Formation of $dt\mu$ muonic molecules in three-body collisions at high temperatures

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New mechanisms for the formation of $dt\mu$ muonic molecules in a deuterium-tritium plasma are analyzed and the formation rates are calculated. At temperatures $T \sim 1$ eV and densities $N \sim 10^{22}$ cm^{-3} of the mixture the formation rate of the $dt\mu$ molecules is $\lambda_{dt\mu} \gtrsim 10^9$ s^{-1}.

1. The formation of $dt\mu$ muonic molecules is the key stage of the muonic catalysis.[1] In the resonance reactions that have thus far been studied[2,3]

$$t\mu + D_2 \rightarrow [(dt\mu)dee] \tag{1a}$$

and

$$t\mu + D_2 + D_2 \rightarrow [(dt\mu)dee] + D_2' \tag{1b}$$

the binding energy $|\epsilon_{11}|$ of a weakly bound state, $j = v = 1$, of a $dt\mu$ molecule that is formed is transferred to excite the rotational-vibrational states of the muonic-molecule complex $[(dt\mu)dee]$. It is obvious that reactions (1a) and (1b) can occur only in a molecular medium, i.e., at temperatures[4] $T \lesssim T_D \approx D/\ln[25/\varphi\sqrt{D(\text{eV})}]$, where $D = 4.55$ eV is the dissociation energy of the D_2 molecule, $\varphi = N/N_0$ is the relative density of the medium, and $N_0 = 4.25 \times 10^{22}$ cm^{-3}. For this reason, it was previously always assumed that at $T > T_D$ $dt\mu$ muonic molecules are formed at a very slow rate and that because of this circumstance, the muonic catalysis is inefficient at these temperatures. [The "in-flight" rate of the nuclear reaction $t\mu + d \rightarrow {}^4\text{He} + n + \mu^-$, i.e., one in which no $dt\mu$ muonic molecules are formed beforehand, is very slow (see Refs. 5–7).]

In the present letter we examine various mechanisms for the formation of $dt\mu$ molecules at $T > T_D$, i.e., in an atomic medium and in a plasma, in three-body collisions of the type

$$t\mu + D + X \rightarrow (dt\mu)e + X', \tag{2}$$

$$t\mu + D + X^+ \rightarrow (dt\mu)e + (X^+)', \tag{3a}$$

$$t\mu + D^+ + X \rightarrow (dt\mu)^+ + X', \tag{3b}$$

$$t\mu + D^+ + e \rightarrow (dt\mu)^+ + e', \tag{4a}$$

$$t\mu + D + e \rightarrow (dt\mu)e + e', \tag{4b}$$

where $X = (H,D,T)$ are the atoms of the hydrogen isotopes.

2. The rate of reaction (4a) is determined by the expression[3]

$$\lambda_{dt\mu} = N_d N_e \int d\epsilon f(\epsilon) dE f(E) V_{fi}^2 \frac{d^3k'}{(2\pi)^3} 2\pi\delta(E' - E - \omega), \tag{5}$$

where $N_d = N_0 \varphi C_d$ and $N_e = N_0 \varphi \alpha_i$ are the number of nuclei d and electrons e per cm^3; $\epsilon = p^2/2\mu_a$, $f(\epsilon)$ and $E = k^2/2M$, $f(E)$ are the kinetic energies and Maxwell's distribution functions for the relative motion of $t\mu + D^+$ and $(t\mu + D^+) + e$, respectively; $\omega = |\epsilon_{11}| + \epsilon$; $\mathbf{p}$, $\mathbf{k}$, and $\mathbf{k}'$ are the relative momenta of $t\mu + D^+$, e and $(t\mu + D^+)$, $(e)'$, and $(dt\mu)^+$, respectively; $E' = k'^2/2M$; $M^{-1} = m_e^{-1} + M_{dt\mu}^{+1}$; and $\mu_a^{-1} = M_{t\mu}^{-1} + M_d^{-1}$.

