

# Universality of the parameters of two-dimensional subbands in the surface layers of Kane's semiconductors

V. F. Radantsev

*A. M. Gor'kiĭ Ural State University*

(Submitted 2 July 1987)

*Pis'ma Zh. Eksp. Teor. Fiz.* **46**, No. 4, 157–159 (25 August 1987)

Experimentally observed similarity of the parameters of 2D subbands in  $\text{Hg}_{1-x}\text{Cd}_x\text{Te}$  of various compositions is attributable to the fact that even with a moderate flexure of the band the larger part of the surface-layer electrons is described by an ultrarelativistic dispersion relation  $\epsilon = ps$ , which is the same for all Kane's superconductors.

The effective mass is the principal parameter which determines the quantum-size levels and the state density in the two-dimensional subbands of the surface layers of semiconductors. At first sight, the results of a comparative study<sup>1)</sup> of 2D electrons in  $\text{Hg}_{1-x}\text{Cd}_x\text{Te}$  of various compositions, shown in Fig. 1, seem to be surprising. In the

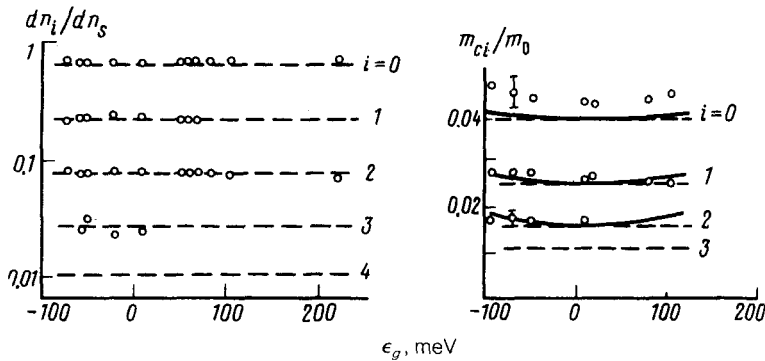


FIG. 1. Filling rates  $dn_i/dn_s$  ( $n_i$  is the population of the  $i$ th subband) and the cyclotron masses  $m_{ci}$  (for  $n_s = 2 \times 10^{12} \text{ cm}^{-2}$ , in units of a free-electron mass  $m_0$ ) for quantum-size subbands of the surface layers of  $\text{Hg}_{1-x}\text{Cd}_x\text{Te}$  plotted as a function of the gap width  $\epsilon_g$ . Points—experimental; solid lines—semiclassical calculation; dashed lines—ultrarelativistic approximation.

tested series of compositions, which correspond to a variation of the gap  $\epsilon_g = \epsilon(\Gamma_6) - \epsilon(\Gamma_8)$  in the range 100–250 meV, the effective mass at the bottom of the conduction band,  $m_n$ , varies by more than an order of magnitude (we varied it by three orders of magnitude in the calculation of  $m_n$ ), whereas the distribution of the carriers in the subbands and the cyclotron masses  $m_{ci}$  remain the same within the errors if the surface electron densities are the same,  $n_s \gtrsim 10^{12} \text{ cm}^{-2}$ . This result shows that the quantum-size levels are closely related to the dispersion in 2D subbands, which can be expected only if the volume dispersion is similar in the samples with different  $m_n$ . Such a situation occurs for an electron gas near the surface of narrow-gap semiconductors (as odd as it may seem from the standpoint of three-dimensional effects determined by the states near the bottom of the conduction band). Because  $m_n$  is small, the magnitude of the surface kink in the bands can be as high as  $\mu_s - \mu_b \sim 250 \text{ meV}$  even at  $n_s \sim 10^{12} \text{ cm}^{-2}$  ( $\mu_s$  and  $\mu_b$  are respectively the surface and bulk chemical potentials) and the condition  $\epsilon \gg \epsilon_g = 2m_n s^2$  ( $s \approx 10^8 \text{ cm/s}$  is the interband matrix element of the velocity operator) holds for the bulk of electrons near the surface in materials with  $\epsilon_g < 100 \text{ meV}$ . At these energies, however, the conduction band of the Kane semiconductors is described by a dispersion relation of the ultrarelativistic type  $\epsilon = \sqrt{p^2 s^2 + m_n^2 s^4} \approx ps$ , which is the same not only for  $\text{Hg}_{1-x}\text{Cd}_x\text{Te}$  of various compositions but also for the entire class of narrow-gap semiconductors by virtue of the proximity of  $s$ .

