Spectral flow in Josephson junctions and effective Magnus force

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Momentum production during the phase-slip process in SNS Josephson junction is discussed. It is caused by the spectral flow of bound states of fermions localized within the junction. This effectively reduces the Magnus force acting on vortices and thus provides an explanation for the experimental observation of a negligible Magnus force in 2D Josephson junction arrays.

Vortex dynamics in 2D Josephson junction arrays has been studied intensively in recent years. The vortices in the arrays could be considered as massive particles with a long-range Coulomb interaction.[1,2] Particular attention was devoted to the forces acting on the vortices, one of them being Magnus force.

In the experiment[3] a straightforward ballistic motion of vortices was observed, which implies the absence of reactive forces acting on a vortex perpendicular to its velocity. This was also confirmed by more recent experiments: the vortices were found to move perpendicular to the driving current,[4] and no Hall effect was detected in the system.[5]

An explanation for the absence of the Magnus force was proposed in Ref. 6. The authors claim that the Magnus force is proportional to the offset charges on the superconducting islands, the effect of which being negligible. In a recent work[7] Gaitan and Shenoy argue that the Magnus force is proportional to the density of superconducting electrons on the islands, averaged over large distances compared to the lattice constant of the array, rather than to the charge of the island, which is given by the difference in the numbers of electrons and protons. On the other hand Zhu, Tan, and Ao[8] have shown the force to be proportional to the *local* superconducting density at the point where the vortex is situated. Since the vortex does not move through the superconducting islands but through the junctions, the Magnus force on the vortex can be substantially reduced.

In the present paper we consider the forces on a vortex moving in a Josephson junction array, and propose a different explanation for the experiments mentioned above. It should be emphasized that one contribution to the force was missed in previous considerations,[6–8] viz., the force from the *spectral flow.* In uniform superfluids there exists the so called zero branch of energy levels of fermions localized within vortex core, which crosses zero as a function of angular momentum.[9] Vortex motion leads to a flow of

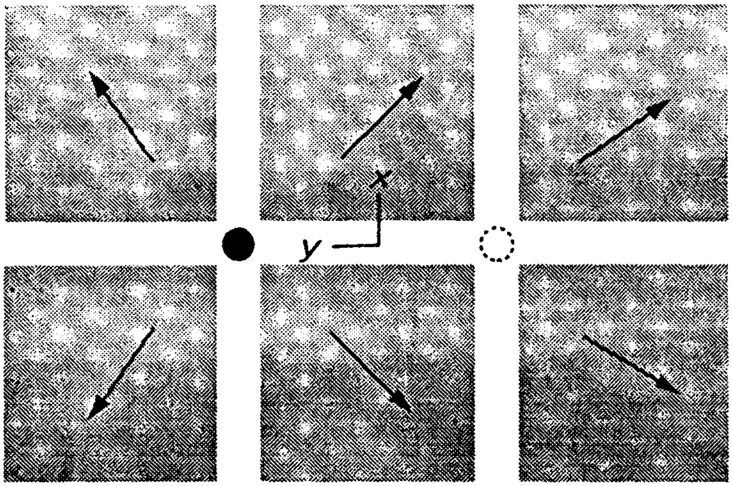

FIG. 1. Josephson junction array. The squares represent superconducting islands and the white spacings between them are normal intermediate layers in junctions. The distribution of phases of superconducting islands for a vortex sitting on a site of the lattice marked by a black circle is shown by arrows. When the vortex moves to the site marked by a dashed circle some levels of fermions localized within the junction between the two central squares cross zero, and some amount of linear momentum k_x is produced. This corresponds to an additional force on the vortex, which compensates the conventional Magnus force.

levels along this branch, and the energy of some levels crosses a value of zero. At low temperature these levels become occupied (or unoccupied, depending on whether they cross zero upwards or downwards). During this process the total number of quasiparticles localized within the core is conserved, while the linear momentum of the quasiparticles is not conserved. This implies a transfer of linear momentum from the vortex to the heat bath and thus an additional force acting on the moving vortex.[10] This force from the spectral flow can almost completely neutralize Magnus force,[11] though in some regimes the spectral flow is suppressed and a net force appears. This scenario reproduces the microscopic calculations by Kopnin and co-authors.[12]

