Stopping of a fast particle in a medium

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Universal expressions are derived for the polarization energy loss of an ultrarelativistic charged particle in a homogeneous, isotropic, and otherwise arbitrary medium by means of the Leontovich dispersion relations.

The polarization of a medium causes the energy lost by a particle per unit path length to vary with the density of the medium in a nonlinear way (the Swann–Fermi density effect[1]). With increasing energy of the particle, the total loss increases logarithmically, and the bounded loss, for which the transfer of square 4-momentum to the medium does not exceed a given value q_0^2 in magnitude, tends toward the constant limit

$$W(q_0) = Q^2 \omega_0^2 \ln(q_0/\omega_0) . \tag{1}$$

Here $\omega_0 = (4\pi ne^2/m)^{1/2}$ is the plasma frequency of the electrons of the medium (e, m, and n are their charge, mass, and density), Q is the charge of the particle, and $\hbar = c = 1$.

These results were originally derived for a very simple model of a medium. Using the Kramers-Kronig dispersion relations, Landau[2] extended them to arbitrary nonmagnetic media without a spatial dispersion. As is shown below, even these limitations can be removed by using some more general Leontovich relations. It then becomes clear that these results are completely universal and that the quantity ω_0 is the effective mass of the quantum of transverse natural vibrations of the medium. An important point is that the incorporation of spatial dispersion, which makes the electrodynamics of continuous media capable of describing all the structural details of a medium, not only expands the range of applicability of the results but also dramatically simplifies their derivation. In particular, it is no longer necessary to distinguish between "short-range" and "long-range" losses.

1. A description of the energy loss of a fast particle requires a relativistic generalization of the Kramers-Kronig relations for the response function of the medium, R:

$$R(\omega, k) = \tilde{R} + \frac{2}{\pi}\int_0^\infty d\zeta\zeta \, \mathrm{Im}R(\zeta, k) / (\zeta^2 - \omega^2 - i\delta\omega) \tag{2}$$

(here and below, the tilde means the limit of the left side as $\omega \to \infty$). A generalization of this sort follows from the relativistic causality condition $R(t, x) = 0$ at $t < x$, which makes the function $R(\omega, |\mathbf{k} - \omega\mathbf{s}|)$ analytic in ω in the upper ω half-plane and which also leads to the Leontovich relations.[3,4] These relations convert into (2) when $s = 0$ (s is an arbitrary vector with $s \leqslant 1$).[1)] In the case $s = 1$, $\mathbf{ks} = 0$, these relations take the form[5]

$$R(\omega, \sqrt{k^2 + \omega^2}) = \tilde{R} + \frac{2}{\pi}\int_0^\infty d\zeta\zeta \, \mathrm{Im}R(\zeta, \sqrt{k^2 + \zeta^2}) / (\zeta^2 - \omega^2 - i\delta\omega) . \tag{3}$$

2. The limited loss of a particle, whose velocity is fixed and close to the velocity of light, is given by (see Ref. 4, for example)

$$W(q_0) = Q^2 \int_0^{q_0} dq qI; \quad I = \frac{2}{\pi}\int_0^\infty d\omega\omega \, \mathrm{Im}R(\omega, \sqrt{q^2 + \omega^2})/(q^2 + \omega^2) , \tag{4}$$

where $R = R_l + R_t$ is the sum of the longitudinal and transverse response functions

$$R_l = -1/\epsilon_l(\omega, k), \qquad R_t = (k^2 - \omega^2)/(k^2 - \omega^2 \epsilon_t(\omega, k)) , \tag{5}$$

ϵ_l and ϵ_t are the longitudinal and transverse dielectric constants of the medium, and $-q^2 = \omega^2 - k^2$ is the square of the 4-momentum transfer to the medium.

From relation (3), the expression $\epsilon_l(\omega, 0) = \epsilon_t(\omega, 0)$, and the asymptotic behavior as $\omega \to \infty$,

$$\epsilon_l(\omega,\sqrt{q^2+\omega^2}) \to 1 + \dots, \qquad \epsilon_t(\omega,\sqrt{q^2+\omega^2}) \to 1 - \omega_0^2/\omega^2 + \dots \tag{6}$$

we find

$$I = R(iq,0) - \tilde{R} = \omega_0^2/(q^2+\omega_0^2).$$

Also using (4), we find expression (1) directly. That expression is therefore applicable for a homogeneous and isotropic medium of a general type.