In such a pseudoultrarelativistic limit  $\mu_s \gg 2m_n s^2$  the classical self-consistent Thomas-Fermi surface potential at  $\mu_b = 0$  (we studied substrates that were slightly doped) can be written, as can easily be shown, in the form

$$\mu(z) = \frac{\gamma e^2}{z} \left( 1 + \frac{\gamma e^2}{\mu_s z} \right)^{-1}, \quad (1)$$

where  $\gamma = 3\pi/2\alpha^3\chi^2$ ,  $\alpha = e^2/s\hbar\chi$  is a modified fine-structure constant, and  $\chi$  is the

dielectric constant. In most of the surface layer, potential (1) is approximately the same as the Coulomb potential, which explains the previously noted<sup>2</sup> adequacy of the semiclassical description of the 2D surface states in the narrow-gap semiconductors. A semiclassical quantization gives rise to the following equation for the dispersion  $\epsilon_i(p_i)$  in 2D subbands:

$$2 \int_{\mu_s - \epsilon_i - p_i s}^{\mu_s} \sqrt{(\epsilon_i - \mu_s + \mu)^2 - p_i^2 s^2} \frac{d\mu}{\mu^2} = \beta \left( i + \frac{3}{4} \right), \quad \beta = \sqrt{8\pi\alpha/3}. \quad (2)$$

Within  $\sim 5\%$  the parameter  $\beta$  is 1 for all narrow-band semiconductors, and Eq. (2) does not contain the parameters of the material, i.e., it determines the structure of 2D subbands which is universal (at  $\mu_s > \epsilon_g$ !) for all the Kane semiconductors.<sup>2)</sup> The values  $dn_i/dn_s$  and  $m_{ci}$ , which are determined in the ultrarelativistic approximation (2) and which are represented by a dashed line in Fig. 1, are in good agreement with the experimental data not only for  $\text{Hg}_{1-x}\text{Cd}_x\text{Te}$  but also for InSb. For the excited subbands the ultrarelativistic approximation is amenable to simple analytic approximations for  $dn_i/dn_s$  and  $m_{ci}$  (the error is no greater than 10% even for  $i = 0$ )

$$dn_i/dn_s = n_i/n_s \approx [e/(e-1)]^\beta \exp -\beta i, \quad e = 2.71 \dots, \quad (3)$$

$$m_{ci} = \frac{\hbar}{s} \delta(i) \sqrt{2\pi n_i} = \frac{\hbar}{s} \delta(i) \left( \frac{2\pi e}{e-1} \right)^{\beta/2} \exp -\frac{\beta i}{2}, \quad (4)$$

where  $\delta(i)$  varies between 1.2 when  $i = 0$  and  $\pi/2$  in the limit  $i \rightarrow \infty$ . Expressions (3) and (4) explain the empirical behavior: the values  $dn_i/dn_s$  and  $m_{ci}^2$  (this also applies to the initial concentrations and the binding energies) are related by a universal relationship in the adjacent subbands; specifically, they differ by a factor of  $2.7 \pm 0.2$  (see Fig. 1).

The invariability of the parameters of the 2D subbands when  $x$  is decreased to values corresponding to the gap-free state, just as the agreement of the experimental data with a theory which ignores the resonant nature of the 2D states at  $\epsilon_g < 0$ , leads to yet another important conclusion in the physical sense: The energy structure of the states of the surface layer cannot be distorted appreciably because of the mixing of these states with the states of the valence band of the volume or with the deformation of the wave functions of the valence band. From the theoretical point of view, this problem has not been analyzed systematically, although in light of the aforementioned analogy with the ultrarelativistic gas<sup>3)</sup> the surface electron system in the Kane semiconductors under study can, in our view, be treated as a one-dimensional analog of the vacuum condensate of electrons near supercharged nuclei.<sup>5</sup> In the latter case, the electron density distribution in the vacuum-shell states is, as was shown in Ref. 6, nearly independent of the fact that such states are at resonance with the lower-continuum states, just as it is nearly independent of the vacuum polarization.

I wish to thank O. A. Pankratov, M. É. Raïkh, E. L. Romyantsev, and Al. L. Éfros for useful discussions.

<sup>1)</sup>A capacitive-spectroscopy method in quantizing magnetic fields, which is not critical toward the magnitude of the gap or the conductivity of the substrate, was used in the experiment.<sup>1</sup> The calculations were

carried out in a semiclassical approximation.<sup>2</sup>

<sup>2</sup>The universality of the 2D subband structure also explains the fact that the effects observed in the inclined magnetic fields do not depend on  $m_n$ , which was pointed out in Ref. 3.

<sup>3</sup>In the three-dimensional case the electron gas in Kane's semiconductors was studied by Landsberg<sup>4</sup> as an analog of the relativistic gas.

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<sup>4</sup>P. T. Landsberg, Phys. Rev. B **33**, 8321 (1986).

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Translated by S. J. Amoretty