# Evaluation of nonperturbative effects in lattice QCD thermodynamics

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Analysis of Monte Carlo calculations in lattice QCD reveals a new nonperturbative contribution to the thermodynamic functions of a quark-gluon system.

**1.** Lattice calculations by the Monte Carlo method are so far the only method for studying nonperturbative effects in quantum chromodynamics (QCD) from the first principles of the theory. In this letter we analyze Monte Carlo data on the thermodynamic functions over the temperature range from $T_c$ to $(4–5)T_c$ ($T_c \cong 200$ MeV is the temperature of the deconfinement phase transition) in order to estimate these effects in QCD at a nonzero temperature. We treat the Monte Carlo results as "experimental data." The reason for our interest in this temperature range is that nonperturbative contributions are negligible at $T \gg T_c$, while at $T < T_c$ the confinement transforms the quark-gluon degrees of freedom into hadronic degrees of freedom, rendering the relationship between the thermodynamics and the original (quark-gluon) Lagrangian QCD unacceptably complicated. Our analysis of the Monte Carlo data reveals several new properties of the QCD equation of state of matter at $T > T_c$.

**2.** We write the energy density $\epsilon(T)$ and the pressure $p(T)$ as

$$\epsilon = \epsilon_0 + \epsilon_1, \qquad p = p_0 + p_1 , \tag{1}$$

where $\epsilon_0$ and $p_0$ are quantities found from perturbation theory, and $\epsilon_1$ and $p_1$ are nonperturbative contributions. We treat $\epsilon_0$ and $p_0$ at $T > T_c$ as thermodynamic functions of a quasi-ideal gas of quarks and gluons:

$$\epsilon_0 = \sigma T^4, \qquad p_0 = \frac{1}{3}\sigma T^4 , \tag{2}$$

where $\sigma \cong$ const is the Stefan-Boltzmann constant "renormalized" to incorporate effects of the finite lattice size[1,2] and single-loop perturbation-theory corrections $\sim g^2(T)T^4$ (in the temperature range under consideration here, the temperature dependence of the coupling constant $g$ is extremely weak).

Satz[3] used a bag model for (1) at $T > T_c$,

$$\epsilon = \epsilon_0 + B, \qquad p = p_0 - B . \tag{3}$$

Finding the following estimate for the function $\delta = \epsilon - 3p$ from Monte Carlo data on Su(2) gluodynamics[4]: $\delta \cong 4B$ ($B^{1/4} \cong 190$ MeV), Satz interpreted that result as confirming the validity of the bag model. However, that result runs into a serious contradiction when it is compared with calculations of perturbations at a finite lattice based

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on a single-loop perturbation theory. The agreement of those calculations for the function[5] $\epsilon(T)$ with the Monte Carlo data is evidence that there is no substantial nonperturbative contribution to the energy density at $T > T_c$. This circumstance, like the need for a linear temperature dependence of $\delta$ for a reconciliation of the Monte Carlo data, was pointed out previously by one of the present authors.[6]

**3.** Let us examine the case of a zero chemical potential, in which the following relation holds:

$$T\frac{dP}{dT} - p = \epsilon. \tag{4}$$

If the function $p(T)$ is known, $\epsilon(T)$ can be found unambiguously from (4). If, on the other hand, the function $\epsilon(T)$ is given, then (4) is a differential equation for $p(T)$, whose general solution is

$$p(T) = T\left[\int \epsilon(T)\frac{dT}{T^2} + C\right], \tag{5}$$

where $C$ is an arbitrary constant. It follows from (5) that the function $p(T)$ will, in general, contain more information on the system than $\epsilon(T)$ will! This basic point has not been noted previously, and the overwhelming majority of the Monte Carlo calculations have dealt exclusively with the function $\epsilon(T)$.

We have analyzed the results of Monte Carlo calculations in SU(2) gluodynamics[4] and in the SU(3) gauge theory with $n_f = 2$ Wilson quarks.[7] Studies[4,7] in which $p(T)$ and $\epsilon(T)$ were calculated simultaneously over a fairly broad temperature range above $T_c$ are essentially the only studies reported in the literature. We find that (1) there are essentially no nonperturbative contributions to the energy density, i.e., $\epsilon_1 \cong 0$; (2) there is a substantial nonperturbative contribution to the pressure, i.e., $p_1 = 0$; and (3) the only possibility for simultaneously satisfying (1) and (2) is, according to (5), a linear $T$ dependence of the nonpertubative pressure $p_1$.

