Maximum in the temperature dependence of the critical current in bulk random Josephson networks

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The superconducting properties of the metastable alloy Zn–Sb are studied in a set of intermediate states of a bulk sample during its transformation to the insulating state. As the temperature is lowered below the transition, the critical current rises till $0.7T_c$, then drops and finally reaches the limiting value. The latter scales with the normal resistance of the sample as R_n^{-2}, whereas at the maximum at $0.7T_c$ the scaling power is different, $R_n^{-3/2}$. A comparison with the properties of small superconducting tunnel junctions is made. © *1995 American Institute of Physics.*

Some substances exhibit weak superconductivity and demonstrate Josephson properties in the bulk. Superconducting ceramics with poor contacts between grains are an example of such type of materials.[1] In the present study, we have investigated the behavior of such substances using the quenched high-pressure phase of alloy Zn–Sb.[2] In storage at liquid nitrogen temperature after quenching, this alloy remained a metastable crystalline superconductor. Slow, careful heating induced a gradual transition into an insulating state. We could trace the evolution of the superconducting response by chopping the transformation by abrupt cooling at different stages and studying the intermediate states at low temperatures. At a practically fixed temperature T_c of the onset of the transition, the resistive transition curves first acquire a low-temperature tail, and then the transition becomes incomplete and, next, quasireentrant. The huge interval of resistivities of the material while spanning this set of states is due to the inhomogeneous character of the transition: The insulating phase appears presumably in a fractal-like fashion, making the current paths along the metallic phase long and confined.[3] Similar behavior has been also found in Ga–Sb (Ref. 4) and Cd–Sb (Ref. 5) alloys.

In the preceding stage, we studied $J-V$ characteristics in a quasireentrant state,[2,6] with a resistance at the minimum of $R_{\min}\approx 0.5R_n$. A maximum of the critical current observed at $T/T_c\approx 0.7$ was the most noteworthy feature of those experiments. In this paper, we continue our investigation of the nature of this maximum.

Consider a 3D lattice of Josephson junctions. There exist two logical possibilities. First, each single junction may have the well-known temperature dependence of the critical current,[7] with the maximum for the bulk sample critical current being caused by the temperature-dependent nonuniformity of the current distribution over the network.[6] Secondly, the observed maximum may reflect the properties of individual junctions. The aim of this paper is to study this second possibility. For this purpose, we selected

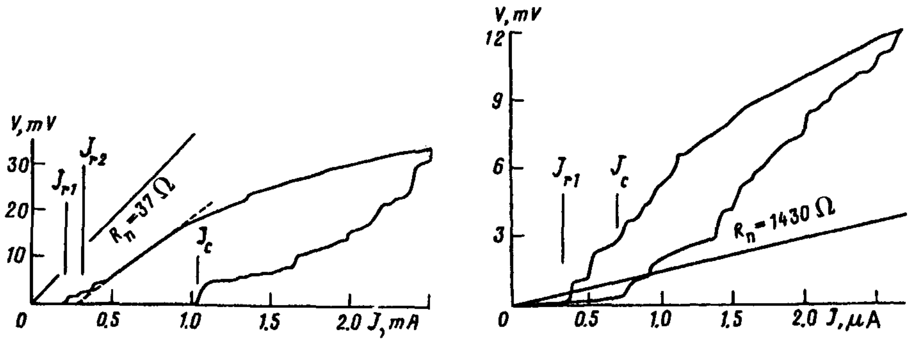

FIG. 1. Hysteresis loops on the $J-V$ curves in two states: that with $R_n=37\ \Omega$ (above) and with $R_n=1430\ \Omega$ (below). $T=1.2$ K. The dashed line on the upper loop is the tangent to the inflection point. Also shown are straight lines which correspond to values of the normal resistance. The positions of the critical currents are marked.

moderate-resistance states of the alloy $Zn_{41}Sb_{59}$ with tails in the superconducting transition and matched their behavior against the behavior of single small superconducting tunnel junctions studied by the Tinkham group.[8,9]

We present here the measurements of a set of ten successive states of a $Zn_{41}Sb_{59}$ sample with normal resistance values R_n from 3 Ω to 1500 Ω (approximate resistivity values, respectively, from 0.3 $\Omega\cdot$cm to 150 $\Omega\cdot$cm). These were the states with tailed transitions. In the states with smaller R_n the transition was sharp, and we could not reach the critical current because of overheating of the contacts. The transition in the last of the presented states, one with $R_n=1500\ \Omega$, was already incomplete at the lowest temperature used (1.2 K).

