An explicit equation for a two-loop measure in a heterotic-string model

V. G. Knizhnik
L. D. Landau Institute of Theoretical Physics, Academy of Sciences of the USSR

(Submitted 21 May 1987)
Pis'ma Zh. Eksp. Teor. Fiz. **46**, No. 1, 8–10 (10 July 1987)

A two-loop vacuum diagram is explicitly calculated in the heterotic-superstring model. The measure vanishes after summation over the boundary conditions.

The calculation of multiloop amplitudes in the superstring theory has recently been the subject of considerable interest. On the basis of single-loop calculations Green and Schwarz[1] have advanced a hypothesis about the absence of discrepancies in the closed oriented superstring theory in all orders of perturbation theory. Despite considerable efforts, however, this hypothesis has so far not been corroborated satisfactorily because of the absence of any explicit equations for a multiloop measure in a superstring, which could be used to determine whether there are no discrepancies. In this letter we present explicit equations for a two-loop measure in the heterotic string theory for fixed boundary conditions for the fermions on a world sheet. We will also show that the choice of the model and the factorization conditions uniquely fix the signs of the contributions from various boundary conditions. As a result, the total measure vanishes in the sypersymmetry models.

The main difficulty in calculating the multiloop amplitudes in the superstring stems from the existence of null ghost modes with a spin 3/2 and the supermoduli corresponding to them. We will use the ansatz used by Martinec,[2] Eq. (43), in which the result of integration over the supermoduli is represented by a correlation function of the corresponding number of operators for the change in the ghost charge, $Q\xi$, which were introduced by Friedan *et al.*[3] To describe type-2 surfaces, we will use the equation

$$y^2=(z-a_1)\ldots(z-a_6) \tag{1}$$

in $C^2=(y,z)$. The coordinates of the branch points are, within the action of the group $SL(2,C)$ of the projective transformations and permutations, the coordinates in the modulus space. We will evaluate the measure as a function of these coordinates. It is known that for surfaces (1) the boundary conditions for the fermions, for which the Dirac operator has no null modes (only those modes give us a nonzero contribution to the partition function), are parametrized by partitioning the points $a_1,\ldots,a_6$ into triads A_1^t, A_2^t, A_3^t and B_1^t, B_2^t, B_3^t; $t=1,\ldots,10$. Using the results of Refs. 4 and 5, in which it was shown that the branch points correspond to the vertex operators, we can write in the following form the contribution to the measure for the heterotic string with the boundary conditions r and s for the two groups of the 16 left fermions and the conditions t for the right fermions:

$$\Lambda_2(r,s,t)=\int \prod_{i=1}^{6} d^2 a_i/dV_{pr}\, T^{-5} \prod_{k<l}^{6} (\bar{a}_k-\bar{a}_l)^{-3}(a_k-a_l)^{-2}\cdot$$

$$\prod_{i<j}^{3} (\bar{A}_i^r-\bar{A}_j^r)^2(\bar{B}_i^r-\bar{B}_j^r)^2(\bar{A}_i^s-\bar{A}_j^s)^2(\bar{B}_i^s-\bar{B}_j^s)^2(A_i^t-A_j^t)(B_i^t-B_j^t)W_t;$$

$$W_t\equiv\langle \xi(x_0)Q\xi(x_1)Q\xi(x_2)\rangle_t, \qquad (2)$$

where

$$T=\int d^2z_1 d^2z_2\,|(z_1-z_2)\,y^{-1}(z_1)\,y^{-1}(z_2)|^2,$$

and the correlation function W_t will be estimated below. Using the arguments of Ref. 3, we can show that (2) does not depend on the choice of the points x_i, within the total derivatives in a_i. We set $x_0=\infty$, $x_1=a_1$, and $x_2=a_2$. Furthermore, we assume $a_1=A_1^t$ and $a_2=A_2^t$ if both points are situated in the same triad and $a_1=A_1^t$ and $a_2=B_1^t$ if they are situated in different triads. After some calculations, we find

$$W_t=W_t^X+W_t^{gh}, \qquad (3)$$

where the contribution from the spin-currents of the "matter" has the form

$$W_t^X=-\frac{5}{8}(a_1-a_2)^{-1}T^{-1}\Big\{(a_2-a_3)\dots(a_2-a_6)\int d^2z_1d^2z_2$$

$$\frac{(a_1-z_1)(a_1-z_2)}{(a_2-z_1)(a_2-z_2)}\Big|(z_1-z_2)y^{-1}(z_1)y^{-1}(z_2)\Big|^2+(a_1\leftrightarrow a_2)\Big\} \qquad (4)$$

and is independent of t, while the ghost part is

$$W_t^{gh}=\frac{1}{4}\sum_{i=1}^{3}(a_1-a_2)^{-1}(a_1-A_3^t)(a_1-B_i^t)(a_2-B_{i+1}^t)(a_2-B_{i+2}^t) \qquad (5a)$$

for $a_2=A_2^t$ and

$$W_t^{gh}=\frac{1}{4}(a_1-a_2)^{-1}(a_1-A_2^t)(a_1-A_3^t)(a_2-B_2^t)(a_2-B_3^t) \qquad (5b)$$

for $a_2=B_1^t$.

