Magnetoexcitons in quantum rings and in antidots

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An opening (antidot) in a two-dimensional (2D) electron gas in a strong perpendicular magnetic field makes possible the formation of ring-shaped excitons stemming from the skipping states of electrons and holes. The binding energy of the ground states of such excitons is logarithmically larger than that of the "bulk" 2D excitons. The possibility of electron-to-hole tunneling around the antidot results in a shift for each of the excitonic levels, which oscillates as a function of the magnetic field.

The problem of Mott excitons in a strong magnetic field has a rather long history (see Refs. 1–5). It has been shown that the exciton binding energy and all the other characteristics (effective mass, dipole moment, dispersion law) are strongly magnetic field dependent. In particular, a very strong magnetic field ($a_H \ll a_B$, where a_H, a_B are the magnetic length and Bohr radius, respectively) makes the 3D exciton effectively 1D with a logarithmically deep ground state.

Recent progress in microstructuring technology has drawn attention to excitons in nanostructures of various dimensionalities (e.g., excitons in quantum dots with parabolic confinement have been considered by the present author and Govorov in Ref. 6).

The aim of this paper is to call attention to the fact that in a strong magnetic field, localized excitons can be formed without any confining potential both for electrons and for holes. Suppose we have an opening of radius R in a 2D electron gas. Such an antidot is regarded in the present work as a region forbidden for both particles to penetrate inside. Thus the contour of the antidot $\rho = R$ (polar coordinates are used) is a hard wall. A magnetic field perpendicular to the plane of the 2D gas presses both types of particles to the wall, forming skipping orbits. Electrons and holes will circulate around the antidot in the opposite directions (as long as the Coulomb interaction is not taken into account). Hence one can expect the formation of a quasi-1D exciton localized in a narrow ring encompassing the antidot.

Let us start with an idealized model: an electron and a hole in a 1D ring placed in a magnetic field. The Hamiltonian of the problem can be written in the form

$$\mathscr{H} = \frac{\hbar^2}{2m_n R^2}\left(-i\frac{\partial}{\partial\varphi_n} + \lambda\right)^2 + \frac{\hbar^2}{2m_p R^2}\left(-i\frac{\partial}{\partial\varphi_p} - \lambda\right)^2 - \frac{e^2}{2\varepsilon R}\left|\sin\frac{\varphi_n - \varphi_p}{2}\right|^{-1}, \quad (1)$$

where $\lambda = \Phi/\Phi_0$, Φ is the magnetic flux through the ring ($\Phi = \pi R^2 B$), Φ_0 is the magnetic flux quantum, m_n, m_p are the effective masses of the electron and hole, respectively; φ_n

and φ_p are the azimuthal coordinates of the particles, and ε is the dielectric constant. To separate the internal motion in the exciton from that of the center of mass, one needs to go over to the variables

$$\varphi_c=\frac{m_n\varphi_n+m_p\varphi_p}{M}, \quad \theta=\varphi_n-\varphi_p; \quad M\equiv m_n+m_p. \tag{2}$$

Then the Hamiltonian reads

$$\mathscr{H}=-B\frac{\partial^2}{\partial\varphi_c^2}+\beta\left(i\frac{\partial}{\partial\varphi}-\lambda\right)^2+U_c(\theta), \tag{3}$$

where $B=\hbar^2/2MR^2$, $B=\hbar^2/2\mu R^2$, $\mu=m_nm_p/M$, and $U_c(\theta)$ is the Coulomb energy [the last term in Eq. (1)]. The total wave function may be written as $\Psi=\exp(iJ\varphi_c+i\lambda\theta)\chi(\theta)$, where J is a real number and $\chi(\theta)$ obeys the 1D Schrödinger equation

$$-\beta\frac{\partial^2\chi}{\partial\theta^2}+U_c(\theta)\chi=(E-BJ^2)\chi\equiv w\chi. \tag{4}$$

Here E is the total energy of the exciton and w is its internal energy.

To determine J and the allowed solutions of Eq. (4) we have to make the total wave function independently periodic in φ_n and φ_p with the period 2π. On the other hand, Eq. (4) has, formally, the Bloch-type solutions

$$\chi=e^{ip\theta}v(\theta); \quad -\frac{1}{2}<p\leqslant\frac{1}{2}, \tag{5}$$

where v is a periodic function of θ with the same period 2π, because this is the period of the potential: $U_c\sim|\sin\theta/2|^{-1}$. By adding 2π to φ_n and φ_p independently we come to the relations

$$J\frac{m_n}{M}+\lambda+p=N_n, \quad J\frac{m_p}{M}-\lambda-p=N_p, \tag{6}$$

where N_n, N_p are arbitrary integers. It follows from Eq. (6) that $J=N_n+N_p$, and thus J is an integer—the rotational quantum number of the exciton as a whole.