We argue that an analogous situation takes place in Josephson junction arrays. Here the role of fermions localized within a vortex core is played by fermions localized within junctions. The phase-slip events in junctions during vortex motion are accompanied by the spectral flow of bound-state fermions. The force from this spectral flow neutralizes the Magnus force, and the reactive force on a vortex becomes negligible.

Since the cancellation of two contributions to reactive force is of topological origin and persists for several different geometries, we consider the simplest case of the SNS junction. Other geometries and possible sources of difference between the Magnus and spectral-flow forces are under investigation and are briefly discussed in the Conclusion. Here we consider a square lattice of superconducting islands as plaquettes, the edges of the lattice representing normal layers in SNS-junctions. The vortices can be regarded as situated at sites of this lattice (Fig. 1). The 2D density of electrons n is assumed to be the same in the superconducting and normal regions.

As in the review[13] and in Ref. 11 (see also Ref. 14) we take it as granted that the conventional Magnus force on a vortex at low temperature is determined by the local n, because n is the variable which is canonically conjugate to the phase of the Bose-condensate:

$$\mathbf{F}_M = \pi n \hat{\mathbf{z}} \times \mathbf{v}_L . \tag{1}$$

Here $\mathbf{v}_L$ is the velocity of the vortex with respect to the superfluid velocity, which is chosen to coincide with the velocity of the heat bath ($\mathbf{v}_s = \mathbf{v}_n = 0$). Note that in the low-temperature limit the superfluid density n_s tends to n irrespective of the magnitude of the order parameter gap Δ (actually, due to the Iordanskii force from the heat bath, Eq. (1) is valid even at nonzero T; see Refs. 13, 14. The same is true for the electrons in "normal" regions with a small magnitude of Δ. This expression does not contradict Ref. 7 at $T=0$, but we want to demonstrate that there is another force which nearly completely compensates the Magnus force.

During one step a vortex moves from a lattice site to an adjacent one. We are going to calculate the production of momentum due to the spectral flow of fermions for such a process, which gives rise to the compensating force. The problem is in many details similar to the evolution of fermionic bound states existing within topological solitons in polymers, superfluid ^{3}He, and other ordered systems in condensed matter containing fermions.[15] The same type of evolution of the fermionic spectrum in topological solitons and sphalerons in particle physics leads to baryogenesis,[16,17] while in our case the spectral flow leads to "momentogenesis."

Let us consider one particular Josephson junction, with x being the coordinate axis normal to the junction (Fig. 1). The dependence of the eigenfunctions of the Bogolyubov–Nambu Hamiltonian

$$\mathcal{H} = \hat{\tau}_3 \cdot \hat{\epsilon} + \hat{\tau}_1 \operatorname{Re} \Delta - \hat{\tau}_2 \operatorname{Im} \Delta \tag{2}$$

on $\mathbf{r}_\perp = (y,z)$ is given by a factor $\exp(i\mathbf{k}_\perp \mathbf{r}_\perp)$. Here $\hat{\vec{\tau}}$ are Pauli matrices in Bogolyubov–Nambu space and $\hat{\epsilon} = (-\nabla^2 - k_F^2)/2m^*$ is the energy operator for quasiparticles in a normal liquid; k_F is the Fermi momentum. We suppose that the order parameter varies slowly on the length scale of the coherence length, while the Fermi momentum remains constant, and that the mean free path is large enough for the electronic levels to be well defined. As we are interested in the energies close to Fermi surface, we suppose that $k_\perp < k_F$, and the eigenfunction is represented in eikonal approximation:

$$\exp(iqx) \cdot \psi(x) \tag{3}$$

where $q^2 = k_F^2 - k_\perp^2$ (q plays the role of the Fermi momentum of a 1D Fermi liquid for given $\mathbf{k}_\perp$), the exponent represents fast oscillations in space, and $\psi(x)$ varies slowly. In this case we may replace $\hat{\epsilon}$ by $q(-i\nabla)/m^*$.