Substituting these equations into (2), we find the contribution to the two-loop measure from the sector with the boundary conditions (r,s,t).

We need now only to choose the weight factors $C(r,s,t)=\pm 1, 0$ for $\Lambda_2(r,s,t)$ which go into the total partition function

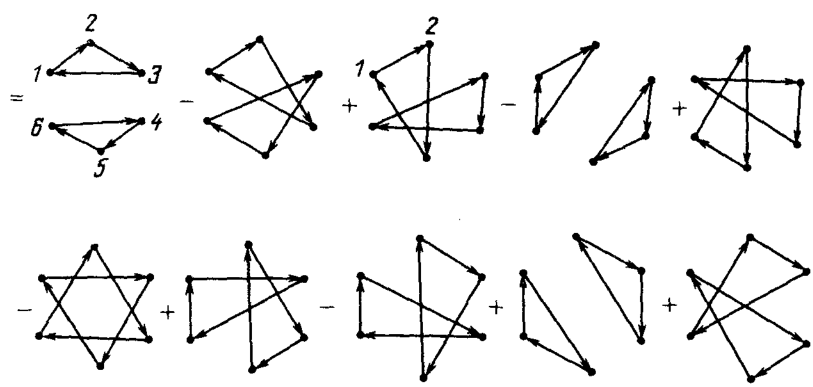
$$\Lambda_2=\sum_{r,s,t} C(r,s,t)\,\Lambda_2(r,s,t). \qquad (6)$$

The specific values of $C(r,s,t)$ are determined by the choice of model and the factorization conditions.[6] For SO(32), for example, the superstrings

$$C(r,s,t)=\delta_{r,s}C(t)\ , \tag{7}$$

where the signs of $C(t)$ can be found from the conditions for the pole cancellation in (2) as $a_k \to a_l$. There is only one choice of signs that satisfies this condition. The contribution to the measure from the terms with W^X in this case is proportional to

$$\sum_t C(t) \prod_{i<j} (A_i^t - A_j^t)(B_i^t - B_j^t)$$



$$\equiv 0\ , \tag{8}$$

where the arrows $i \to j$ correspond to the factors $(a_i - a_j)$ and each diagram denotes the product of six factors corresponding to the drawn arrows. The ghost part of the measure can be constructed by using Eqs. (5). For the given choice of signs it too vanishes. To derive these identities, we had to show that the number of zeros in the argument a_6 of the polynomial, which is the identity, in each case exceeds its power in a_6. We use graphic representation (8) in this case. Representation (8) is a modified version of the Riemann identity $\Sigma_m C(m)\theta_m^4 = 0$, but the ghost component is cancelled because of a more complex identity, whose generalization to a larger order has, to our knowledge, not been done.

For the $E_8 \times E_8$ model the summing over the boundary conditions r and s is done independently

$$C(r,s,t)=C(t)\ , \tag{9}$$

and Λ_2 again vanishes. As a result, for each supersymmetry model we have

$$\Lambda_2 = 0\ , \tag{10}$$

consistent with Green and Schwarz's hypothesis.

We note in conclusion that the ansatz used by Martinec[2] was also used for the measure by Verlinde and Verlinde,[7] who have not been able, however, to trace cancellation (10).

[1]M. B. Green and J. H. Schwarz, Phys. Lett. **109B**, 444 (1982).
[2]E. Martinec, Nucl. Phys. **B281**, 157 (1987).
[3]D. Friedan, E. Martinec, and S. Shenker, Phys. Lett. **160B**, 55 (1985).
[4]M. Bershadsky and A. Radul, Intern. J. Mod. Phys. **A2**, 165 (1987).

[5]V. G. Knizhnik, Comm. Math. Phys., 1987 (in press).
[6]N. Seiberg and E. Witten, Nucl. Phys. **B276**, 272 (1986).
[7]E. Verlinde and H. Verlinde, Multiloop calculations in covariant superstring theory, Utrecht Preprint, January, 1987.

Translated by S. J. Amoretty