Ponderomotive effect in above-threshold ionization

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(Submitted 16 July 1987)
Pis'ma Zh. Eksp. Teor. Fiz. **46**, No. 5, 173–175 (10 September 1987)

"Distorted Volkov solutions" are derived. These solutions incorporate in a consistent quantum-mechanical manner the vibrational and translational motion of an electron in a nonuniform light field. The solutions are used to discuss the shape of the spectrum of above-threshold ionization.

The experiments by Bucksbaum *et al.*[1] resulted in the first direct observation of the scattering of electrons by a ponderomotive potential which arises from a nonuniformity of the amplitude of the scattering light field. Several investigators had previously pointed out the importance of the ponderomotive effect for explaining nonlinear effects which arise in the interaction of electrons and atoms with an intense field.[2] In the present letter we solve the one-dimensional Schrödinger equation for an electron in a nonuniform electric field $E(x,t) = E(x)\cos\omega t$, which is directed along the x axis. If the wavelength of the electron, λ_e, is small in comparison with the length scale of the variations in the field, R, we can use the semiclassical approximation, and the wave function of the electron is, within terms $\sim\lambda_e/R$, a distorted Volkov solution ($\hbar = m_e = e = 1$)

$$\psi_e(x,t) = \exp\{-i\epsilon t + i\int^x p(x')dx'\}\exp\left\{i\,\frac{U(x)}{2\omega}\sin 2\omega t + i\,\frac{E(x,t)}{\omega^2}\,p(x)\right\}, \quad (1)$$

where ϵ is the quasienergy, $U(x) = E^2(x)/4\omega^2$ is the ponderomotive potential, and $p(x) = \sqrt{2(\epsilon - U(x))}$ is the semiclassical momentum. The first factor in (1) stems from the translational motion in the potential U; the second describes oscillations at the field frequency which adjust to accommodate the varying conditions during the translational motion.

In problems with focused radiation, R is the transverse dimension of the focus, outside which $E(x)$ tends toward zero. Outside the focus, at $x \gg R$, solution (1) converts into the wave function of a free particle with a momentum $p(\infty) = \sqrt{2\epsilon}$. If we take the limit $R \to \infty$ and consider finite distances x, we have $U(x) = \text{const}$, and (1) converts into a Volkov solution in a uniform field with a constant momentum $\tilde{p} = \sqrt{2(\epsilon - U)}$.

We wish to emphasize that the distorted Volkov solutions are indexed by the quantum numbers of the electron which has left the focus. These solutions are accordingly particularly convenient for describing an interaction of electrons with spatially bounded photon beams. A spatial nonuniformity turns out to be important for effects involving the emission of a photon by an electron in a light field, stimulated multiphoton bremsstrahlung in collisions with atoms, above-threshold ionization, etc. Solution

(1) obviously gives a natural description of the observed reflection of electrons with $\epsilon < U$ from a light beam.[1]

As an example we consider above-threshold ionization. A pulsed ionizing field of duration τ_i is simulated by a field of the type selected if the condition $R/v_e \ll \tau_i$ holds (v_e is a characteristic velocity of the electron) and no significant ionization occurs during the time the field is applied. We also assume that the ionization does not reach saturation during the time τ_i. The procedure used to calculate the probability for a transition from a bound state with an energy ϵ_0 to state (1) is the same as in Refs. 4 and 5. For a transition involving the absorption of n photons, the conservation law

$$\epsilon_n = \epsilon_0 + n\omega, \tag{2}$$

is found. This law leads to the experimentally observed positions of the peaks in the spectrum of photoelectrons. Empirical considerations supporting (2) were discussed in Ref. 6. The probability for n-photon ionization, $w_n(x_0)$, depends on the position of the atom which is being ionized, x_0 (we ignore the thermal motion). It can be found from an expression which applies to a uniform field by making the replacement $p \rightarrow p(x_0)$. The ionization channel involving the production of an electron with an above-barrier energy $\epsilon_n > U_{\max}$ is open for atoms at any point in the focus. The channel with $\epsilon_n < U_{\max}$ is partially closed since the ionization matrix element for atoms between the turning points defined by the conditions $U(x_n) = \epsilon_n$ is exponentially small. The reason it is small is the decay of the ψ function of the final state below the barrier.

We are reporting here results calculated on ionization from a δ-function well (cf. Ref. 5). We restrict the discussion to the limiting case in which the condition $N = |\epsilon_0|/\omega \gg 1$ holds, and the ionization regime is a multiphoton regime, $s_0 = U_{\max}/\omega \ll N$, at all points in the focus. Denoting by n_0 the threshold number of photons required for ionization in a weak field, and by $n_0 + k_0$ the number of photons whose absorption results in entrance into the first completely open channel, we can describe the emission from the focus of electrons with an energy $\epsilon = \epsilon_0 + (n_0 + k_0 + m)\omega$, $m \geqslant 0$ per unit time by

$$W_{n_0 + k_0 + m} = c_m \left(\frac{es_0}{2N}\right)^N \left(\frac{e^2 s_0}{2N}\right)^{\delta + m}, \tag{3}$$

where $\delta = \Delta + k_0 - s_0 < 1$, $\Delta = n_0 - N < 1$, $e = 2.71$, and c_m is a factor which depends weakly on m. With increasing m, the emission described by (3) falls off comparatively slowly; the heights of adjacent peaks differ by an amount $e^2 s_0/2N$. The exponent which determines the dependence of $W_{n_0 + k_0 + m}$ on the field intensity is smaller by an amount s_0 than would be expected on the basis of perturbation theory.

The relative emission for partially closed channels with an energy $\epsilon = \epsilon_0 + (n_0 + k)\omega$, $0 \leqslant k < k_0$, is determined primarily by the ratio of the channel energy to the barrier height,

$$\frac{W_{n_0+k}}{W_{n_0+k_0}} \approx \left(\frac{\Delta + k}{s_0}\right)^N . \tag{4}$$

and it falls off rapidly with decreasing k by virtue of the condition $N \gg 1$. The weak dependence of the absolute value of W_{n_0+k} on the intensity is determined by the shape of the spatial envelope of the field. In particular, this would be a logarithmic dependence in the case of a Gaussian envelope.

In this approach, the simple one-dimensional model thus conveys at a qualitative level the basic observed[3] aspects of above-threshold ionization: The spectral positions of the peaks do not depend on the intensity. The envelope of the peaks has an asymmetric maximum near $\epsilon \gtrsim U_{\max}$. The number of relatively suppressed channels is on the order of $U_{\max}/\omega$. The dependence of the emission yield on the intensity is weaker than the perturbation-theory prediction.

The reason for the relative suppression of ionization channels with an electron energy $\epsilon_n < U_{\max}$ has been determined: It is the exponentially small value of the matrix element for ionization involving the production of an electron in the region of the focus which is inaccessible to the electron classically.

We wish to thank N. B. Delone and M. V. Fedorov for a useful discussion of the results.

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Translated by Dave Parsons