We shall investigate the dependence of the energy spectrum on the phase difference $\Delta\phi$ between two superconducting islands. For simplicity we suppose that the order parameter is real $\Delta(x) = |\Delta(x)|$ on one side of the junction ($x<0$), and on the other side ($x>0$) the order parameter is given by $\Delta(x) = |\Delta(x)| \cdot \exp(i\phi)$. For $\phi = \pi$ the order parameter is an odd real function of x (Fig. 2) and the Hamiltonian is supersymmetric:

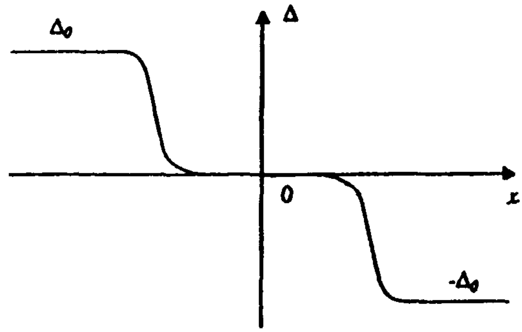

FIG. 2. Dependence of the superconducting order parameter on the coordinate x normal to the junction for a phase difference of π between superconducting islands. The corresponding Bogolyubov–Nambu Hamiltonian has zero-energy eigenvalues, i.e., at the moment when ϕ crosses π the levels cross zero.

$\{\mathcal{H},\tau_2\}=0$. This $0-\pi$ soliton corresponds to the sphaleron in particle physics,[16,17] i.e., the intermediate state between vacua with different topological charges. Due to supersymmetry the Hamiltonian has a eigenfunction

$$\tilde{\psi}_0=\mathrm{const}\begin{pmatrix} 1 \\ -\mathrm{sign(q)}i \end{pmatrix}\psi_0,$$
$$\psi_0=\exp\left(-\frac{m^*}{|q|}\int_0^x dx'\Delta(x')\right), \tag{4}$$

with zero eigenvalue. So, at $\phi=\pi$ for each q one energy level crosses zero. For small $\phi-\pi$ the perturbation of the Hamiltonian is

$$\mathcal{H}_{\mathrm{int}}=-(\phi-\pi)\Delta(x)\hat{\tau}_2 \tag{5}$$

for $x>0$ and $\mathcal{H}_{\mathrm{int}}=0$ for $x<0$. The energy level is shifted to

$$E=\ \mathrm{sign}(q)(\phi-\pi)\omega(|q|), \tag{6}$$

where

$$\omega(|q|)=\frac{\int_0^\infty dx|\Delta(x)|\psi_0^2(x)}{\int_0^\infty dx\psi_0^2(x)}. \tag{7}$$

From (6) it follows that at $\phi=\pi$, i.e., at the sphaleron, the energy level crosses zero upwards for $q>0$ and downwards for $q<0$. A similar phenomenon (cf. Refs. 18, 19 as well) has been found for sphalerons in particle physics.[16,17]

This leads to a production of the x component of the linear momentum

$$\Delta P=2\cdot\frac{1}{2}A\int\frac{d^2k_\perp}{(2\pi)^2}2|q|=\frac{k_F^3}{3\pi}A. \tag{8}$$

The prefactor 2 takes into account the double spin degeneracy, and the 1/2 removes the double counting of particle and hole momenta. Here A is the cross-sectional area of the junction, which is given by the product of the lattice constant a and the thickness l of the film along the z axis (which is perpendicular to the plane): $A=al$.

