Superconductivity and charge density waves of the electron liquid in highly anisotropic polar semiconductors

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The example of the electron liquid in a polar semiconductor with a large number of electron waves ($\nu \gg 1$) is used to show that such an electron liquid is superconducting at densities $n > n_{cr}$. The effective electron-electron interaction increases dramatically as $n \to n_{cr}$. At $n < n_{cr}$ there is a transition accompanied by the formation of a charge density wave.

The model of a highly anisotropic electron liquid and of an electron-hole liquid is intriguing because the correlation energy in it is comparable to the kinetic energy at high densities, where the random-phase approximation is valid.[1] It was shown in Refs. 2–4 that a strong correlation interaction of this sort in an electron-hole liquid results in several interesting properties, including superconducting properties. It was pointed out in those studies that in the absence of a dispersion of the dielectric constant an electron liquid does not have the properties characteristic of an electron-hole liquid. In the present letter we analyze an electron liquid in a doped semiconductor with a large number of electron valleys, $\nu \gg 1$, with an electron dispersion law which is the same for all valleys [$\epsilon(\mathbf{p}) = \mathbf{p}^2/2m$, where m is the effective mass of the electron] and whose dielectric constant has a strong frequency dispersion, with the result that an electron liquid of this sort has a superconductivity. We know[5] that incorporating the interaction of electrons with the optical phonon mode described in polar semiconductors by the coupling constant $g(k) = [(2\pi e^2/k^2)(\kappa_0 - \kappa_\infty/\kappa_0\kappa_\infty)\hbar\omega_0]^{1/2}$ acts along with the Coulomb interaction between electrons, to cause a frequency dispersion of the dielectric constant: $\kappa(i\omega) = \kappa_\infty \kappa_0(\omega_0^2 + \omega^2/\kappa_\infty \omega_0^2 + \kappa_0 \omega^2)$, where ω_0 is the frequency of a longitudinal optical mode, and κ_0 and κ_∞ are the static ($\omega \to 0$) and high-frequency ($\omega \gg \omega_0$) dielectric constants, for which we assume $\kappa_0 \gg \kappa_\infty$. For the discussion

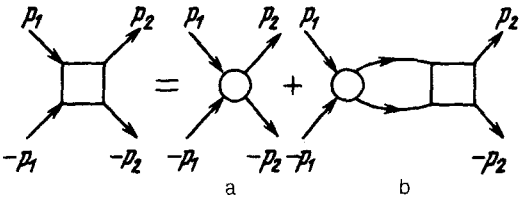

FIG. 1. Dyson equation for the vertex $\Gamma(p_1 p_2)$.

below it is convenient to choose a system of units with $e^2/\kappa_\infty = m = \hbar = 1$. The most interesting region of densities of an electron liquid, which we will be discussing below, is the region $\nu \ll n \ll \nu^4$. In this region (first) the random phase approximation is valid, and (second) the momenta and frequencies determining the renormalizations of the chemical potential and of the interaction vertices are typically high: $k \sim n^{1/4} \gg p_F$, $\omega \sim n^{1/2} \gg \epsilon_F$, where $p_F = (3\pi^2 n)^{1/3}$ and $\epsilon_F = p_F^2/2$ are the Fermi momentum and Fermi energy. For the latter we assume $\epsilon_F \gg {}^{\nu}\omega_0$. We consider the Dyson equation for a set of ladder diagrams with a zero net momentum (Fig. 1). An integration over the internal momenta in the diagram (Fig. 1b) is carried out near the Fermi surface. Diagrams which are irreducible by the Cooper channel contribute to the vertex $\Gamma_0(p_1 p_2)$ (Fig. 1a), as do reducible diagrams with an integration over the internal momenta far from the Fermi surface. A selection of diagrams for $\Gamma_0(p_1 p_2)$ was also carried out in Refs. 2–4. Their sum is

$$\Gamma_0(p_1 p_2) = (V(\mathbf{q}\omega) + \gamma)[1 - \nu\Pi_0(\mathbf{q}\omega)(V(\mathbf{q}\omega) + \gamma)]^{-1},$$