We thus find a new equation of state for QCD matter at $T > T_c$:

$$\epsilon = \epsilon_0 = \sigma T^4, \quad p = p_0 + p_1 = \frac{1}{3}\sigma T^4 - AT; \qquad A = \text{const} > 0. \tag{6}$$

Figures 1 and 2 show a description of the Monte Carlo data of Refs. 4 and 7 by means of (6). We find these results extremely convincing. For the parameter $A$ we find [expression (7b) incorporates a correction factor to eliminate effects of the finite lattice size]

$$A^{1/3} \cong 36.3\,\Lambda_L^{SU(2)} \cong 0.8\,T_c, \tag{7a}$$

$$A^{1/3} \cong 230\,\Lambda^{SU(3)}_{L(n_f=2)} \cong 1.5\,T_c. \tag{7b}$$

**4.** Expressions (3) for the thermodynamic functions are widely used, along with the assumption of an ideal gas of $\pi$ mesons at $T < T_c$, in various phenomenological calculations. We do not believe that the popularity of this model (a bag model) is a

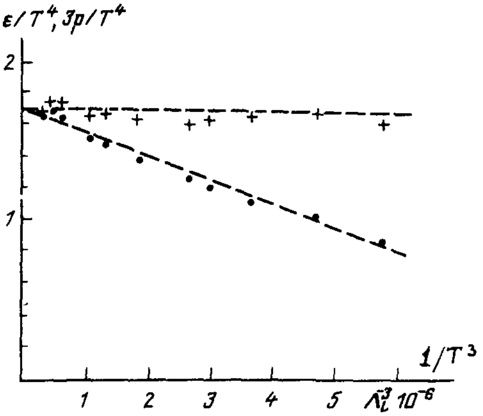


FIG. 1. Monte Carlo data for a SU(2) gauge system without quarks on a $10^3\times 3$ lattice.[4] $+$ —$\epsilon/T^4$; $\bullet$—$3p/T^4$. The dashed lines show the functions $\sigma$ and $\sigma - 3A/T^3$ found from (6); the parameter $A$ is taken from expression (7a).

consequence of its solid theoretical foundation but simply a reflection of the fact that it provides an economical way of parametrizing the functions $p(T)$ and $\epsilon(T)$ for a first-order phase transition. In this regard, the use of (6), instead of (3), could be regarded as a new model, just as economical as the bag model. The temperature ($T_c$) of the phase transition from the ideal gas of "massless" $\pi$ mesons with $\gamma_h = 3$ internal degrees of freedom to quark-gluon plasma (6) with $u$ and $d$ quarks ($\gamma_q = 37$) can then be determined by the expression

$$T_c = \left[\frac{90A}{\pi^2(\gamma_q - \gamma_h)}\right]^{1/3} \cong 0.64A^{1/3}, \qquad (8)$$

in agreement with result (7b), which was found from an analysis of Monte Carlo data. The jump in the energy density at the point of the phase transition is $\Delta\epsilon = 3AT_c \cong 2$ GeV/fm$^3$.

A question which logically arises is that of the physical meaning of the nonperturbative increment $p_1$ in the pressure in (6). On the basis of simple dimensionality

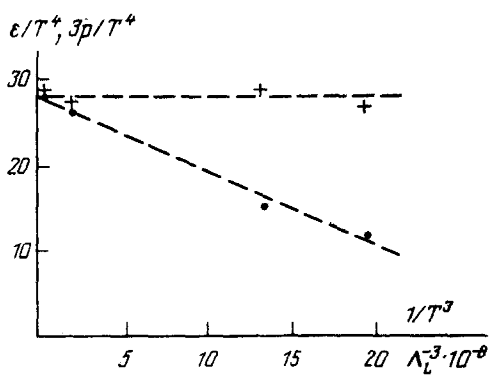


FIG. 2. Monte Carlo data for an SU(3) system with Wilson fermions of two types on an $8^3\times 3$ lattice.[7] The notation is the same as in Fig. 1; the parameter $A$ is taken from (7b).

considerations we would tend to interpret the parameter $A$ as the number density of certain "quasiparticles" which would have to carry a zero energy and a zero momentum but which would make a nonzero contribution to the pressure. There would be about three such quasiparticles in a volume of 1 $\mathrm{fm}^3$, according to (7b).

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[1]J. Engles, F. Karsch, and H. Satz, Nucl. Phys. **B205**, 239 (1982).
[2]M. I. Gorenshteĭn and O. A. Mogilevskiĭ, Yad. Fiz. **46**, 302 (1987) [Sov. J. Nucl. Phys. **46**, 186 (1987)].
[3]H. Satz, Phys. Lett. **1138**, 245 (1982).
[4]J. Engels, F. Karsch, I. Montvay, and H. Satz, Nucl. Phys. **B205**, 545 (1982).
[5]U. Heller and F. Karsch, Nucl. Phys. **B251**, 254 (1985).
[6]O. A. Mogilevskiĭ, Proceedings of the International Seminar on Problems of High-Energy Physics, Dubna, 1986, D1,2-86-668, p. 227.
[7]T. Celik, J. Engels, and H. Satz, Nucl. Phys. **B256**, 670 (1985).

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