Peculiarities of the drag-induced thermoelectric power at the topological transition points

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The drag-induced thermoelectric power reveals an abrupt change or a logarithmic peculiarity at the Lifshitz topological transition.

Study of the systematic features of the kinetic coefficients near the topological Lifshitz transitions has recently attracted increasing attention (see Refs. 1–4 and the bibliography cited there). Vaks *et al.*[1] identified the distinguishing features of the conduction σ and of the diffusive thermoelectric power α_e. We consider here the drag-induced thermoelectric power α_{ph}, concerning which contradictory assertions have been published in the literature: Vaks *et al.*[1] showed that the systematic feature behaves as $\epsilon^{1/2}\theta(\epsilon)$, where ϵ is the distance before the energy transition. Abrikosov and Pantsulaya[2] suggested that there are no characteristic features of any kind. In this letter we show that the nature of the characteristic feature of α_{ph} can be determined by systematically analyzing the scattering anisotropy, which can always be done under the conditions conducive for observation of the topological transitions: The characteristic feature in this case has the form $\theta(\epsilon)$ or $1/\ln|\epsilon|$. Evidence for the existence of characteristic features of α_{ph} has recently been obtained experimentally[4] using In under pressure.

At low temperatures, when phonons are scattered primarily by electrons, α_{ph} can correctly be described by[5]

$$\alpha_{ph} = \frac{\pi}{90}\,\frac{T^3}{\sigma}\sum_{\lambda}\int \frac{d\Omega_{\hat{\mathbf{q}}}}{(\hbar s_{\hat{\mathbf{q}}\lambda})^2}\,\frac{\mathbf{j}^A_{q\lambda}\cdot\hat{\mathbf{v}}_{\hat{\mathbf{q}}\lambda}}{\Gamma_{\hat{\mathbf{q}}\lambda}}\,, \tag{1}$$

where $s_{\hat{\mathbf{q}}\lambda}$ and $\mathbf{v}_{\hat{\mathbf{q}}\lambda}$ are the phase and group velocities of the phonons with a quasimomentum $\mathbf{q}$ and polarization λ, $\hat{\mathbf{v}}_{\hat{\mathbf{q}}\lambda} = \mathbf{v}_{\mathbf{q}\lambda}/v_{\mathbf{q}\lambda}$; the integration is carried out in the direction of $\mathbf{q}$; and $\Gamma_{\hat{\mathbf{q}}\lambda}$ and $\mathbf{j}^A_{\hat{\mathbf{q}}\lambda}$ are the absorption coefficient of sound and the acoustoelectric coefficient:

$$\Gamma_{\hat{\mathbf{q}}\lambda} = \frac{1}{(2\pi)^2\hbar\rho v_{\hat{\mathbf{q}}\lambda}}\int dS_{\mathbf{k}}\,\frac{\Lambda^2_{\mathbf{k}}}{v^2_{\mathbf{k}}}\,\delta(\hat{\mathbf{v}}_{\mathbf{k}}\cdot\hat{\mathbf{q}})\,, \tag{2}$$

$$\mathbf{j}^A_{\hat{\mathbf{q}}\lambda} = \frac{e}{(2\pi\hbar)\rho v_{\hat{\mathbf{q}}\lambda} s_{\hat{\mathbf{q}}\lambda}}\int dS_{\mathbf{k}}\,\frac{\Lambda^2_{\mathbf{k}}}{v^2_{\mathbf{k}}}\,\frac{\partial \mathbf{l}_{\mathbf{k}}}{\partial k_q}\,\delta(\hat{\mathbf{v}}_{\mathbf{k}}\cdot\hat{\mathbf{q}})\,, \tag{3}$$

where ρ is the density of the metal; $\mathbf{v}_{\mathbf{k}}$ and $\mathbf{k}$ are the group velocity and quasimomentum of the electron; $\Lambda_{\mathbf{k}}$ is the deformation potential; the integration is over the Fermi

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surface; and $\mathbf{l}_\mathbf{k}$ is the vector path length, which is the solution of the kinetic equation

$$\frac{\partial f_\mathbf{k}^0}{\partial \epsilon_\mathbf{k}} \mathbf{v}_\mathbf{k} = \hat{L}\, \mathbf{l}_\mathbf{k} \tag{4}$$

($\hat{L}$ is the collision operator) and which determines the conductance

$$\sigma = \frac{2e^2}{3\,(2\pi)^3} \int \frac{dS_\mathbf{k}}{\hbar v_\mathbf{k}} \mathbf{v}_\mathbf{k} \times \mathbf{l}_\mathbf{k} \tag{5}$$

In the case of the scattering of electrons by impurities, $\mathbf{l}_\mathbf{k}$ has the following characteristic features near the topological transition[1]:

$$\mathbf{l}_\mathbf{k} = \mathbf{l}_\mathbf{k}^{(1)} + \mathbf{l}_\mathbf{k}^{(2)} \epsilon^{1/2} \theta(\epsilon)\,, \tag{6}$$

where ϵ is the distance before the energy transition. Since the characteristic features of α_{ph} are more pronounced, we will ignore the second term in (6) and assume $\mathbf{l}_\mathbf{k}$ to be regular.