where $\mathbf{q} = \mathbf{p}_2 - \mathbf{p}_1$, $\omega = \epsilon_2 - \epsilon_1$, $|\mathbf{p}_1| = |\mathbf{p}_2| = p_F$, $\Pi_0(\mathbf{q}\omega)$ is the zeroth polarization operator, $V(\mathbf{q}\omega) = 4\pi\kappa_\infty/\mathbf{q}^2\kappa(\omega)$, and the vertex γ is determined by the sum of diagrams (Fig. 2). The wavy line on these diagrams represents the screened Coulomb interaction in the random-phase approximation. The diagrams in Fig. 2 are dominated by integration momenta and frequencies $p \sim n^{1/4} \gg p_F$ and $\omega \sim n^{1/2} \gg \epsilon_F$; the vertex γ can therefore be assumed independent of the external momenta and frequencies. Furthermore, since the relation $\epsilon_F \gg \omega_0$, holds, this vertex is determined by κ_∞. A simple calculation yields $\gamma = -\pi^{3/4}[\Gamma(1/4)]^{-2} n^{-3/4}$. The contribution of the vertex γ—an attraction—to the effective interaction $\Gamma_0(p_1 p_2)$ is important only at frequency transfers $\omega \ll \omega_0$, because in the region of densities under consideration here the relation $V(p_F; \omega \gg \omega_0) = 4\pi/p_F^2 \gg |\gamma|$, holds, and at a sufficiently large value of κ_0 we have $V(p_F; \omega \to 0) = (4\pi\kappa_\infty/p_F^2\kappa_0) \ll |\gamma|$. At this point we assume

$$4\pi\kappa_\infty/p_F^2\kappa_0 \ll |\gamma|. \qquad (1)$$

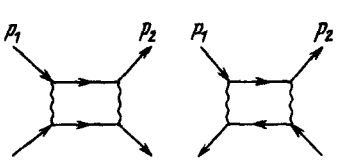

FIG. 2. Diagrams for the vertex γ.

Let us examine the vertex $\Gamma_0(\mathbf{p}_1\mathbf{p}_2;\omega=0)$ at a zero transfer frequency. We see from $\Gamma_0(p_1p_2)$, that it has a pole if

$$F(q) = [1-\nu\gamma\Pi_0(\mathbf{q},0)]\,\mathbf{q}^2 - \nu\Pi_0(\mathbf{q},0)\frac{4\pi\kappa_\infty}{\kappa_0} = 0. \tag{2}$$

