Nonadiabatic passage through a second-order phase transition

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Under certain conditions a system in a weak external field can pass through the point of a phase transition without the creation of macroscopic domains of a metastable phase in the course of the transition. These conditions are found.

When the point of a second-order phase transition is crossed (by slowly reducing the temperature, for example), the medium ordinarily becomes partitioned into a mixture of domains of the two phases. A weak external field lifts the degeneracy in terms of the sign of the real order parameter below the transition point and causes one of the asymmetric phases to become metastable. In an external field, and after a sufficiently long time, the system is always in the uniform single-phase state imposed by this field. In the present letter we show that under certain conditions it is possible to choose a rate of passage such that no macroscopic domains of a metastable phase arise as the transition is crossed in the presence of an external field, and the system immediately goes into a single-phase state.

The behavior of fluctuations of the real order parameter near the point of a second-order phase transition is described by the stochastic differential equation[1]

$$\dot{\eta} = -\gamma\frac{\delta F}{\delta\eta(\mathbf{r},t)} + f(\mathbf{r},t). \qquad (1)$$

It is convenient to use $1/\gamma$ as the unit of time. Ginzburg-Landau equation (1) then becomes

$$\dot{\eta} = a\eta - b\eta^3 + g\Delta\eta + h + \tilde{f}(\mathbf{r},t), \qquad (2)$$

where h is the exernal field, and the random force $\tilde{f}$ has the correlation function

$$\langle\tilde{f}(\mathbf{r},t)\tilde{f}(\mathbf{r}',t')\rangle = 2T\delta(\mathbf{r}-\mathbf{r}')\delta(t-t'), \qquad (3)$$

so that its intensity is determined by the temperature of the medium, T.

We assume that the bifurcation parameter a increases linearly over time and that at the initial time the system is directly at the transition point:

$$a = ct. \qquad (4)$$

We assume that at $t = 0$ we have $\eta = 0$ throughout the medium (this assumption is not important; the final results are applicable in the general case in which the point of the phase transition is crossed from the region of symmetric states).

At thermal equilibrium, nonuniform fluctuations of the order parameter are characterized by a correlation radius r_c, which becomes infinite at the point of the transition. A certain amount of time is required to reach thermal equilibrium. This thermal relaxation time becomes longer as we move closer to the point $a = 0$.

If the system starts at $t = 0$ from a symmetric state ($\eta \equiv 0$), then after a finite time t spatial correlations in the system can be established only in regions with a dimension no greater than $r_d(t) = \sqrt{gt}$. It is instructive to compare this dimension with that correlation radius $r_c(t)$ which corresponds at thermal equilibrium to the instantaneous value of the bifurcation parameter $a = ct$.

Since r_d increases, while r_c decreases, over time, there is an instant t^* at which we have $r_d(t^*) = r_c(t^*)$. At $t \gg t^*$, the adiabatic approximation is valid: The behavior of the fluctuations is the same as at thermal equilibrium with the given value of $a = ct$. The region $0 < t < t^*$ is nonadiabatic; in it, the state of the system is far from thermal equilibrium.

From the theory of phase transitions we have the Ginzburg-Levanyuk criterion[2]: Under the condition

$$a \gg (T^2 b^2 / g^3) \tag{5}$$

the fluctuations are small at thermal equilibrium (this is the region of applicability of the mean-field theory). In the opposite case, the fluctuations are large (the fluctuation region).

The most interesting situation is that in which the system leaves the nonadiabatic region at $t = t^*$ and goes immediately into the mean-field region.[1)] Since we have $r_c = \sqrt{g/a} = \sqrt{g/ct}$ in this region, by equating r_c and r_d we find $t^* = c^{-1/2}$ and $a^* = ct^* = c^{1/2}$. If the system is to be in the mean-field region at $t = t^*$, the value of a^* must satisfy inequality (5). The effect is to impose a lower limit on the passage rate c.

In the initial stage the order parameter η is small, and we can replace (2) by the simpler equation

$$\dot{\eta} = g\Delta\eta + h + \tilde{f}(\mathbf{r}, t). \tag{6}$$

It follows from the equation that the mean value of the order parameter increases over time in accordance with $\langle \eta \rangle = ht$, while the mean square value of the fluctuations in the order parameter, $\delta\eta = \eta - \langle \eta \rangle$, in a volume element V at time t is

$$\langle (\delta\eta^2)_V \rangle = Tt/V. \tag{7}$$

Since r_d actually plays the role of a correlation radius at $t \ll t^*$, we set $V \sim (gt)^{3/2}$ to find an estimate of the strength of the fluctuations. We then see from (7) that the mean square value of the fluctuations in the order parameter in a volume with the correlation radius falls off over time in accordance with $[\langle (\delta\eta^2)_V \rangle]^{1/2} \sim t^{-1/4}$. After a sufficiently long time has elapsed, this mean square value thus becomes smaller than the mean value $\langle \eta \rangle = ht$.

We impose the requirement that the relative fluctuations be small when the system leaves the nonadiabatic region (i.e., at $t = t^*$) and that the system goes immediately into the mean-field region. It can be seen from the discussion above that this requirement means that the two following conditions must hold:

$$(h^2 g^{3/2}/T)^{4/5} \gg c/\gamma \gg (T^2 b^2/g^3)^2 \qquad (8)$$

(here we have returned to our original units of time).

If these conditions do not hold, then once it reaches the mean-field region the system will become partitioned into a mixture of domains of two phases. The final attainment of thermal equilibrium (i.e., the expelling of the domains of metastable phase) will occur through a displacement of domain walls. Since the velocity of this displacement decreases as the external field h is weakened, the final stage of the transition may take an exceedingly long time in very weak fields.

In contrast, the satisfaction of conditions (8) guarantees that no macroscopic domains of a metastable phase will arise at any point during the passage through the transition point (more precisely, the probability for the formation of such domains will be exponentially small), and the system will immediately be in a uniform single-phase state.

In this case, for nearly any macroscopic region (with a dimension greater than the correlation radius) the value of the order parameter at the exit time t^* will differ only slightly from ht^* and will therefore have the sign imposed by the external field. The subsequent evolution of an exponential instability will then bring the system to a single-phase state with specifically this sign of the order parameter. The second inequality in (8) guarantees that at $t > t^*$ thermal fluctuations in the steady state will not be capable of creating macroscopic domains of a metastable phase or affecting the outcome of the process.

If the two inequalities in (8) are to be consistent with each other, the external field must not be too weak:

$$h \gg T^3 b^{5/2} / g^{9/2} . \qquad (9)$$

In weaker fields, there is no way to take the system through the point of the phase transition without giving rise to macroscopic domains of a metastable phase along the way.

[1)] When a system goes into the fluctuation region, the particular fluctuations it experiences in arriving there are not important, since strong fluctuations will nevertheless subsequently arise even in the adiabatic regime.

[1]S. Ma, Critical Phenomena, W. A. Benjamin, Reading, Massachusetts, 1976 (Russ. transl. Mir, Moscow, 1980).

[2]L. L. Landau and E. M. Lifshitz, Statisticheskaya fizika, Nauka, Moscow, 1976, Part I (Statistical Physics, Pergamon, Oxford, 1980).

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