Two typical $J-V$ curves with hysteresis loops are shown in Fig. 1. Both branches of the loops have random steps, presumably due to the discrete structure of the network. Voltage jumps up (down) indicate the switching off (on) of the shorting Josephson currents in some bonds of the network. The $J-V$ loops display two critical currents, J_c on the rising branch and J_r (called the recapture critical current[8]) on the descending branch of the loop. For J_c rather poor reproduction was typical: The value of J_c changed from run to run. The value of J_r reproduced much better, but it had its own disadvantage. Figure 1 shows that there are different possible ways of defining it. Point J_{r1} is defined as the position of the last jump down of the voltage, while point J_{r2} is the J-axis intercept of the tangent to the inflection point $\partial^2 V/\partial^2 J=0$. However, this uncertainty did not affect our conclusions.

The initial part of the $J-V$ curve in the high-resistance state has a finite slope because the resistance in this state does not reach zero even at 1.2 K. Second, in this state most of the (J,V) points on the loop correspond to resistances $R=V/J$ much larger than R_n. Hence, the resistance in the loop region is controlled by the single-particle tunneling between superconducting parts of the network.

The loops presented in Fig. 1 were obtained at low temperature. On rising temperature the loops become narrower. Then both the hysteresis and the steps disappear. At

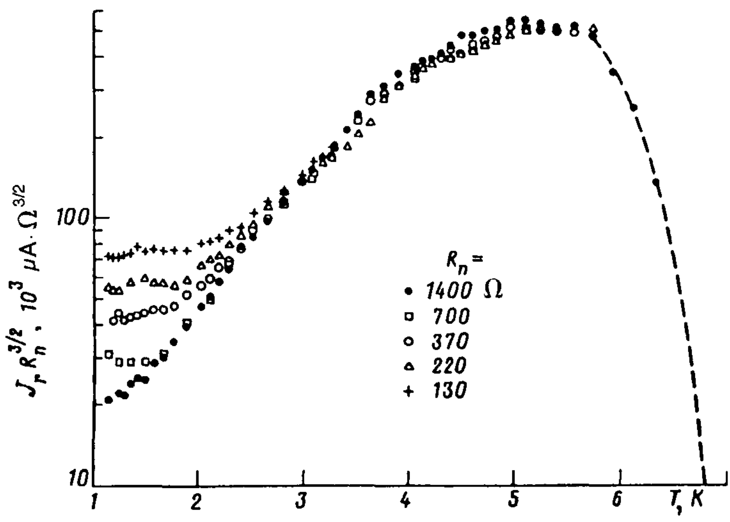

FIG. 2. Temperature dependence or the critical current J_r normalized by the normal resistance R_n to the power 3/2 in five different states.

$t \equiv T/T_c \gtrsim 0.5$, the critical currents coincide: $J_c = J_r$. At $t \gtrsim 0.8$, the precision of measurements of J_r falls down because the curve $V(J)$ becomes very smooth, and the inflection point shifts to higher voltages and its position becomes uncertain.

In Fig. 2 we display the temperature dependence of $J_r(T)$ for a number of different states. When normalized by the factor $R_n^{3/2}$ the curves coincide precisely above $t \gtrsim 0.3$, in the vicinity of maximum $J_r(T) = J_{r,\max}$. At low temperatures the curves diverge, and each one approaches its own limiting value $J_{r,\min}$. According to Fig. 3,

$$J_{r,\max} \propto R_n^{-3/2}, \quad (1)$$

$$J_{r,\min} \propto R_n^{-2}. \quad (2)$$

Relations (1), (2) are the main experimental results of this paper.

