex axis from the integration, since $n\nabla\Theta$ has a singularity there if $n_0 \neq 0$, and our adiabatic action is not applicable close to the core axis. The fact that $n_0 \neq 0$ means that not all the particles participate in the superfluid motion and there are normal excitations inside the core.⁹ As the Magnus force arises from the nondissipative transfer of momentum from the vortex to infinity, the other term $n_0\hbar\mathbf{v}_L \times \mathbf{z}$ is due to the transfer of momentum from the condensate to the normal excitations inside the core. This term is just the term obtained by Volovik¹⁰ who, starting from BCS theory, derived an effective action describing the transfer of the momentum from the condensate to the fermionic modes in the vortex core. The subsequent absorption of this momentum by the heat bath due to impurity scattering leads to the Bardeen–Stephen dissipation. Thus the hidden assumption in the derivation of Eq. (6) is that impurity scattering is so strong that all the momentum transferred to the normal excitations is absorbed by the heat bath. However, for BCS superconductors we have $n_\infty - n_0 \ll n_\infty$, and the Magnus force is compensated almost completely.¹⁰ In this case the impurity scattering should be considered in more detail. Such a calculation has been done long ago³ without allowance for the difference between n_∞ and n_0 . Below we take into account both effects simultaneously and show that their combination can lead to a sign change in the Hall conductivity.

An accurate microscopic treatment of the scattering processes in the adiabatic action approach will be published elsewhere.¹¹ Instead, we consider a simple phenomenological model similar to that of Nozières and Vinen,¹² but here we take into account both impurity scattering and changes in density. We model the core as completely normal with a carrier density n_0 and a sharp boundary at a radius $r_c \approx \xi$. The superconducting material has a density n_∞ .^{12,13} We denote the velocity of the normal carriers inside the core in the laboratory frame as \mathbf{v}_c and consider the transfer of the momentum in the system at $T \ll T_c$. In the presence of a transport current \mathbf{j}_T and electric and magnetic fields, the equation for conservation of momentum in the electron system, $d\mathbf{P}/dt=0$, reads

$$\mathbf{j}_T \times \mathbf{B}/c + n_\infty e\mathbf{E} - mn_0\mathbf{v}_c f(B/H_{c2})/\tau = 0. \quad (7)$$

The first two terms describe the momentum transfer due to the Lorentz and electric-field forces. The third term takes impurity scattering into account (τ and m are the transport time and the effective mass). For $B > H_{c2}$, $f(B/H_{c2}) = 1$, $n_\infty e \mathbf{v}_c = \mathbf{j}_T$, and Eq. (7) gives the Drude formulas for the longitudinal and Hall conductivities. For $B \ll H_{c2}$ impurity scattering takes place only in the vortex core, and $f \propto B/H_{c2}$. If $n_0 = n_\infty$ the transport current \mathbf{j}_T is equal to $n_0 e \mathbf{v}_c$, and we obtain $\omega_0 \tau \mathbf{z} \times (\mathbf{v}_L - \mathbf{v}_c) = \mathbf{v}_c$ (Refs. 12,14), where we have introduced $\omega_0 = eB/mcf(B/H_{c2})$ ($\omega_0 \approx \Delta^2/\varepsilon_F$ at $B \ll H_{c2}$ in clean materials). Solving this equation, one finds

$$\mathbf{v}_c = \frac{\omega_0 \tau}{1 + (\omega_0 \tau)^2} \mathbf{z} \times \mathbf{v}_L + \frac{(\omega_0 \tau)^2}{1 + (\omega_0 \tau)^2} \mathbf{v}_L, \quad (8)$$

which agrees with the result of the microscopic calculations in the relaxation-time approximation.³ If, however, $n_0 \neq n_\infty$, then $n_0 e \mathbf{v}_c$ is not equal to \mathbf{j}_T . In the reference frame moving with the vortex, the current conservation gives $n_0 e \tilde{\mathbf{v}}_c = \tilde{\mathbf{j}}_T$. Going to the laboratory frame we have $n_0 e \mathbf{v}_c = \mathbf{j}_T + \delta n e \mathbf{v}_L$, where $\delta n = n_0 - n_\infty$. Inserting in (7) one finds that the equation for \mathbf{v}_c (8) remains unchanged, and

