Precision of the eikonal approach in nuclear scattering

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The physical causes for the high accuracy of the eikonal approach in nuclear scattering at relatively low energies are determined.

Interest in the precision of the eikonal approach and the extent to which it can be used in nuclear scattering[1,2] at intermediate energies has recently increased sharply, on the one hand, because of a remarkably successful description of the proton-nucleus scattering at 1 GeV (see, e.g., Ref. 3) and antiproton-nucleus scattering at energies of about[4] 50 MeV and, on the other, because of the search for possible manifestations of nonnucleon degrees of freedom in the nuclei which can be identified by simply using reliable calculation methods in the conventional models. The study of nonadiabatic effects poses the most serious difficulties in going beyond the scope of the eikonal approxmation. The reason for this is that the effects associated with the divergence from the geometric optics (generally known as the Fresnel effects) are studied in the context of the scattering from a system of fixed centers, i.e., essentially in the context

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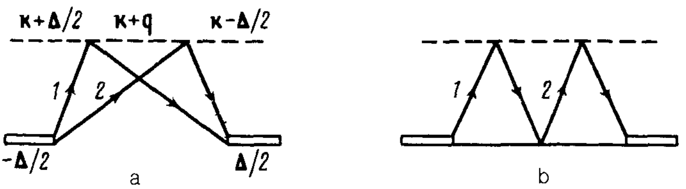

FIG. 1.

of a simple two-body problem (see the review in Ref. 5). With an increase in the energy, these effects die off. The study of nonadiabatic effects based on the assumption that scattering occurs from the fixed centers (with allowance for the nucleon recoil, the nuclear binding energy, the scattering of nucleons of the nucleus by each other between the times the incident particle interacts with different nucleons), which do not disappear even at high energies, is an essentially many-body problem.

A partial cancellation of various nonadiabatic effects in the scattering by a deuteron was detected by Kolybasov and Kondratyuk.[6] This partial cancellation stems from the fact that both the binding of nucleons in the deuteron and their redistribution (diagram b in Fig. 1) are due to the same *NN*-interaction potential (see also Refs. 7–9). Taking diagrams a and b into account and assuming that the *NN*-interaction potential can be localized, we see that the first-order nonadiabatic effects with respect to the parameters $v_N/v_0 \sim 1/mR$ and $\Delta^2 R/m$ are identically zero in the elastic scattering by a deuteron (v_0 is the velocity of the incident particle, v_N is the characteristic velocity of the nucleon in the deuteron, m is the nucleon mass, R is the radius of the deuteron, and Δ is the momentum transfer). This result was later confirmed by Alberi *et al.*[10] By using the method of Alberi *et al.*,[10] however, this study can be extended further to include second-order nonadiabatic effects. We have carried out such a study, the results of which we are reporting in this letter.

The formal scheme can be described as follows. The amplitude corresponding to the sum of diagrams a and b can be expressed[7,10] in terms of the complete Green's function G_{np} of a two-nucleon system, which is convoluted with two deuteron vertices and with the elementary interaction amplitudes. Here

$$G_{np} = (\hat{H}_{np} - E - i\eta)^{-1} = \frac{2\omega}{2\mathbf{kq} - \delta - i\eta}, \quad \delta = \delta_F + \delta_n,$$

$$\delta_F = -q^2 + \Delta^2/4, \quad \delta_n = 2\omega\,(-\epsilon - q^2/4m + \Delta^2/16m - \hat{H}_{np}), \tag{1}$$

where $\hat{H}_{np}$ is the Hamiltonian of the *np* system. The symbol δ_F is related to the Fresnel effect and δ_n is linked with the nonadiabatic effects. The meaning of the variable $\mathbf{q}$ is clear from diagram a (all the momenta are given in the Breit system), ω is the total energy of the incident particle, $\mathbf{k}$ is the average momentum of this particle before and after the scattering, and ϵ is the deuteron binding energy. Let us expand (1) in powers of δ up to terms of order δ^2. The first term of the expansion gives us the standard eikonal result. It can be shown that the second-order nonadiabatic term of most interest to us is proportional to the combination

$$\frac{\omega^2}{Sm^2}\int\frac{d\mathbf{p}d\mathbf{q}}{(2\mathbf{kq}-i\eta)^3}\phi(\mathbf{p}+\frac{\mathbf{q}}{2})\phi(-\mathbf{p}+\frac{\mathbf{q}}{2})[(\mathbf{p\Delta})^2-4(\mathbf{pq})^2], \qquad (2)$$

where $\phi(\mathbf{p})$ is the wave function of the deuteron in the momentum representation. The second term in (2), which contains $(\mathbf{pq}^2)$ within the integral, gives a correction of the type $(v_N/v_0)^2$, which does not vanish as a result of forward scattering. It turns out, however, that this term vanishes identically, regardless of the type of wave function. The correction $\sim(v_N/v_0)^2$ thus vanishes, and we are left with just the term of the type $\Delta^2/m^2v_0^2$, which implies that, in contrast with the general understanding, a correct picture of the fixed centers requires that the "recoil" rate, $\Delta/2m$, rather than v_N, must be small in comparison with v_0. In other words, the state of the nuclear system must change only slightly as a result of the interaction with the incident particle. The first term in (2) is expressed in terms of $\langle 1/r^2\rangle$—the mean square of the nucleon-nucleon backscattering, i.e., in terms of the same combination as that used in the eikonal expression.

Using a similar procedure for the other terms of the expansion G_{np} in powers of δ, we find the following expression which describes the relationship between the sum of diagrams a and b and the eikonal term:

$$M_2=M_2^{eik}\left[1+\frac{i(a+b\Delta^2)}{k}+c\frac{\Delta^2}{k^2}+d\frac{\omega^2\Delta^2}{k^2m^2}\right],$$

$$a=-\frac{2\pi\phi^2(\mathbf{r}=0)}{\langle 1/r^2\rangle},\quad b=\frac{1}{8}\frac{\langle 1/r\rangle}{\langle 1/r^2\rangle},\quad c=-\frac{1}{128\langle 1/r^2\rangle},\quad d=-\frac{1}{32}. \qquad (3)$$

The last three terms in parentheses give the parameters which determine the accuracy of the eikonal approximation. If the amplitudes of the elementary interactions versus the momentum transfer are taken into account, the coefficient a will contain the wave function of the deuteron, which is "smeared" radially with a smearing radius on the order of $\sqrt{B}$, where B is the inclination of the cone in the elementary event. The coefficient d is generally independent of the wave function. There are no first-order nonadiabatic effects and the second-order nonadiabatic effects vanish in the limit $\Delta\to 0$. Expression (3) takes on a very simple and discernible form with a Gaussian wave function of the deuteron $\phi(\mathbf{p})\sim\exp(-R^2\mathbf{p}^2/2)$ (single terms of this expression were written out elsewhere[6,10–12]):

$$M_2=M_2^{eik}\left[1+\frac{i}{\sqrt{\pi}kR}\left(\frac{R^2\Delta^2}{8}-1\right)-\frac{1}{256}\frac{\Delta^4R^2}{k^2}-\frac{\omega^2\Delta^4}{32m^2k^2}\right]. \qquad (4)$$

At a momentum transfer $\Delta\lesssim k$ the nonadiabatic correction is small even at low energies. This small correction is apparently one of the reasons for the unexpected success of the eikonal description of the antiproton-nucleus scattering[4] at 50 MeV.

The results of this study show that the parameter describing the nonadiabatic effects has an auxiliary numerical factor, regardless of the type of the wave function and that these effects can be ignored even at low energies. In other words, these results show that the eikonal method is highly accurate.

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