Relativistic self-focusing in a plasma

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The general properties of stationary waveguides formed in plasma by relativistic self-focusing of ultrapowerful laser pulses are investigated on the basis of a modified nonlinear Schrödinger equation. It is shown that the waveguides can transport arbitrary amounts of energy and that their characteristic size has a lower limit. The formation of the waveguides is studied. It is found that for ultrahigh laser power levels a qualitatively new regime of self-excited oscillations of the "vacuum channel" appears. © *1995 American Institute of Physics.*

1. Relativistic self-focusing of light in the strongly nonlinear regime, when the quiver velocity of electrons is comparable to the velocity of light, has been studied extensively (see, for example, Refs. 1 –5). To realize this phenomenon experimentally, the laser radiation power P must exceed the critical value $P_{cr}=1.7\times 10^{10}$ (n_c/n) W. Here n is the density of the plasma and $n_c=\omega_0^2 m/4\pi e^2$ is the critical density corresponding to the frequency ω_0 of the light. In the last few years the case in which the radiation power exceeds P_{cr} by not more than an order of magnitude has mainly been studied. The impressive progress made in laser technology now makes it possible to obtain much higher radiation fluxes. For example, at the Livermore National Laboratory a $\sim 10^{14}$-W laser has already been put into operation and work on a petawatt design is being completed (Fast Ignitor Project),[6,7] which for $n \sim n_c$ corresponds to power levels tens of thousands of times higher than the critical level. All this progress makes it necessary to study relativistic self-focusing in the supercritical regime. In the present letter we show how the structure of the steady-state solutions changes with increasing power level. Qualitative arguments and numerical modeling are used to demonstrate the formation in the plasma of a stable "vacuum" waveguide, in which unlimited laser radiation power can in principle be transported.

2. We shall study the evolution of powerful laser pulses whose duration is limited by the conditions

$$\omega_{pi}^{-1} \gg \tau \gg \omega_{pe}^{-1} .$$

When these conditions hold, the motion of the ions and the generation of comoving fields (wake fields) can be ignored.

The equation describing in the quasistationary approximation the self-effect of a field, with allowance for the relativistic effects, has the form[1,2]

$$2ia_z+\Delta a_\perp+\left(1-\frac{n}{\gamma}\right)a=0. \quad (1)$$

Without loss of generality, we assume that the radiation is circularly polarized. Here $a=eA/mc^2$ is the envelope of the vector potential in dimensionless form, $\gamma=\sqrt{1+|a|^2}$ is the relativistic factor, the transverse coordinate $\mathbf{r}_\perp$ is normalized to $(\omega_p/c)^{-1}$, and the z coordinate in the direction of propagation is $z\to z(\omega_p^2/\omega_0 c)$, $\omega_p/\omega_0=\sqrt{n/n_c}\ll 1$. The dimensionless electron density is

$$n=\begin{cases}1+\Delta_\perp\gamma, & 1+\Delta_\perp\gamma>0\\ 0, & 1+\Delta_\perp\gamma\leqslant 0.\end{cases} \quad (2)$$

This expression describes the "cavitation" — expulsion of electrons from the region where the light field is strong.

In the limit $a\ll 1$ Eq. (1) becomes the classical nonlinear Schrödinger equation (NSE), describing, specifically, the self-focusing of light in a Kerr medium. The properties of the two-dimensional NSE have been studied in detail. We call attention to the classical papers of Talanov[8] and Vlasov *et al.*[9] and reviews of Litvak,[10] Zakharov,[11] and Petviashvili *et al.*[12]. In this limit, there is no cavitation and the stationary waveguides are unstable. As the radiation intensity increases, the mechanism of self-focusing of light changes. In our letter we examine the properties of self-focusing of ultrastrong light beams, when cavitation becomes an important factor.

In the NSE limit a laser pulse with power

$$P=\frac{1}{2\pi}\int |a|^2 d\mathbf{r}_\perp$$

separates at $P>P_{cr}$ into fragments, in each of which $P\approx P_{cr}$. Each fragment collapses to scales on which dissipative mechanisms which stop the development of a singularity come into play. These strongly nonstationary processes undoubtedly complicate the transfer of laser radiation in the plasma. We shall show for the radially symmetric case that in the strongly nonlinear ("relativistic") case stable waveguides which can carry an arbitrarily large amount of energy can form. Most of the energy propagates inside the "vacuum tube" from which electrons are completely expelled by the field. Stimulated scattering by the plasma waves, which could destroy the beam, is suppressed in this case because of the expulsion of the electrons and the weakness of the interaction with the ions.

