Intensified particle diffusion in the field of a plasma wave in a transverse magnetic field

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The dynamics of passing particles in the field of a plasma wave, which is propagating across a weak magnetic field, has been studied. A numerical integration shows that even a very slight perturbation of a monochromatic wave results in a sharp intensification of the particle diffusion in transverse energy.

The basic features of the dynamics of particles which are moving in the field of an intense plasma wave, which is itself propagating across a weak magnetic field, were found in Ref. 1. This problem has recently attracted increased interest, however, in connection with the rapidly developing research on the appearance of stochastic behavior in deterministic dynamic systems. This onset of stochastic behavior plays an important role in the acceleration and heating of particles. Karney[2] has taken a stochastic approach to this problem for the case of a wave of moderate amplitude set [$\Omega_0^2|\omega\,\omega_H \ll 1, \Omega_0 = (e\,E_0 k/m)^{1/2}$ is the bounce frequency] in a study of ion heating by lower hybrid waves.

The case of large amplitudes $\Omega_0^2/\omega\,\omega_H \gg 1$ was studied in detail in Refs. 1, 3, and 4. In particular, Sagdeev and Shapiro[1] found an interesting feature in the motion of particles which are not trapped by the wave: As a particle revolves in the magnetic field, the changes which occur in the modulus of its velocity, $\Delta v \sim \Omega_0/k$, upon the crossing of successive resonances $v_x = \omega/k$ are precisely equal but opposite in sign in the case in which we have $\Omega_0^2/\omega\,\omega_H \to \infty$. Accordingly, over each revolution in the magnetic field a particle with a velocity $v > \omega/k$ undergoes no change in energy as it crosses two resonance points $v_x = \omega/k$, $v_y = \mp\sqrt{v^2 - (\omega^2/k^2)}$. At finite values of the parameter $\eta = \Omega_0^2/\omega\,\omega_H \gg 1$, the change in the energy of the particle consists of, in addition to the part $\Delta v \sim \Omega_0/k$, which cancels out and which does not depend on the phase of the particle in the wave, $\psi = kx - \omega t$ [since this quantity tends toward $\pi(\mathrm{mod}\,2\pi)$ at the point of the resonance], an increment $\Delta v' \sim \Delta v(1/\eta)\ln\eta$, which does not cancel out for successive resonances. This increment depends on the phase $\psi(t_i)$ at the point of the resonance as it would depend on a genuinely random quantity, since a large shift in the phase ψ arises in the course of the motion from one resonance to another. A diffusion of the particles in the velocity v thus arises. This diffusion is slight, however, by virtue of the relation $\Delta v'/\Delta v \sim (\ln\eta/\eta) \ll 1$. On the other hand, the cancellation of the resonances is associated with the adiabatic nature of the wave-particle interaction[3,4] near the point of the resonance. As we will see, this adiabatic nature may be violated by even extremely small perturbations, e.g., in the form of a second wave of small amplitude. The increments in the velocity of the particle, which are $\sim\Omega_0/k$ (as before), cease to cancel each other out, and the diffusion coefficient, increases substantially (by a factor up to η^2).

We have studied the dynamics of electrons in the field of two plasma waves with approximately equal phase velocities by numerical integration.

The equation of motion of an individual particle in the field of waves which are propagating along the x axis and across the magnetic field, which is directed along the z axis, can be written

$$\ddot{x} + \omega_H^2 x = -\frac{e}{m} E_0 \sin(kx - \omega t) - \frac{e}{m} E_1 \sin(k_1 x - \omega t + \theta)\,. \qquad (1)$$

To study the motion of a particle near the resonance, $\dot{x} = \omega/k$ it is convenient to introduce a fast time $\tau = \sqrt{2}\Omega_0 t$ and a phase $\psi = kx - \omega t$. In terms of these variables, in Eq. (1) becomes

$$\frac{d^2\psi}{d\tau^2} + \frac{1}{2}\sin\psi + \frac{1}{2\eta} q + \frac{\Omega_1^2}{2\Omega_0^2}\sin[(1+\delta)\psi + \beta\tau + \theta] = 0, \qquad (2)$$

where $\delta = (k_1 - k)/k, \beta = \delta(\omega/\sqrt{2}\omega_0)$, $\Omega_1^2 = eE_1 k/m$, and $q = kv_y/\omega = (k\omega_H/\omega)x$ is a function which varies slowly near the resonance and which may be regarded as depending on only the slow time t.

In the case of a single wave ($E_1 = 0$) the trajectory of a particle can be described well by an integral of Eq. (2), which can be found easily by assuming that the slow time remains constant near the point of the resonance ψ_i (see Ref. 4 for more details):

$$\frac{1}{2}\left(\frac{d\psi}{d\tau}\right)^2 + \sin^2\frac{\psi}{2} + \frac{1}{2\eta} q(t)(\psi - \psi_i) = \sin^2\frac{\psi_i}{2}\,. \qquad (3)$$

Since $\eta \gg 1$, the turning point ψ_i is always close to $\pi(\mathrm{mod}\,2\pi)$. Figure 1 shows a phase portrait of Eq. (2) and a profile of the potential energy in the case $E_1 = 0$. Figure 2a