The matrix element $V_{fi}(\epsilon, E, E')$ of the transformation operator $V = \mathbf{dr}/r^3$, where $\mathbf{d}$ is the dipole moment of the $dt\mu$ molecule, and $\mathbf{r}$ is the coordinate of the electron e reckoned from the c.m. of the $t\mu + D^+$, is calculated from the wave functions of the initial state (ψ_i) and the final state (ψ_f) (see Refs. 7 and 8):

$$\psi_i = e^{i\mathbf{pR}} \psi_{\mathbf{k}}^{(+)}(\mathbf{r}),$$
$$\psi_f \approx \frac{C_a}{R^2\sqrt{\kappa_a}}(1 + \kappa_a R) e^{-\kappa_a R} Y_{JM_J}(\mathbf{R}/R) \psi_{\mathbf{k}'}^{(-)}(\mathbf{r}). \tag{6}$$

Here $\kappa_a = (2\mu_a|\epsilon_{11}|)^{1/2} \approx 10.3$ a.u., $C_a = 0.574$, R is the distance between $t\mu$ and d, $\psi_{\mathbf{k}}^{(+)}(\mathbf{r})$ and $\psi_{\mathbf{k}'}^{(-)}(\mathbf{r})$ are the Coulomb functions which describe the relative motion of e and $(t\mu + D^+)$ and e' and $(dt\mu)^+$, respectively. [Equations (6) and the expression for V are valid in the region $r \gg R$ which accounts for the largest contribution to V_{fi}.] Using the results of Refs. 7–9 and the contribution of channel (4b), which is equal in order of magnitude to the contribution of channel (4a), we find ($\beta_a = M_{t\mu}/M_{dt\mu}$):

$$\lambda_{dt\mu} = \frac{2^7\pi^2 N_0^2 \beta_a^2 C_a^2}{\sqrt{3}\,\kappa_a^6} \sqrt{\mu_a M}\, G(\gamma) \varphi^2 \alpha_i C_d = 0{,}6 \cdot 10^{10} \varphi^2 \alpha_i C_d G(\gamma)\,\mathrm{s}^{-1}, \tag{7}$$

where α_i is the degree of ionization of the D atoms, and the function

$$G(\gamma) = \gamma^2 \int_0^\infty dx \sqrt{x}\, e^{-\gamma x} \frac{3x^2 + 2x + 3}{(x+1)^4}; \tag{8}$$

$$\gamma = |\epsilon_{11}|/T, \qquad x = \epsilon/|\epsilon_{11}|$$

is illustrated in Fig. 1.

3. In reactions (2) and (3) the attraction of the pairs (D, X), (D^+, X), and (D, X^+) is important. As a result of this attraction, the principal contribution to V_{fi} comes from the region of small r. For this reason, the effect of electron screening on the transformation operator $\hat{V}$ in this case can be ignored. For the same reason, perturbation theory cannot be used in this case, and the calculation of the reaction rates should be based on the principle of detailed balancing which has the following form[4] for reaction (3a) ($\hbar = e = 1$):

$$\frac{\lambda_{dt\mu}}{\lambda_{\text{decay}}} = 3N_D \left(\frac{2\pi}{MT}\right)^{3/2} \exp\{|\epsilon_{11}|/T\}, \tag{9}$$

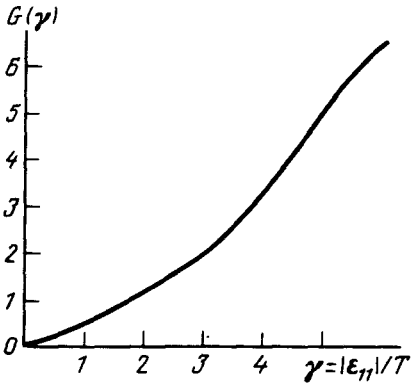

FIG. 1.

