Nonlocalizability of contact phenomena in composite, two-dimensional charged systems

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(Submitted 19 October 1995; resubmitted 9 November 1995)
Pis'ma Zh. Éksp. Teor. Fiz. **62**, No. 11, 879–884 (10 December 1995)

A two-imensional Corbino disk is used as an example to show that contact of an initially neutral, two-dimensional electronic system (2DEG) with "external" metal electrodes, whose internal characteristics are different from the 2DEG (the work functions are different), leads to breakdown of local neutrality of the two-dimensional part of the disk over the entire width of the disk. A classical description is given of anomalously long contact nonuniformities in the distribution of the 2D electron density. The role of quantum corrections to the classical theory is discussed. The effect of contact nonuniformities of the density on the linear part of the current-voltage characteristic of a Corbino sample under the conditions of the quantum Hall effect is discussed. The available experimental data for a Corbino disk, which indicate the existence of an equilibrium nonuniformity of the 2D electron density in the disk, are evaluated. In particular, the contact potential difference in this system is estimated. © *1995 American Institute of Physics.*

It is well known that when two 3D metals with different work functions W_i come into contact with one another, electrons are transferred from metal into the other so as to equalize their electrochemical potentials.[1] In good metals local neutrality correspondingly breaks down on scales of the order of interatomic distances and in low-conductivity samples over a distance of the order of the Debye length. A similar problem also arises for contacts between 2D and 3D conducting systems. In this case it is found that in a 2D system there is no characteristic length over which local neutrality of the 2DEG is destroyed; i.e., the contact perturbation of the electron density encompasses virtually the entire accessible 2D region. Taking into account the sensitivity of many two-dimensional problems [especially in the quantum Hall effect (QHE) regime] to the local density of a 2DEG, it can be assumed that the anomalous extent of the contact perturbation of the electron density in a 2DEG should play an appreciable role in the effective behavior of the 2DEG. It will be shown below that this is indeed so for a Corbino disk. Specifically, the current-voltage (I–V) characteristic of the Corbino sample is modified in the QHE regime. There appears in the problem an effective width $2a$ of the 2D region of the Corbino disk, where the condition that the filling factor be an integer actually holds. This width may be appreciably less than the nominal dimensions $2w$ of the 2D Corbino region, if the difference in the work functions between the 2DEG and its metal "edges" is sufficiently large. It is obvious that the width $2a$ appears in the definition of the I–V

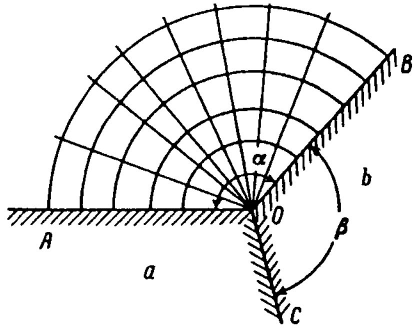

FIG. 1.

characteristics, controlling the total voltage drop between the Corbino "edges." The existence of an effective region $a<w$ for a Corbino disk under QHE conditions has recently been observed experimentally.[2]

1. In presenting specific results, we recall first the classical electrostatics of the contact of two metals with different volume characteristics, which is effectively taken into account by introducing the contact potential difference ϕ_{ab} (Ref. 1),

$$e\phi_{ab}=W_a-W_b, \tag{1}$$

where W_i are the so-called work functions of the corresponding metals, and e is the elementary charge.

Now, if the metals A and B with open boundaries are in contact (see Fig. 1), then an electric field with potential ϕ appears in the vacuum gap between the faces AO and OB:[1]

$$\phi(\theta)=\phi_{ab}\frac{\theta}{\alpha}, \tag{2}$$

where α is the angle between the faces OA and OB.

The intensity of the field E is

$$E_r=-\frac{1}{r}\frac{\partial\phi}{\partial\theta}=\frac{\phi_{ab}}{\alpha r}, \quad E_r=4\pi e\,\delta n(r), \tag{3}$$

and it decreases inversely as the distance r from the point O. The surface charge density $\delta n(r)$, distributed along the free faces OA and OB, is related to this field.

Now, let a 2D electronic system play the role of the ray OB, and for simplicity let the angles α and β in the figure be equal to one another. The additional surface charge density from the relation (3) is also a correction to the uniform electron density n_s of this system. In other words, the contact of a 2DEG with conducting electrodes (2D or 3D) can disrupt appreciably, even at large distances, the spatial uniformity of the 2D electron density, if the uniformity is present in the 2D sample with no contacts. It is natural to call the disruption of the uniformity of the 2D electron density as a result of contact with the external electrodes a Coulomb proximity effect.

If a 3D thin film is deposited along the ray OB, then the charges of the type (3) with density that decays into the film over some Debye length influence the boundary conditions that determine the electron distribution inside the film. Therefore, even in this case we can legitimately talk about long-range Coulomb proximity effects.

2. The divergence of the field E from Eq. (3) at small distances and the integral divergence for the total effective surface charge, which also follows from Eq. (3), can be eliminated by obvious means. The first singularity is removed by introducing into the problem quantum corrections to the condition of thermodynamic equilibrium. The integral divergence of the charge is eliminated by limiting the dimensions of the 2D system, which is obvious for a Corbino disk.