3. The derivation of (1) which we have just been through is based implicitly on the property that the quantity ω_0 determined by (6) does not depend on q. This property allows us to interpret ω_0 as an effective mass of the quantum of the transverse natural vibrations of the medium (in the high-frequency limit), whose spectrum $\omega = \sqrt{k^2+\omega_0^2}$ is determined by the pole of R_t in (5) with the help of (6). The proof that ω_0 is independent of q is based on the circumstance that the transverse dielectric constant ϵ_t, although not a response function, is nevertheless analytic in the upper ω half-plane and satisfies relation (2) (Ref. 6, for example). After substituting in $k = \sqrt{q^2+\omega^2}$, this relation, combined with (6), leads to the expression

$$\omega_0^2 = \lim_{k\to\infty} \frac{2}{\pi}\int_0^\infty d\omega\,\omega\,\mathrm{Im}\,\epsilon_t(\omega,k),$$

which is indeed independent of q. This expression can be written in the form $4\pi ne^2\langle 1/\epsilon\rangle$ (ϵ is the relativistic energy of the particles of the medium; the angle brackets mean the expectation value over its state). In the nonrelativistic limit this expression becomes the expression given at the beginning of this letter.

4. The arguments above hold if q_0 is not too large. As this quantity increases, recoil effects and other quantum-mechanical effects come into play. There are some special methods[7] for describing these effects. In those methods, the characteristics of the medium are incorporated in the Green's function of the photon (in the Coulomb gauge)[8]:

$$D_l = 1/(k^2\epsilon_l), \quad D_t = 1/(k^2 - \omega^2\epsilon_t).$$

The limit $q_0 \to \infty$ corresponds to the total energy loss, which is finite because of the "self-cutoff" of the corresponding integral over q at the value $q_{\max}$, because of the conservation law $\delta[\omega - E(\mathbf{p}) + E(\mathbf{p}-\mathbf{k})]$ [$E(\mathbf{p})$ is the energy of the particle, whose momentum is $\mathbf{p}$]. In deriving (4), on the other hand, we used a classical analog of this law, $\delta(\omega\text{-}\mathbf{kv})$ ($\mathbf{v}$ is a fixed velocity of the particle), which appears in the expressions for the charge and current densities of the particle. The quantity $q_{\max}$ increases with increasing energy of the particle, E, so that the leading (logarithmic) term in the total loss can be found (1) by simply replacing q_0 by $q_{\max}$:

$$W(E) = Q^2\omega_0^2 \ln(q_{max}/\omega_0). \tag{7}$$

In the limiting cases $E \gg \langle\epsilon\rangle$ (the total loss occurs in an elementary event) $E \ll \langle\epsilon\rangle$ (the maximum momentum transfer corresponds to backscattering) we have $q_{\max} \sim \sqrt{E}$ and $q_{\max} \sim E$, and the logarithm in (7) is replaced by $(1/2)\ln(E/\omega_0)$ and $\ln(E/\omega_0)$, respectively.

5. We note in conclusion that the imaginary parts of the response function are nonzero in absorption bands (which correspond to the successive excitation of "rigid-body" degrees of freedom, i.e., lattice and electronic, intranuclear, and intranucleon degrees of freedom), which are separated by wide transparency windows. The characteristics corresponding to certain values of ω and k (and for temperatures and pressures which are not too high) are thus insensitive to absorption bands which are further away along the energy scale. Correspondingly, the ∞ in the equations of this letter (including the equations that determine ω_0) should refer to that transparency window which holds q_0 and E in (1) and (7).

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[1)]The relativistic causality condition reflects the absence of signals with velocities greater than the velocity of light in a vacuum. It is possible that there exists a more stringent causality condition (and a broader range of analyticity of the response functions), because of the absence of signals of velocities greater than the velocity of light in the medium.

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