If $F(q)$ vanishes at real $|\mathbf{q}|\neq 0$, the homogeneous ground state of the electron liquid becomes unstable, and a charge density wave forms in it. This instability is accompanied by a structural instability, as can be seen easily by examining the phonon Green's function. The renormalized phonon frequency tends toward 0 as $F(q)\to 0$. The existence of a solution of Eq. (2) is determined by the quantity $\alpha(q)=1-\nu\gamma\Pi_0(q)$. If the density of the electron liquid satisfies $n\geqslant n_1=(3/8)^{12/5}n_0$ {n_0 is the density for which the ground-state energy has a minimum, its value, derived in Ref. 1, is $n_0=\nu^{8/5}(2^3/3^{2/3})\pi^{-7/12}[\Gamma(1/4)]^{-2}$; n_1 is determined from the condition $\alpha(0)=0$}, then we have $\alpha(q)\geqslant 0$ for arbitrary q. We then also have $F(q)>0$ for all q. If $n\gg n_1$, then $\alpha(q)=1$. It is easy to see that there exists a maximum density n_{cr} for which Eq. (2) can have real solutions. At densities $n<n_{cr}$ there will be a transition accompanied by the formation of a charge density wave, while at $n>n_{cr}$ the homogeneous ground state of the electron liquid will be stable. In the present letter we are considering only densities $n>n_{cr}$, and we are interested primarily in densities $n_{cr}<n<n_1$. Let us analyze $F(q)$ under the condition $q\ll p_F$ and at densities close to n_1, for which the relation $|\alpha(0)|\ll 1$ holds. At these densities it follows from (1) that we have $\beta=[\nu\Pi_0(0)/p_F^2](4\pi\kappa_\infty/\kappa_0)\ll 1$. Under the condition $q\ll p_F$ we have $\Pi_0(q)=(-p_F/\pi^2)(1-q^2/8p_F^2)$. Substituting this expression for $\Pi_0(q)$ into (2), and expressing q in terms of the angle (θ) between $\mathbf{p}_1$ and $\mathbf{p}_2$ $\mathbf{q}^2=p_F^2\theta^2(\theta\ll 1)$, we find $F(\theta)=(p_F^2/\pi^2)\{[\theta^2-4|\alpha(0)|]^2+8\beta-16\alpha(0)\}$. At $r^2=8\beta-16\alpha^2(0)>0$ we have $F(\theta)>0$ for any θ, while at $r^2\leqslant 0$, there exist values of θ for which the relation $F(\theta)=0$ holds. This result means that $r^2=0$ is the equation which determines n_{cr}, and the conditions $n>n_{cr}$ and $r^2>0$ are equivalent. From the relation $F(\theta)=0$ at $r^2=0$ we can find the momentum which determines the period of the charge density wave: $|\mathbf{q}_0|$. It turns out to be $q_0=2p_F\sqrt{|\alpha(0)|}$. From the equation for n_{cr} and condition (1) we see that the relation $n_1-n_{cr}\ll n_1$ holds. Assuming that the densities n are close to n_{cr}, we consider the case $r^2\ll\beta$. For angles θ close to $\theta_0=2\sqrt{|\alpha(0)|}$ we thus have $F(\delta\theta)=\frac{1}{2}\theta_0^2p_F^2[(\delta\theta)^2+(r^2/4\theta_0^2]^{-1}$, where $\delta\theta=\theta-\theta_0$, $\delta\theta\ll\theta_0$, and the vertex $\Gamma_0(\theta)$ is given by

$$\Gamma_0(\delta\theta) = \frac{2}{p_F^2\theta_0^2}\,\frac{(4\pi\kappa_\infty/\kappa_0)-|\gamma|p_F^2\theta_0^2}{(\delta\theta)^2+(r^2/4\theta_0^2)}. \tag{3}$$

Substituting θ_0 into (3), and using inequality (1) and the inequality $|\alpha(0)|\ll 1$, we easily see that the relation $\gamma p_F^2\theta_0^2\gg(4\pi\kappa_\infty/\kappa_0)$ holds. This result means that the effective interaction $\Gamma_0(\theta)$ is attractive. Let us calculate the harmonic of the effective interaction with an orbital angular momentum $l=0$: $\Gamma_0^{(l=0)}=\int_0^\pi\Gamma_0(\theta)\sin\theta d\theta$. This integral is dominated by angles $\theta\sim\theta_0$. Substituting (3) into it, we find $\Gamma_0^{(l=0)}=-|\gamma|(2\pi\theta_0^2/r)$. From this expression for the static effective interaction we see that this interaction increases greatly in the limit $r\to 0$, i.e., as $n\to n_{cr}$. At densities $n\gg n_1$

this weak interaction is described by $\Gamma_0^{(l=0)} = -c(|\gamma| p_F/\nu)$, where $c \sim 1$ is a number which arises as a result of an averaging over angles. The superconducting gap Δ at $n \gg n_1$ can be determined in the BCS theory: $\Delta \sim \sqrt{\kappa_\infty/\kappa_0}\omega_0 \exp\{-\nu/(c|\gamma| p_F^2)\}$. The question regarding the gap at n values close to n_{cr} require an additional analysis.

The transition to a superconducting state, the transition accompanied by the formation of a charge density wave at $n < n_{cr}$, and the growth of the effective interaction as $n \to n_{cr}$—all these properties are characteristic of a quasi-two-dimensional electron liquid with a distance $d \ll \alpha_B$ between planes (α_B is the first Bohr radius) and of an electron liquid in a semiconductor with highly anisotropic electron valleys (this assertion also includes the case $\nu \sim 1$).

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