$$\mathbf{j}_T = e n_0 \left[\frac{\omega_0 \tau \mathbf{z} \times \mathbf{v}_L}{1 + (\omega_0 \tau)^2} + \left(\frac{(\omega_0 \tau)^2}{1 + (\omega_0 \tau)^2} - \frac{\delta n}{n_0} \right) \mathbf{v}_L \right], \quad (9)$$

where the two terms on the right-hand side map to the Bardeen–Stephen longitudinal conductivity and the Hall conductivity. The δn term, rewritten as the transverse force, is just the term derived in Eq. (6) from the adiabatic action. The transverse force can be rewritten as $\pi \hbar [n_\infty - n_0 / (1 + (\omega_0 \tau)^2)] \mathbf{v}_L \times \mathbf{z}$, where the first term is the Magnus force and the second is the force due to impurity scattering, which cancels the Magnus force almost completely in a conventional situation.^{3,10} Equation (9) is valid at $T \ll T_c$. When $T \rightarrow T_c$, Eq. (8) holds as well and describes the transfer of momentum for quasiparticles with $\epsilon < \Delta$. Quasiparticles with $\epsilon > \Delta$ give the normal state contribution to the Hall conductivity. Thus in Eq. (9) n_0 should be multiplied by the factor $g(\Delta/T)$, where the function $g(x \rightarrow \infty) \rightarrow 1$ and $g(x \rightarrow 0) \approx x$ (see Ref. 15). From Eq. (9) we obtain the Hall conductivity

$$\sigma_{xy} = \frac{n_0 e c}{B} g \frac{(\omega_0 \tau)^2}{1 + (\omega_0 \tau)^2} - \frac{\delta n e c}{B} + \sigma_{xy}^n (1 - g). \quad (10)$$

The above discussion refers to a model uncharged superconductor, where $\delta n/n \sim (\Delta/\varepsilon_F)^2$.¹⁰ In real superconductors, Coulomb screening suppresses inhomogeneities in the charge density, and the total charge of the vortex becomes zero. Below we show that screening does not change the value of the Hall conductivity, though the latter cannot be expressed any longer in terms of the density difference δn . To include screening, we supplement the Lagrangian \mathcal{L}_{sc} by the Coulomb term, $\mathcal{L}_{\text{tot}} = \mathcal{L}_{sc} + \mathcal{L}_C$, with

$$\mathcal{L}_C = \frac{1}{8\pi} \int (d^3 q) \mathbf{E}_\parallel(\mathbf{q}) \mathbf{E}_\parallel(-\mathbf{q}) \varepsilon(\mathbf{q}) \quad (11)$$

and $\mathbf{E}_\parallel(\mathbf{q})$ is the Fourier component of the longitudinal electric field $\mathbf{E}_\parallel(\mathbf{r}) = -\nabla \phi$. Here $\varepsilon(\mathbf{q})$ is the dielectric function, with $\varepsilon(q \rightarrow 0) = 1 + (r_D q)^{-2}$, where r_D is the screening length (usually $r_D \ll \xi_0$). The electric potential around the vortex is determined by the

equation $\delta\mathcal{L}/\delta\phi(r) = -e\delta\tilde{n}(r) + (\phi/r_D^2 - \nabla^2\phi)/4\pi = 0$, whereas the charge density $e\delta n = -\nabla^2\phi/(4\pi) = e\delta\tilde{n} - \phi/(4\pi r_D^2)$, and we have introduced for future convenience the notation $e\delta\tilde{n}(r) = -\delta\mathcal{L}_{sc}/\delta\phi(r)$. In the weak-screening limit with $r_D \gg \xi$ we find $\delta n(r) \approx \delta\tilde{n}(r)$, whereas for $r_D \ll \xi$ the density is almost constant $\delta n(r) = \delta\tilde{n}(r)(r_D/\xi)^2 \sim n_0(\Delta/\varepsilon_F)^4$ (Ref. 16), and $\phi(r) = 4\pi r_D^2 \delta\tilde{n}(r)$. The key point is that the Hall conductivity can be expressed via the value $\delta\tilde{n} = \delta\tilde{n}(0)$, which *does not depend on screening*. This follows from the fact that the Coulomb part of the Lagrangian \mathcal{L}_C depends upon the longitudinal electric field only, and thus does not contain a contribution from the singular vortex-induced phase $\chi_v(\mathbf{r}) = \Theta(\mathbf{r} - \mathbf{r}_L)$. Therefore the 'topological' contribution to the Hall conductivity is determined by the effective action S_I , Eq. (3), with $n(r)$ replaced by $\delta\tilde{n}(r)$, and the result stated in Eq. (10) is recovered up to the replacement of δn by $\delta\tilde{n} = -\partial\Omega_{sc}/\partial\mu|_0^\infty$. In leading order of $(T_c - T)$, $\delta\tilde{n}$ can be expressed via experimentally accessible quantities:

$$\delta\tilde{n} = -\frac{H_c^2(T)}{4\pi} \frac{\partial \ln(T_c - T)}{\partial \mu}. \quad (12)$$

We will show now that the $\delta\tilde{n}$ term in the conductivity we found above is just the term which has been obtained from TDGL.^{4,5} We follow the approach of Ref. 17, where it has been proposed that the imaginary part of the relaxation time can be obtained from the dependence of the transition temperature T_c on the chemical potential μ . The first term in the GL thermodynamic potential is modified to $\Omega_{sc} = -\alpha(T_c + e\phi\partial T_c/\partial\mu - T)\psi^*\psi + \dots$. Introducing the gauge invariant combination $(2e\phi - i\partial/\partial t)$, one obtains the imaginary part of the relaxation time in the TDGL,^{5,17} $\gamma_2 = -(\alpha/2)\partial T_c/\partial\mu$. Without the Coulomb interaction the change in density can be obtained in the same way as before: $\delta n(r) = -\partial\Omega_{sc}/\partial\mu = \text{const} - 2\gamma_2|\psi(r)|^2$. The δn contribution to the Hall conductivity in Eq. (9) coincides with the result of Refs. 4 and 5 if the numerical parameter β ($-\alpha_2$ in the notation of Ref. 4) is equal to 1, corresponding to a value of the TDGL parameter $u \ll 1$ (Ref. 5). For large values of u the analysis of Refs. 4 and 5 gives a similar result but with an additional coefficient of order unity in front of the δn term. In these papers the condition of local electrical neutrality ($\nabla \cdot \mathbf{j} = 0$, i.e., $\delta n(r) = 0$) has been imposed in order to incorporate Coulomb screening. Actually a consistent treatment of Coulomb effects requires adding a term ϕ^2/r_D^2 to the GL free energy and allowing for local density variations. A microscopic calculation for superconductors with paramagnetic impurities¹⁸ shows that these numerical corrections to the $\delta\tilde{n}$ term become small for low enough concentrations of paramagnetic impurities.

The effect of vortex charge on the Hall effect was recently considered in Ref. 19. Although the treatment of the static charge distribution in the vortex core is the same as ours, the transverse force and the Hall conductivity found in Ref. 19 are smaller by a factor $\sim (r_D/\xi)^2 \ll 1$ and have the *opposite* sign as compared to our Eqs. (6) and (10), explicitly contradicting the well-known result for the Magnus force in the Galilean invariant case, where $n_0 = 0$.

The crucial point in the discussion of the experiments is the sign of $\delta\tilde{n}$. Making the estimates $\delta\tilde{n}/n = \text{sign}(\delta\tilde{n})(\Delta/\varepsilon_F)^2$ and $\omega_0 = \Delta^2/\varepsilon_F \ll \tau^{-1}$, one arrives at

$$\sigma_{xy} \approx \frac{n_0 e c}{B} \frac{\Delta^2}{E_F} [(\Delta \tau)^2 g - \text{sign}(\delta \tilde{n})] + \sigma_{xy}^n (1 - g). \quad (13)$$