It is well known that on the basis of the classical nonlinear Schrödinger equation with a cubic nonlinearity the development of an instability of an arbitrary initial distribution of high power results in self-focusing of light and divergence of the amplitude over a finite distance. In our model a singularity is not formed. The growth of the field amplitude results in the expulsion of the electrons from the central part of the light beam. This means that the main part of the pulse propagates in a "vacuum tube," where there is no nonlinearity, and therefore a singularity of the field is impossible. In such a situation it is natural to expect stationary solutions, for which the radiation (ponderomotive) pressure is balanced by the action of the electric field produced by the charge separation. In Ref. 13 it was proved for simple forms of a saturating nonlinearity that stationary solitons are stable, and in Ref. 14 it was shown that under the conditions of stationary self-focusing a singularity of the wave field does not appear and an oscillating waveguide is formed. In our case, in contrast to the case considered in Refs. 3 , 13 , and 14 , the nonlinearity is nonlocal.

Axisymmetric profiles of the electric field and electron density were first obtained in Refs. 1 and 2 . It was also shown in those studies that there exists a countable number of characteristic modes. We shall consider the family of stationary waveguides and their properties.

3. The steady-state solutions $a \sim a(r)\exp(i\lambda^2 z)$ are described by the equation

$$\Delta_\perp a+(1-2\lambda^2)a-\frac{n(a)a}{\sqrt{1+|a|^2}}=0 \tag{3}$$

with the boundary conditions

$$\left.\frac{da}{dr}\right|_{r=0}=0; \quad a\to 0 \text{ as } r\to\infty.$$

We shall analyze the solutions of Eq. (2) which take into account the cavitation in a region of size R. Inside the cavitation region we have

$$a(r)=AJ_0(\sqrt{1-2\lambda^2}r).$$

At $r=R$ this solution must be joined to the solution of the nonlinear problem in the "outer region" under the condition that $a\to 0$ as $r\to\infty$:

$$a(r)\Big|_R=AJ_0\sqrt{1-2\lambda^2}R), \tag{4}$$

$$\left.\frac{da}{dr}\right|_R=-A\sqrt{1-2\lambda^2}J_1(\sqrt{1-2\lambda^2}R). \tag{5}$$

Vanishing of the electron density, $n=0$, at $r=R$ gives the condition

$$1-\frac{A^2(1-2\lambda^2)J_0^2(\sqrt{1-2\lambda^2}R)}{\sqrt{1+A^2J_0^2(\sqrt{1-2\lambda^2}R)}}+\frac{A^2(1-2\lambda^2)J_1^2(\sqrt{1-2\lambda^2}R)}{(1+A^2J_0^2(\sqrt{1-2\lambda^2}R))^{3/2}}=0. \tag{6}$$

We restrict the analysis below to the main mode of Eq. (3): Numerical simulation of Eq. (1) shows that an arbitrary initial condition (as first noted in Ref. 2) evolves into this mode.

The power of the steady-state solution is the sum of two components:

$$P_{sol}=P_{cav}+P_{out},$$

$$P_{cav}=\int_0^R|a|^2rdr=\frac{A^2}{1-2\lambda^2}\int_0^{s_0}J_0^2(s)sds,$$

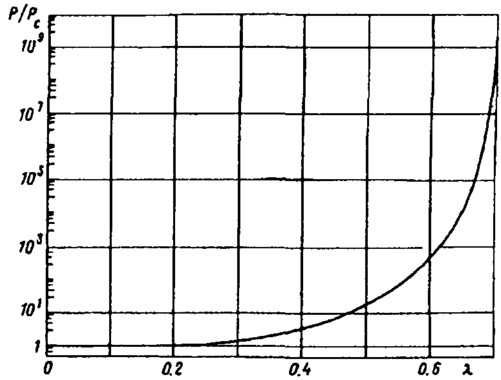

FIG. 1. Plot of the power P/P_{cr} in the stationary channel as a function of λ.

where $s_0 = \sqrt{1-2\lambda^2}R$. It follows from Eqs. (4) and (5) that s_0 cannot vanish if $\lambda^2 \to 1/2$. Therefore P_{cav} is a monotonic function of λ. This means that unlimited power can in principle be transported along the stationary "channel." Similarly, it is easy to show that the radius of the "cavity" increases as $\lambda^2 \to 1/2$.