We shall implement this program for a specially prepared, unscreened, degenerate heterostructure with a stepped donor density distribution $n_d(x)$. The donors are arranged in the $z=0$ plane according to the law

$$n_d(x)=N_d, \quad |x|>w; \quad n_d(x)=n_d, \quad |x|<w. \tag{4}$$

Here $2w$ is the width of a step in the donor distribution along the x axis. This step plays the role of the central part of the Corbino sample in the one-dimensional approximation. The electrons lie in the same plane as the donors; i.e., the thickness of the spacer between the electrons and the donors is set equal to zero.

The condition of equilibrium in the electronic system is

$$e\phi(x)+\frac{\pi\hbar^2}{2m_*}n(x)=\text{const}, \tag{5}$$

$$e\phi(x)=\frac{2e^2}{\kappa}\int_{-\infty}^{+\infty}\delta n(s)\ln(x-s)ds, \tag{6}$$

$$\delta n(x)=n(x)-n_d(x). \tag{7}$$

Here κ is the dielectric constant, $n_d(x)$ is taken from Eq. (4), and the thermodynamic part of the electrochemical potential is written in the standard Thomas–Fermi approximation.

To solve Eq. (5) for $\delta n(x)$, we differentiate this equation with respect to x. As a result, we obtain

$$\frac{\pi\hbar^2}{2m_*}\frac{d\delta n(x)}{dx}-\frac{\pi\hbar^2(N_d-n_d)}{2m_*}[\delta(x+w)-\delta(x-w)]+\frac{2e^2}{\kappa}\int_{-\infty}^{+\infty}\frac{\delta n(s)ds}{(x-s)}=0. \tag{8}$$

Fourier transforming Eq. (8) gives δn_q

$$\delta n(x)=\frac{1}{\sqrt{2}\pi}\int_{-\infty}^{+\infty}\delta n_q\exp(iqx)dq, \tag{9}$$

$$\sqrt{2}\pi\delta n_q=2a_b^*(N_d-n_d)\frac{\sin(qw)}{(4+a_b^*q)}, \quad a_b^*=\frac{\kappa\hbar^2}{m_*e^2}, \quad q\geqslant 0. \tag{10}$$

It is obvious that the finiteness of the effective Bohr radius a_b^* guarantees that the integral (9) will converge at the "dangerous" points $x=\pm w$. The above-noted integral divergences are absent to the extent that the width w is finite. In particular, the excess electron density $\delta n(0)$ is found to be

$$\delta n(0)=\frac{1}{\sqrt{2}\pi}\int_{-\infty}^{+\infty}\delta n_q dq=\frac{2}{\pi}(N_d-n_d)[\mathrm{ci}(\gamma)\sin(\gamma)-\mathrm{si}(\gamma)\cos(\gamma)], \tag{11}$$

$$\gamma=4w/a_b^*. \tag{11a}$$

Here ci(x) and si(x) are the integral cosine and sine.

The classical case, which is analogous to Eqs. (1) and (2), is obtained from this model if

$$a_b^*\ll w,\quad \frac{\pi\hbar^2}{2m_*}(N_d-n_d)=e\phi_{ab}. \tag{12}$$

Here

$$\delta n_0(x)=\frac{a_b^* w}{2\pi}\frac{(N_d-n_d)}{(w^2-x^2)}=\frac{\kappa w\phi_{ab}}{\pi^2 e(w^2-x^2)}. \tag{13}$$

In the case $a_b^*\ll w$ the expression (13) works well far from the points $x=\pm w$. Using Eq. (13), it is easy to show, for example, that at large distances $|x|>w$ the correction $\delta n(x)$ is inversely proportional to x^2, i.e., it is integrable. The scale of the nonuniformity in depth of 2DEG is also obvious. Specifically, at $x=0$ Eq. (13) agrees with the definition of $\delta n(0)$ in Eq. (11).

3. Among the characteristics of a 2D Corbino disk that are sensitive to the Coulomb proximity effects, we mention first the current-voltage (I–V) characteristic. Let us consider, for example, the I–V characteristic under conditions of an integer quantum Hall effect. Which part of the disk satisfies the requirement that the filling factor be an integer if the 2D system is initially spatially nonuniform? To answer the question, we turn to the results of Ref. 3 . It follows from Ref. 3 that if the curvature $n''(0)$ of the classical electron density distribution is known at the extremal point of the distribution [the point where the first derivative vanishes, $n'(0)=0$], then width $2a$ of the plateau on which the filling factor remains an integer $\nu_l=1,2,3,\dots$ is given by

$$n''(0)a^2/4=[\nu(0)-\nu_l]n_H,\quad n''(0)=d^2n(0)/dx^2, \tag{14}$$

$$\nu(0)=n(0)/n_H,\quad n_H^{-1}=\pi l_H^2,\quad l_H^2=\frac{c\hbar}{eH}. \tag{14a}$$

Here H is the intensity of the magnetic field which is oriented perpendicular to the plane of the disk.