The new term we have found is important in the dirty case $\Delta \tau < 1$ and can lead to a sign change if $\delta \tilde{n} > 0$ (the carrier density in the core is larger than outside). Let us consider this case in more detail in application to HTSCs. In these materials $\Delta \tau > 1$ at low temperature and $\Delta \tau \rightarrow 0$ at T_c . Note that what enters in Eq. (6) is $\Delta(T)$ rather than $\Delta(0)$. Thus at low temperatures we can neglect the $\delta \tilde{n}$ contribution, and the sign of σ_{xy} is positive (as in the normal state). As the temperature approaches T_c , $\Delta^{3/2} \tau / T_c^{1/2} \sim 1$ and σ_{xy} becomes negative. At high fields the normal-state Hall conductivity becomes important and the Hall effect changes sign back to the normal value at $B \sim H_{c2}(T) / \Delta \tau$ if $\Delta \tau > 1$. If $\Delta(T) \tau < 1$, the second sign change happens close to $H_{c2}(T)$, since the $\delta \tilde{n}$ contribution goes to zero $\propto (H_{c2} - B)$.⁵ These are just the two sign changes observed in Bi- and Tl-based materials. In 90-K YBCO the low temperature sign change back to the normal sign is usually not observed, since ρ_{xy} is unmeasurably small due to pinning. However, in those experiments where pinning is suppressed either by a high current²⁰ or by high frequencies²¹ the second sign change seems to be observed at low temperatures. The temperature dependence of the Hall conductivity (10) is in very good agreement with the data of Samoilov *et al.*,²² who found for TBCCO that the B^{-1} term in the Hall conductivity changes sign around 83 K and is proportional to $(T_c - T)$ at higher temperature.

In Ref. 23, Hall-angle evidence for the superclean regime in 60-K YBCO has been reported. In this material the Hall angle changes sign and becomes large ($\Theta_H \approx -1/2$) at low temperature. There are two different ways to treat these data in our scheme. The one taken in Ref. 23 is that in 60-K YBCO the superclean limit is realized with $\omega_0 \tau \gg 1$, and the Magnus force has a “wrong” sign due to the complicated structure of the Fermi surface. Another possible scenario is that this material is not superclean but moderately clean, with $\omega_0 \tau \ll 1$, and has the same sign of $\delta \tilde{n}$ as the 90-K material, but with a larger numerical value (the 60-K compound is closer to half filling, and the dependence on chemical potential is sharper than for the 90-K compound). To estimate the value of $\delta \tilde{n} / n$ we note that the additional term in the Hall conductivity is $-\delta \tilde{n} e c / B$, whereas the normal-state Hall resistivity is $\rho_{xy}^n = B / n e c$. Multiplying these two quantities, we obtain an estimate for $\delta \tilde{n} / n$. Analysis of experiments^{22,23} gives $\delta \tilde{n} / n = 10^{-3}$ for the Tl compound and 0.03 (0.07) for 90-K (60-K) YBCO, respectively. Thus the difference between these two Y-based materials seems to be much smaller than between Y- and Tl-based compounds. The experimental data²³ for 60-K YBCO can be understood under the assumption that at low temperatures $\omega_0 \tau \approx \delta \tilde{n} / n \approx 0.1 \ll 1$; in that case only the second term in (10) is important, and the Hall angle takes a value of order unity (since the longitudinal conductivity contains a factor $\omega_0 \tau$), although the material can still be rather dirty (note that the 60-K material is believed to be dirtier than the 90-K material). On the other hand, the 90-K YBCO is expected to have a larger low- T value of $\omega_0 \tau$ and a smaller value of $\delta \tilde{n} / n$ (see the above estimate), which makes the observation of the second sign change in this material^{20,21} quite natural. The second of the proposed scenarios seems preferable to us since it does not involve any *ad hoc* hypothesis involving a complicated Fermi surface and suggests a unified description of both the 60-K and 90-K compounds.

In a 3D BCS model, T_c depends upon the density of states and increases with

increasing density, leading to a positive $\partial T_c / \partial \mu$ and thus $\delta \tilde{n} < 0$. A simple example of a situation with $\delta \tilde{n} > 0$ is a 2D system with a spectrum $\epsilon_k = k^2/2m + k^4/4m^2 \epsilon_0$. In this case $\delta \tilde{n} \sim 2\Delta^2/\epsilon_0 \Lambda$, with the BCS coupling constant Λ . The case of HTSC is complicated by the fact that the normal state Hall effect has a hole-like sign, although from simple electron counting the Fermi surface should have an electron-like shape. It would be tempting to relate the $\delta \tilde{n}$ term with the doping dependence of T_c via Eq. (12), which would lead to the conclusion that a sign change should occur for overdoped material. This is dangerous, however, since in some versions of RVB-like theories²⁴ the doping dependence of T_c and that of the superconducting energy away from T_c may have opposite signs.

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