The existence of a limiting value of λ^2 means that the gradient of the steady-state solution cannot become very large, so that the paraxial approximation [and, correspondingly, Eq. (3)] remains valid as the field increases.

Figure 1 is a plot of the power in the stationary channel as a function of λ. This plot was obtained by solving Eq. (3) numerically. It confirms that there is no limit on the power in the stationary channel.

Figure 2 shows that as the intensity of the radiation increases, an increasingly larger fraction of the power is "trapped" in the cavity formed by electronic cavitation.

Another important conclusion follows from Fig. 3: There exists a minimum radius R_0 of the focused light beam. We underscore that in the dimensionless variables

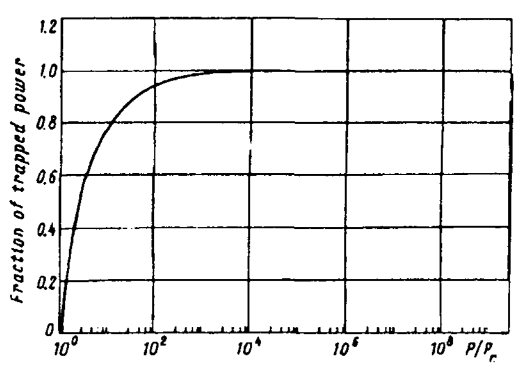

FIG. 2. Fraction of the power trapped in a cavity as a function of the total power P/P_{cr}.

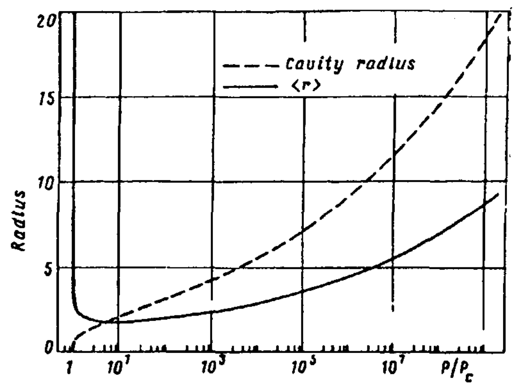

FIG. 3. Plot of the characteristic width $\langle r\rangle=(1/P\int_0^\infty|a|^2r^3dr)^{1/2}$ of the waveguide and the radius of the cavitation region as a function of the power P/P_{cr}.

$R_0/\lambda_{rad}=1.75\sqrt{n_{cr}/n}\gg 1$, and therefore the paraxial approximation remains valid even in this case.

The numerical solution of Eq. (1) shows (see also Ref. 2) that the manner in which a steady-state solution is established can be very different, depending on the amplitude and characteristic size of the initial distribution of the field, and also on the degree of focusing of the laser beam. However, the main parameter determining the evolution of ultrashort pulses is the radiation power. A light pulse of "moderate intensity," $P/P_{cr}<100-150$, leads to the formation of a stationary channel, in which the structure of the field and the electron density distribution are close to the corresponding solutions of the steady-state problem. Most of the power in the initial beam is trapped in this channel, and the remaining fraction is "radiated to infinity" in the process of the establishment of the steady-state solution.

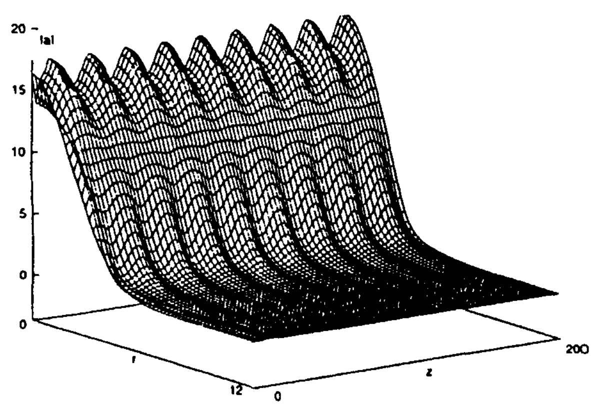

FIG. 4. Evolution of the amplitude $|a(r,z)|$ of the vector potential for a pulse with power $P/P_{cr}\approx 160$ and the initial distribution $a(z=0,r)=16.33\exp(-r^2/9)$.

Ultrastrong pulses $P/P_{cr}>150$, which have already been obtained experimentally,[6] give rise to a qualitatively new regime of undamped, periodic, self-excited oscillations of the "vacuum-tube" wall and of the distribution of the light field in the channel (see Fig. 4). The period and amplitude of the self-excited oscillations are determined now not only by the power level of the laser beam, but also by the local intensity.

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