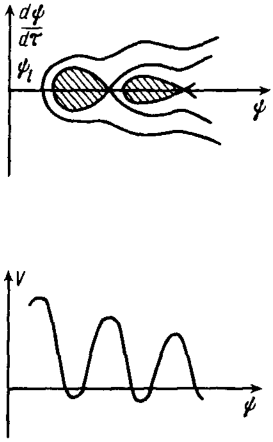

FIG. 1. Phase portrait and profile of the potential in Eq. (3).

shows the result of a numerical integration of Eq. (2) under the condition $E_1 = 0$. Even in the case $E_1 = 0$, however, not all the trajectories behave as shown in Figs. 1 and 2a. Because v_y varies, if only slightly, as a particle moves between the humps of the potential well, a small fraction of the particles can be trapped by the wave at $v_y = -v_0 < 0$ and will then move in the (v_x, v_y) plane along a straight line $v_x = \omega/k$,

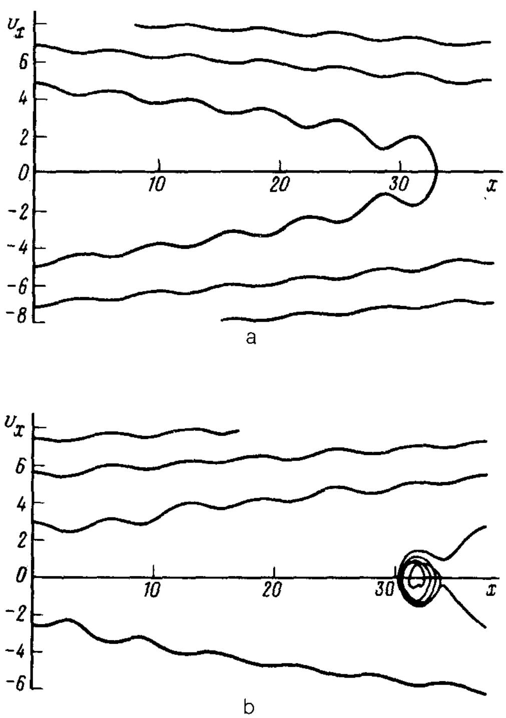

FIG. 2. Trajectories of particles in the (x, v_x) phase plane for $\omega_p/\Omega_0 = 10$, $\omega_H/\Omega_0 = 0.01$. a—$E_1 = 0$; b—$E_1/E_0 = 0.3$.

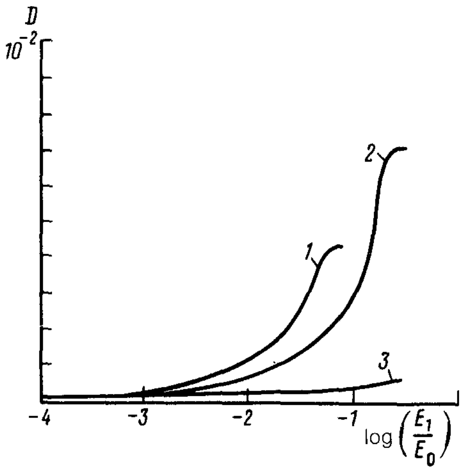

FIG. 3. Mean square increment in the velocity of a particle per revolution in the magnetic field versus E_1/E_0 under the conditions $\omega_p/\Omega_0 = 10$, $\omega_H/\Omega_0 = 0.01$. 1—$\beta = 0.51$; 2—$\beta = 0.88$; 3—$\beta = 3.5$.

instead of along a circular arc. After they reach a velocity $v_y \cong v_0 > 0$, the particles become transit particles again. The number of such particles is small [a simple analysis shows that their number is $\sim (\omega_H \omega/\Omega_0 v_0 k)\ln(\eta\omega/\pi k v_0)$ of the total number of particles having a velocity $v = \sqrt{v_0^2 + (\omega^2/k^2)}$]. However, their number increases dramatically when a second wave, even of small amplitude, comes into play, since now there is the possibility that particles will be trapped in the potential wells (and, incidentally, escape from them) by virtue of perturbations which disrupt the separatrix of Eq. (3). The motion near the resonance will become extremely irregular. A blurring of the separatrix near the hyperbolic point over a width on the order of the distance between the individual loops of the separatrix (Fig. 1) occurs under the condition $1/\eta \lesssim E_1/E_0$, as follows from Eq. (2). Here we are assuming $\beta \sim 1$, since at very small values of b dynamic system (2) remains approximately conservative, and the motion of the particles differs only slightly from that in the field of a single wave. At $b \gg 1$, on the other hand, the perturbation is a high-frequency perturbation, and its effect is again insignificant. Consequently, even at $1/\eta \lesssim E_1/E_0 \ll 1$, with $\beta \sim 1$ (Fig. 2b), we find a pronounced irregularity, and we find that the increments in the energy of the particle as it crosses successive resonances no longer cancel out. Under these conditions the mean square increment in the velocity of a particle per revolution should be determined by the average increment in the velocity at each resonance, $\Delta v \sim \Omega_0/k$. Figure 3 shows plots of the mean square increments versus E_1/E_0 for various values of the parameter β; these results agree with the qualitative arguments above. In summary, the presence of even a very small perturbation leads to a sharp increase (by a factor of about η^2) in the diffusion coefficient, because the cancellation of resonances no longer occurs.

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