where $N_D = N_0 \varphi C_d (1-\alpha_i)$, $\lambda_{\text{decay}} \approx \lambda_{\text{capt}} W_{\text{decay}}$ is the rate of the reverse reaction, and $\lambda_{\text{capt}} \approx 2\pi(\alpha/M)^{1/2} N_0 \varphi \alpha_i$ is the rate of the capture $(dt\mu)ee + X^+ \to (dt\mu)eX^+$ in the Thomson model with a polarization interaction: $\alpha/2r^4$, where $\alpha = 4.5$ a.u. It can be shown that $W_{\text{decay}} \approx 1$; i.e., each $(dt\mu)e + X^+$ collision leads to a decay $(dt\mu)e \to t\mu + D$. Since the probabilities for the processes (3a) and (3b) are approximately the same, we find the following expression for the total rate of these processes:

$$\lambda_{dt\mu} = 2\times 6\pi \left(\frac{\alpha}{M}\right)^{1/2} \left(\frac{2\pi}{MT}\right)^{3/2} N_0^2 \varphi^2 C_d \alpha_i (1-\alpha_i)$$
$$\approx 0{,}6 \cdot 10^{11}\, T^{-3/2}\,(\text{eV})\, \varphi^2 C_d \alpha_i\, (1-\alpha_i)\, \text{s}^{-1}. \qquad (10)$$

In a corresponding way, we obtain the following expression for the rate of process (2) ($C_6 = 6.5$ a.u. is the van der Waals attraction between D and X, and α_D is the degree of dissociation of the D_2 molecules):

$$\lambda_{dt\mu} \approx 3\pi(3/2)^{2/3} (3C_6/T)^{1/3} (2T/M)^{1/2} (2\pi/MT)^{3/2} N_0^2 \varphi^2 \alpha_D^2 C_d$$
$$\approx 1.6\times 10^{10}\, T^{-3/2}\,(\text{eV})\, \varphi^2 \alpha_D^2 C_d\ \text{s}^{-1}. \qquad (11)$$

It thus follows that at $\varphi \gtrsim 0.2$ and $T \sim 1$ eV the rate of formation of $dt\mu$ molecules in the atomic medium and in a plasma is $\lambda_{dt\mu} \gtrsim 10^9\ \text{s}^{-1}$. Since these conditions can easily be achieved in pulsed discharges, muon catalysis in strong electric fields should be studied in greater detail, especially since the sticking coefficient ω_s can be reduced in this case, as was suggested in Ref. 10.

[1]Muon Catalyzed Fusion, Vol. 1, 1987; Proc. Intern. Conf., CF-87, Tokyo, 1–3 September 1986.
[2]S. I. Vinitskiĭ, L. I. Ponomarev, I. V. Puzynin *et al.*, Zh. Eksp. Teor. Fiz. **74**, 849 (1978) [Sov. Phys. JETP **47**, 444 (1978)].
[3]L. I. Men'shikov and L. I. Ponomarev, Phys. Lett. **167B**, 141 (1986).
[4]L. D. Landau and E. M. Lifshitz, Statisticheskaya Fizika (Statistical Physics), Nauka, Moscow, 1976 [Pergamon Press, Oxford (1980), 2 Vols., 3rd ed.].
[5]W. P. S. Tan, Nature **263**, 656 (1976); B. O. Hinks, M. K. Sundaresan, and P. J. S. Watson, Nature **269**, 584 (1977).
[6]L. N. Bogdanova *et al.*, Yad. Fiz. **34**, 1191 (1981) [Sov. J. Nucl. Phys. **34**, 662 (1981)].

[7]L. I. Men'shikov, Yad. Fiz. **42**, 1184 (1985) [Sov. J. Nucl. Phys. **42**, 750 (1985)].
[8]L. I. Men'shikov, Yad. Fiz. **42**, 1449 (1985) [Sov. J. Nucl. Phys. **42**, 918 (1985)].
[9]H. A. Bethe, Intermediate Quantum Mechanics, Benjamin, New York, 1964.
[10]L. Bracci and G. Firoentini, Nature **297**, 134 (1982).

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