Note the remarkable similarity between properties of our samples and those of single small high-resistance tunnel junctions: For such junctions the critical current also has maximum in the temperature dependence at $T \approx 0.8T_c$, and which is more pronounced in the states with high normal resistance;[8] it also ceases to decrease and reaches a limiting value at low temperature,[9] and the low temperature critical current is also inversely proportional to the square of the normal resistance.[8] That is why we start our discussion by listing the main expressions and relations for a small single Josephson junction which has a shunting capacitance C and a shunting resistance $r(T)$. We will use lower case for the currents, resistances, and voltages for a single junction to distinguish them from those in the bulk samples. The dynamics of a junction depends on the relations between the temperature T and two energies: the Josephson coupling energy

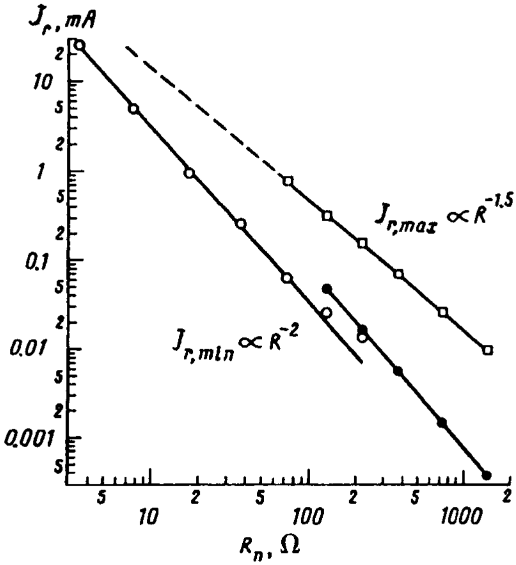

FIG. 3. Dependence of the critical currents J_r on the normal resistance R_n of the state. Squares—at the maximum of the $J_r(T)$ curve, approximately at $T=0.7T_c$. Circles—the low temperature limiting value, open circles—defined by steps, full circles—defined by the derivative.

$$E_J=\frac{\pi}{4}\left(\frac{\hbar/e^2}{r_n}\right)\Delta \tag{3}$$

(here $\Delta(T)$ is the superconducting gap), and the Coulomb energy $E_C=e^2/2C$. It is clear from (3) that when the normal resistance is small, i.e., when

$$r_n \ll \hbar/e^2 \approx 4\ \mathrm{k}\Omega, \quad \text{then} \quad E_J \gg \Delta > T_c, \quad \text{and hence one has } E_J \gg T \tag{4}$$

below the transition temperature. Then the thermal fluctuations may be neglected and the critical current j_{c0} is[7]

$$j_{c0}(T)=\frac{\pi}{4}\frac{2\Delta(T)}{er_n}\tanh\frac{\Delta(T)}{2T}=\frac{2e}{\hbar}E_J\ \tanh\frac{\Delta(T)}{2T}. \tag{5}$$

The phase difference φ of the superconducting order parameter across a junction biased with a current i satisfies the equation[10]

$$C\ddot{\varphi}+\dot{\varphi}/r+(2e/\hbar)^2U'(\varphi)=0, \quad U(\varphi)=-E_J\cos\varphi-(\hbar/2e)i\varphi. \tag{6}$$

Equation (6) has two solutions for the average voltage across the junction in some interval of currents $j_{c0}>j>j_{r0}$, i.e., it describes hysteretic behavior of the junction. The recapture critical current j_{r0} can be expressed through the dimensionless damping parameter β_c (Ref. 10)

$$\beta_c=(2e/\hbar)j_{c0}r^2C. \tag{7}$$

When the parameter is large, $\beta_c \gg 1$, then

$$j_{r0}=(4/\pi)j_{c0}\beta_c^{-1/2}=\frac{2}{\pi}\left(\frac{2\hbar}{eC}\right)^{1/2}\frac{j_{c0}^{1/2}}{r}. \tag{8}$$

If r in expression (4) were determined by the single-particle tunneling current, it would be given by[5,11]

$$r=r_{\text{tun}}=r_n(T/\Delta)\exp(\Delta/T). \tag{9}$$

Then j_{r0} would tend to zero at low temperature.

For junctions with higher r_n, when inequalities (4) no longer hold, fluctuations become important. Then a term with a Gaussian fluctuating current should be added into Eq. (6). The fluctuations shift the currents j_{c0} and $j_{r\,0}$ toward each other and bring the other critical currents j_c and j_r into line. According to Ref. 8, the shift of j_{c0} is larger:

$$j_{r\,0}\lesssim j_r\leqslant j_c<j_{c0}, \tag{10}$$

so that in the nonhysteretic regime, when $j_r=j_c$, the measured critical current should be interpreted as $j_{r\,0}$ (Ref. 8).