Now consider Eq. (4). In a small vicinity of the point $\theta=0$, as well as at $\theta=\pm 2\pi$, $\pm 4\pi$,..., one can expand the Coulomb energy: $U_c\approx e^2/\varepsilon R\theta$, so we have a 1D hydrogen "atom" with charge e^2/ε and effective mass μ. It is known (see, e.g., Ref. 4 and references therein) that the ground state of such a system is logarithmically deep:

$$E_0=-\frac{2\mu e^4}{\varepsilon^2\hbar^2}\ln^2\left(\frac{a_B}{a_0}\right), \tag{7}$$

where a_0 is a "cutoff" radius. For a 3D exciton in a strong magnetic field this radius a_0 equals the magnetic length a_H. In our case a_0 is obviously related to the size of the skipping orbits and will be determined below.

Besides the ground state, a 1D exciton has the hydrogen series of energy levels $E_n=-\mu e^4/2\varepsilon^2\hbar^2n^2$, $n=1, 2, 3, \ldots$. All these levels form (approximately!) the set of allowed values for the eigenvalue w in Eq. (4) provided that the effective Bohr radius $a_B=\varepsilon\hbar^2/\mu e^2$ is much smaller than the distance $2\pi R$ to the next well in the Coulomb energy U_c. For the antidots reported up till now the condition $a_B \ll R$ is very well satisfied, and we may solve Eq. (4) in the tight-binding approximation.

Any solution of the type (5) corresponds to the energy

$$w=E_n-\Delta_n \cos 2\pi p, \quad n=0,1,2,\ldots, \quad \Delta_n>0, \tag{8}$$

where the "quasimomentum" should be determined by Eq. (6). Then for the total energy of the exciton one obtains:

$$\begin{aligned} E(J,n)&=BJ^2+E_n-\Delta_n \cos 2\pi(\Phi/\Phi_0+Jm_n/M)\\ &=BJ^2+E_n-\Delta_n \cos 2\pi(\Phi/\Phi_0-Jm_p/M); \quad n=0,1,2,\ldots \end{aligned} \tag{9}$$

The quantity Δ_n in Eq. (9) is an amplitude corresponding to the tunneling of the electron to the hole (or vice versa) around the antidot. In order of magnetic it is $\Delta_n \sim |E_n|\exp(-2\pi R/na_B)$ for $n \geq 1$ and $\Delta_0 \sim |E_0|\exp[(-2\pi R/a_B)\ln(a_B/a_0)]$ for the ground state. The possibility of such tunneling shifts the energy levels from their unperturbed positions E_n and results in a periodic dependence of the binding energy of the ring exciton on the magnetic flux, with the period Φ_0. The latter statement has a general character and is not connected with the tight-binding approximation used. The eigenvalues w of Eq. (4) are always periodic functions of p from Eq. (5), and via Eq. (6) they are periodic functions of the magnetic flux.

Consider now a more realistic model and take into account the radial motion of the electrons and holes in the skipping states. To find the energy spectrum in this regime we will use the small parameter a_H/R. The above-introduced quantity λ is of order $(R/a_H)^2 \gg 1$. The radial coordinates of the particles can be written as $\rho_n=R+x_n$, $\rho_p=R+x_p$. Then x_n, x_p are replaced by X (the center of mass) and $x=x_n-x_p$ (the relative radial distance), and the Hamiltonian is expanded in X and x. By substituting $\Psi=\chi\exp(i\kappa\theta+iJ\phi_c)$ with

$$\kappa=\frac{2J\mu}{M}\frac{x}{R}-\lambda\left(1+\frac{2X}{R}+\frac{2\gamma x}{R}\right), \tag{10}$$

we get the Schrödinger equation to first order in X and x:

$$\begin{aligned} &\left[-\frac{\hbar^2}{2}\left(\frac{1}{M}\frac{\partial^2}{\partial X^2}+\frac{1}{\mu}\frac{\partial^2}{\partial x^2}\right)+2(\beta\lambda^2-BJ^2)\frac{X}{R}+(2\gamma\beta\lambda^2+4JB)\frac{x}{R}\right]\chi+\mathscr{H}_{\text{int}}\chi\\ &\quad=(E-BJ^2)\chi, \end{aligned} \tag{11}$$

where

$$\gamma=\frac{m_p-m_n}{M},$$

$$\mathscr{H}_{\text{int}}=-\beta\left(1-\frac{2X}{R}-\frac{2\gamma x}{R}\right)\frac{\partial^2}{\partial\theta^2}-\frac{e^2}{\varepsilon}\left(1-\frac{X}{R}-\frac{\gamma x}{2R}\right)\left(x^2+4\sin^2\frac{\theta}{2}\right)^{-1/2}. \tag{12}$$