The width $2a$ is maximum when the additional Coulomb energy, which is produced as a result of the deformation of the initial classical electron density $n(x)$, is equal to the cyclotron energy $\hbar\omega_c$. Here

$$a_{\max}^3 = \frac{3\kappa\hbar\omega_c}{\pi e^2|n''(0)|}. \tag{15}$$

Substituting into the definition (15) the quantity $n''(0)$ which follows from Eqs. (9) and (13), we obtain

$$(a_{\max}/w)^3 = \frac{3\pi\hbar\omega_c}{2e\phi_{ab}}. \tag{16}$$

Therefore, $a_{\max}$ is noticeably less than w if $\hbar\omega_c < e\phi_{ab}$. The linear part of the I–V characteristic for a Corbino disk in this case is given by

$$j_x = \sigma_{xx}V/2a, \tag{17}$$

where j_x is the current density, σ_{xx} is the diagonal part of the conductivity tensor under the conditions of the QHE, V is the drift potential difference between the Corbino edges, and $2a$ is the width of the "incompressible" strip from Eq. (14). Equation (17) is true as long as $a < a_{\max}$ from Eq. (16). The standard formula for j_x, disregarding proximity effects, in this case would have the form (17) with a replaced by w.

Continuing the discussion of an "incompressible" strip, we now turn to Ref. 2 . In these interesting experiments, performed with the help of a linear electro-optic effect, it was shown that under symmetric excitation (the ac voltage is applied symmetrically to the two contacts) a Corbino disk with nominal width of the 2D region $2w = 700$ μm is electrically active under QHE conditions only at the edges of the 2D system which are adjacent to these contacts. The width of the active regions is of the order of 100 μm and does not depend (in a definite range of frequencies of the order of 5 kHz) on the measurement frequency. The central part of the Corbino disk of width $2a = 500$ μm and with an integer filling factor $\nu_l = 2$ has virtually no diagonal conductivity, and in the experiments of Ref. 2 it remained an equipotential. Our interpretation of the picture observed in Ref. 2 can be summarized as follows. Because the fact that the contact potential difference between the 2DEG and the metal Corbino edges is different from zero, the 2D system loses spatial uniformity. Under QHE conditions the effective strip with an integer filling factor has dimensions $a < w$ [Eq.(16)] which depend on the contact potential difference. The ratio w/a from Ref. 2, together with Eq. (16), therefore allow us to estimate the scale of the contact potential difference for the Corbino disk from Ref. 2:

$$e\phi_{ab} = \hbar w_c (w/a)^3 = 2.74\hbar\omega_c. \tag{18}$$

For a magnetic field $H = 8.5$ T, which corresponds in Ref. 2 to $\nu_l = 2$, the contact potential difference from Eq. (18) is of the order of 390 K.

The nonuniformity of the electropotential along an equilibrium 2D system with $H = 0$ is a more delicate Coulomb proximity effect. This effect, also qualitatively recorded in Ref. 2 , is of quantum origin, i.e., it arises only when the zero-point vibrational energy is included in the equilibrium equation (5). Unfortunately, nothing is said in Ref. 2, aside from qualitative assertions that there is a nonuniformity in the distribution $\phi(x)$ in the absence of a magnetic field and that the scale of this nonuniformity is on the order of the dimensions w of the 2D system. Consequently, the quantum solution (8)–(11) of the problem of equilibrium in the composite system, which indicates that the potential ϕ along the 2DEG is nonuniform, is an existence theorem for this effect. An approximate formula, which follows from an analysis of the quantum solution (5)–(11) of the problem of equilibrium in a Corbino disk, can be used to estimate the scales of the nonuniformity of $\phi(x)$ far from singularities:

$$e\phi(x)=\text{const}-\pi\hbar^2\delta n_0(x)/2m_*, \quad (19)$$

where $\delta n_0(x)$ is a classical electron density distribution corresponding to a piecewise-smooth behavior of the electropotential. This quasiclassical determination of the degree of nonuniformity of $\phi(x)$ in charged 2D systems was also used previously in Ref. 4.

In this study we have called attention to the existence of long-range Coulomb proximity effects in low-dimension charged systems. The basic factors responsible for their appearance are similar to contact phenomena in 3D metals. However, the specific nature of the solution of Poisson's equation for low-dimension charge distributions leads to anomalous spreading of the perturbation of the electron density over regions far removed from the contact zones. Analysis of these anomalies leads to several conclusions which are of interest from the experimental and theoretical standpoints. The results obtained above make it possible, in particular, to invoke Columb proximity effects to explain the experiments of Ref. 2. We found that the scale of the contact potential difference (on the order of 400 K) for an unscreened Corbino disk with metal contacts could be estimated in this case. Systematic information about the contact potential difference between 2D and 3D conducting systems, to the best of our knowledge, is not yet available. Coulomb proximity effects make it possible to fill this gap.

We thank V. F. Gantmakher and V. T. Dolgopolov for a discussion of the results obtained in this study and for useful remarks.

This work was supported by INTAS (Grant No. 93-933) and by the Russian Fund for Fundamental Research (Grant No. 95-02-06108a).

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Translated by M. E. Alferieff