We can successfully apply this interpretation to our results. Since the dissipative channel of the junction is determined by a single-particle tunneling current, by taking the upper-case variables for the j's and r's in expressions (2)–(5) we obtain from Eqs. (2), (4), and (9) the scaling relation (1). Note that this does not necessarily mean that our sample is in essence a chain of small junctions. If it were so, it would follow from (4) that the number of junctions in the chain and the capacitance C of each of them remain the same during the transformation of the sample. This seems improbable. Here (and below) we want only to emphasize that the behavior of a bulk sample fits the theory intended for a single junction.

The comments upon the second relation, Eq. (2), are not so straightforward.

To start with, $J_{r\,0}$ does not tend to zero with $T\to 0$ despite Eqs. (4) and (9). A similar behavior of single junctions[9] is explained by the effect of the loading impedance Z_{ac} of the leads. According to Ref. 9, there exists a minimum value $j_{r\,0}^{\min}$ which is determined by the balance between the dc power fed to the junction and the power dissipated at high frequency in the leads. The Josephson channel is the source of an ac current $j_{c0}\sin(\omega t)$, and the average power dissipated at ω is of the order of $j_{c0}^2 Z_{ac}$. Since the power in the dc single-particle tunneling channel is $j_{r\,0}^{\min}(2\Delta/e)$, we get the relation[9]

$$j_{r\,0}^{\min}\propto e j_{c0}^{2}Z_{ac}/\Delta. \tag{11}$$

Relation (2) follows from Eq. (6) if we assume that Z_{ac} is independent of r_n and $j_{c0}\propto(1/r_n)$ in accordance with Eq. (2), and after we return to the upper-case variables.

The scaling law $\propto r_n^{-2}$ has been found in Ref. 8 also, although for j_{c0} instead of $j_{r\,0}$. The explanation which was proposed in Ref. 8 used different terms.

Equation (6) also describes a particle moving with friction along the φ axis in a tilted cosinusoidal potential U (the "tilted washboard model") —see, for example, the review.[12] According to the Josephson relation $v=(\hbar/2e)\dot{\varphi}$, the trapping of the phase-ball in a potential well means zero voltage across the junction ($v=0$) and a current without dissipation. In the classical description, the particle can be shifted toward the neighbor well due to either a regular force $\partial U/\partial\varphi$ or thermoactivated fluctuations. However, if the capacitance C is small and the Coulomb energy E_C is comparable to E_J, then the quantum description comes into play. Then the phase-ball may tunnel to the neighboring well or even become completely delocalized as an electron in a periodic potential. The energy of the ground state (of the "bottom of the Bloch band") was calculated in Ref. 8 in the limit $E_C \gg E_J$ and turned to be $-E_J^2/8E_C$, instead of $-E_J$ in the opposite limit. A similar result was obtained in Ref. 13. It was argued in Ref. 8 that, because of this crossover, E_J should be replaced by $E_J^2/8E_C$ in the expression (2) for j_{c0} of a single junction with small C. This implies a scaling law similar to relation (2).

These arguments relate to j_{c0} as well as to experimental observations obtained at very low temperatures.[8] Our experimental relation (2) contains J_r. Hence, the resemblance at this point is not so obvious.

In summary, a similarity is established between the properties of the bulk metastable Zn–Sb alloy in different intermediate states and of small high-resistance, low-capacitance Josephson junctions. Hence, the equations describing these two subjects should be similar. We doubt that our substance should be treated as a multitude of separated and well-defined small Josephson junctions. In particular, this would mean that for all the states under consideration the network of junctions and their density are the same, and that only the parameters of each of them vary from state to state. The existing structure model of the transformation in the Zn–Sb alloy[3] more favors thin superconducting wires and constrictions. Note that a network of phase-tunneling effects is possible for such objects as well.[14]

It is also unclear to what extent the discreteness of the substance is important and whether the finite probability of phase slipping cannot be attributed to each point of the 3D space, as happens when thin long 1D wires are considered. This is the central point of the problem, and we plan to study other similar materials from the same point of view.

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