The characteristic features of $\mathbf{j}_{\hat{\mathbf{q}}\lambda}^A$ and $\Gamma_{\hat{\mathbf{q}}\lambda}$, which are determined by the second power of $\mathbf{v}_\mathbf{k}$ in the denominator, have the form[6]

$$A_{\hat{\mathbf{q}}} + B_{\hat{\mathbf{q}}} \ln |\epsilon| \tag{7}$$

in the case of the formation of a neck and these features have the form

$$C_{\hat{\mathbf{q}}} + D_{\hat{\mathbf{q}}} \theta(\epsilon) \tag{8}$$

in the case of the formation of a cavity. The coefficients $A_{\hat{\mathbf{q}}}$, $B_{\hat{\mathbf{q}}}$, $C_{\hat{\mathbf{q}}}$, and $D_{\hat{\mathbf{q}}}$ for $\mathbf{j}_{\hat{\mathbf{q}}\lambda}^A$ are different from those for $\Gamma_{\hat{\mathbf{q}}\lambda}$ and they can have characteristic features that are amenable to integration[7] over $\mathbf{q}$. The coefficient $B_{\hat{\mathbf{q}}}$ is nonvanishing over the angular interval which does not depend on ϵ. Substituting these coefficients in the expression in Ref. 1, we find

$$\alpha_{ph} = \begin{cases} a + b/\ln|\epsilon| & \text{(neck)} \qquad (9a) \\ a + b\theta(\epsilon) & \text{(cavity)}\,. \qquad (9b) \end{cases}$$

The first result is valid only in the immediate vicinity of the transition, when the second term in (7) is the dominant term. In the opposite situation, i.e., at some distance from the transition, the result for the neck is

$$\alpha_{ph} = a + b \ln|\epsilon|. \tag{10}$$

A few remarks about the other scattering regimes are in order.

1. In the impurity-free limit, we have a Peierls situation, i.e., a free drift of the electron and phonon subsystem, in the case of a closed Fermi surface if (1) is applicable. In this case, the drift approximation is valid for $\mathbf{l}_\mathbf{k}$ ($\mathbf{l}_\mathbf{k} \sim \mathbf{k}$), $\mathbf{j}_{\mathbf{q}\lambda}^A$ and $\Gamma_{\hat{\mathbf{q}}\lambda}$ are

proportional to each other (the Weinreich relation),[8] and for α_{ph} we find the well-known Ziman formula[9]

$$\alpha_{ph} = \frac{1}{3} \frac{C_{ph}}{N_e e}, \tag{11}$$

where C_{ph} is the lattice specific heat, and N_e is the total number of electrons. If the topological transition occurs as a result of a change in the spectrum with $N_e = \text{const}$, α_{ph} will have no special features. If the topological transition occurs as a result of a change in N_e, α_{ph} will have only barely noticeable features of the type $\epsilon^{3/2}\theta(\epsilon)$.

2. A pure metal for open Fermi surfaces requires a special consideration, since the features characterizing the behavior of $\mathbf{l}_{\mathbf{k}}$ become more pronounced in comparison with those in (6) and may even compete with those in (9).

3. If the time required to bring about a relaxation of phonons by means of electrons, $\tau_{\text{ph},e}$, is comparable to the time of their relaxation achieved by other mechanisms, $\tau_{\text{ph,etc.}}$, then by describing these mechanisms in a τ approximation we find an equation which differs from (1) in that $\Gamma_{\hat{q}\lambda}$ is replaced by[5] $\Gamma_{\hat{q}\lambda} + (s\tau_{\text{ph,etc.}})^{-1}$. The characteristic features of α_{ph} do not change [Eqs. (9) and (10)], but the region of applicability of Eq. (9a) diminishes and as $\tau_{\text{ph,etc.}}/\tau_{\text{ph},e} \to \infty$, it disappears entirely: Eq. (10) becomes applicable in the entire region. The last result can be obtained without using the τ approximation: This can be done by assuming that in the calculations of Ref. 5 the nonequilibrium phonon distribution is specified and independent of the properties of the electron subsystem (cf. Ref. 3).

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