The dependence of the wave function on φ_c is as before, but there still remains the question of how to separate the radial center-of-mass motion from the internal one. In a homogeneous electric field the internal and the center-of-mass degrees of freedom can be completely separated (see Refs. 3 and 4). Our case is spatially nonuniform, and an exact separation is not possible. Nevertheless such a procedure can be carried out approximately for sufficiently strong magnetic fields. As one can see from Eq. (11), the frequencies of motion of the particles in the skipping states are governed by the effective potentials

$$W_1(X)=2(\beta\lambda^2-BJ^2)X/R,$$

$$W_2(x)=(2\gamma\beta\lambda^2+4JB)x/R,$$

$$x,X>0. \tag{13}$$

The skipping states exist, of course, if the rotational energy BJ^2 is not larger than $\beta\lambda^2$, but this does not cause any problem on account of the condition $\lambda\gg 1$ (besides, the probability of exciton formation by optical absorption is proportional to the factor $|\int\Psi(X,\varphi_c)d\varphi_c dX|^2$, which is nonzero only for $J=0$; we are keeping J in the calculations just for generality). For not too close effective masses of electron and hole one has $\gamma\sim 1$, and the frequency corresponding to the x and X degrees of freedom are estimated as

$$\omega_x\sim\omega_X\sim\hbar\lambda^{4/3}/\mu R^2. \tag{14}$$

The internal azimuthal motion of the exciton is characterized by the Bohr frequency $\omega_B\sim\hbar/\mu a_B^2$, and the rotation around the antidot is the slowest motion: $\omega_R\sim\hbar/MR^2$. Hence, in a magnetic field for which

$$a_H^2\ll\sqrt{Ra_B^3} \tag{15}$$

we have ω_x, $\omega_X\gg\omega_B$, and the situation is quite similar to the one in molecules when considering the vibrational–rotational spectra. For GaAs with $a_B=100$ Å, $R=1000$ Å, the magnetic field determined by Eq. (15) must exceed 4T.

In the spirit of the theory of molecules we have to solve the "internal" equation $\hat{H}_{\text{int}}\chi=w(x,X)\chi$ for given x and X (like rotation at fixed nuclei) and then to find corrections stemming from the fast degrees of freedom x and X (like the interaction of rotation and vibrations). These corrections renormalize the effective mass and the charge of the hydrogen-like system described by the Hamiltonian $\hat{H}_{\text{int}}$ from Eq. (12):

$$\mu\to\tilde{\mu}=\mu\left(1-\frac{2X}{R}-\frac{2\gamma x}{R}\right)^{-1},\quad e^2\to\tilde{e}^2=e^2\left(1-\frac{X}{R}-\frac{\gamma x}{2R}\right). \tag{16}$$

A key fact is that in the effective Rydberg energy $\tilde{\mu}\tilde{e}^4/2\hbar^2\varepsilon^2$ the X-dependent terms cancel out to the adopted accuracy (there are no terms linear in X). That means that all the higher energy levels E_n ($n=1,2,\ldots$) are independent of X, and only the ground state energy E_0 contains X, in a logarithm via the effective Bohr radius $\varepsilon\hbar^2/\tilde{\mu}\tilde{e}^2$. Thus to "logarithmic accuracy" (meaning by that a very weak dependence) the center-of-mass radial motion is separated from the internal degrees of freedom.

The term x^2 in the square root in Eq. (12) gives the above-mentioned cutoff radius of the quasi-1D exciton. An estimate of this radius follows from the potential $W_2(x)$ in Eq. (13): $a_0 \sim R\lambda^{-2/3}$—the size of ground state of a particle in the triangular well. Hence the lowest energy level of the exciton equals $E_0 = -(2\mu e^4/\varepsilon^2\hbar^2)\ln^2 A$, where $A = a_B R^{1/3}/a_H^{4/3} \gg 1$, in accord with Eq. (15).

The shift of the exciton levels due to tunneling depends both on X and x and should be averaged with appropriate wave functions of the radial motion in the skipping states [λ, which determines the magnetic flux, is now also renormalized; see Eq. (10)]:

$$w = E_n + \left\langle \Delta_n(x,X)\cos 2\pi\left[\lambda\left(1+\frac{2X}{R}+\frac{2\gamma x}{R}\right)-\frac{2J_\mu}{M}\frac{x}{R}-J\frac{m_n}{M}\right]\right\rangle. \tag{17}$$

By modeling the wave function of the triangular well in the form $\psi \sim X\exp(-X/a_0)$ we obtain for $J=0$

$$w - E_n \sim \frac{\langle\Delta_n\rangle\cos 2\pi\lambda}{\lambda^{2/3}}. \tag{18}$$

This is a quite clear result: in comparison with the 1D case given by Eq. (9), a decreasing factor $\lambda^{-2/3}$ has appeared because the magnetic flux linked by the electron–hole trajectories is now not strictly fixed at the value $\pi R^2 B$ but fluctuates in the skipping orbit states.

In conclusion, it has been shown that in a strong magnetic field, ring-shaped excitons can be formed around antidots due to skipping states of electrons and holes. The ground state energy level of such excitons is logarithmically deep in comparison with that of the "bulk" 2D exciton. Besides, there is an exponentially small contribution to all the exciton levels, which oscillates as a function of magnetic flux with the fundamental period hc/e.

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