Using this equation, one can find the spectral-flow force experienced by a vortex moving with respect to the heat bath. Since the velocity of the vortex is $v_L=a/\Delta t$, where Δt is the period of time during which the vortex crosses one junction, one has for the spectral-flow force in the x direction

$$F_{\text{sp.flow}}=\frac{\Delta P}{\Delta t}=\frac{\Delta P v_L}{a}=\pi C_0 v_L, \tag{9}$$

where $C_0=lk_F^3/3\pi^2$. In vector notation

$$\mathbf{F}_{\text{sp.flow}}=\pi C_0 \mathbf{v}_L\times\mathbf{z}. \tag{10}$$

C_0 is very close to the 2D particle density $n=\rho l$, since the 3D density is given by $\rho\approx k_F^3/3\pi^2$ to an accuracy of the order of $(\Delta/E_F)^2$. Therefore, the force induced by the spectral flow nearly neutralizes the Magnus force, as anticipated.

Discussion. We have shown that in an ideal situation the energy levels for all values of q cross zero simultaneously at the moment when the phase difference between the islands equals π. In the general case the energy levels may cross zero at different times for different q. However, the production of linear momentum is still given by (8): The number of levels which cross zero is related to the asymmetry index, which is half the difference between the numbers of negative and positive energy levels at given q. This index $N(q)$ can be expressed in terms of the Green's function $\hat{G}=(i\omega-\hat{\mathcal{H}})^{-1}$:

$$N(q)=\int\frac{d\omega}{2\pi}\,\mathrm{Tr}\,\hat{G}=-\frac{1}{2}\sum_n\ \mathrm{sign}\,E_n(q), \tag{11}$$

where the summation is performed over eigenvalues for given q (Ref. 20). The change in $N(q)$ cannot depend upon the details of phase slip event but only on the initial and final states. Therefore, our derivation for a particular case is generalized to arbitrary phase-slip events. One gets the same result by calculating the Green's function in a gradient expansion.

Our discussion shows that the production of momentum due to spectral flow is of topological origin, and we suppose that the result would not change with impurities taken into account when the levels cannot be classified in terms of q. This question is under investigation.

Let us now make some remarks for a more general geometry of the Josephson junction array, when the superconducting islands do not cover almost all the area of the sample. If the core size of a vortex r_c is small compared to the lattice spacing a, then one has the situation discussed in Ref. 8: the vortex can be considered as a point-like object moving in a locally homogeneous environment with a slowly changing density of electrons. In this case one can apply the bulk results:[11] the *en route* Magnus force will be canceled by the *en route* spectral-flow force for any route.

Let us consider another extreme limit, $r_c\gg a$, which takes place if the Josephson coupling between the islands is small or if the superconducting islands cover a small part of the area. In this case, as was discussed in Ref. 7, one can average over distances of the order of the core size r_c and thus obtains the case of a conventional homogeneous superfluid with r_c as the coherence length and a as the "interatomic distance." The Magnus force in this case will be determined by the average number density $\bar{n}$, as was stated in Ref. 7; however, the spectral flow force on a vortex will be also proportional to average value of C_0. Thus the two forces will again nearly compensate each other, but possibly to a relative accuracy of the order of $(a/r_c)^2$.

Thus there can be at least 3 different factors which disrupt the fine tuning between the Magnus force and the spectral flow force: (1) The particle–hole asymmetry gives a difference $\bar{n}-\bar{C}_0\sim\bar{n}(\Delta/E_F)^2$ (Ref. 10). (2) The finite spacing ω_0 between the quasiparticle energy levels, which suppresses the spectral flow of fermions, gives $\bar{n}-\bar{C}_0\sim\bar{n}(\omega_0\tau)^2$ (Refs. 11, 21). In conventional Abrikosov vortices the spacing is $\omega_0\sim\Delta^2/E_F$, while in the Jospehson junction the discreteness arises due to the dimensional quantization along the axis y, which is parallel to the junction, or due to impurities. (3) The inhomogeneity discussed above may also give $\bar{n}-\bar{C}_0\sim\bar{n}(a/r_c)^2$: for example, the vortex can prefer paths with appreciable $\omega_0\tau$, which leads to different *en route* values of n and C_0.

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