Regge trajectories and the shape of hadrons

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Arguments that hadrons have an elongated shape are presented.

Regge trajectories, i.e., curves showing the dependence of the angular momentum on the square mass of a particle, have two surprising features: they are (quite accurately) straight lines, and for baryon curves their slope is almost the same as that of meson trajectories.

We know that a linear trajectory can be interpreted in the following way. In the case of a meson, a gluon string is stretched between a quark and an antiquark. The energy of the quarks determines only the "end effect," which is independent of the string length l. The part of the mass which depends on l and on the angular momentum is determined by the energy of the gluon field. For large angular momenta the following relations are valid:

$$M = 2\int_0^{l/2} \frac{\sigma(x)\,dx}{\sqrt{1-\omega^2x^2}}\ ; \qquad I = 2\int_0^{l/2} \frac{\sigma(x)\,\omega x^2 dx}{\sqrt{1-\omega^2x^2}}\ , \tag{1}$$

where $\sigma(x)$ is the tension in the string, which may depend at the ends on the distance (x) from the center of the string, and ω is the angular velocity. The length of the string is found from the condition for a minimum of M at a given I. Ignoring the x dependence of σ, we easily find that the minimum value of M^2/I is reached at $\omega l/2 = 1$. In this case we have

$$M^2 = 2\pi\sigma I = (\pi\sigma l/2)^2; \qquad I = \pi\sigma l^2/8\ . \tag{2}$$

The observed value $\alpha' = dI/dM^2$ yields $\sigma = (400\text{ MeV})^2$.

The linearity of the meson trajectories can be explained in this way, but why is the slope of meson trajectories the same as that of baryon trajectories?

Apparently the only way to explain the parallel course of the baryon and meson trajectories, while preserving the idea of a gluon string, is to suggest that upon the formation of a string in a baryon, two quarks lie predominantly at one end of the string, while the third lies at the other end. Since two quarks, as sources of a gluon field with $N_c = 3$, are equivalent to a single antiquark, a baryon string is the same, within end effects, as a meson string, so the trajectories would have the same slope.

It follows from an analysis of the spectra of heavy mesons that the tension in a string between heavy quarks is approximately that which has been found from Regge trajectories. The tension in a string is thus independent of the mass and quantum numbers of the quarks.

There is yet another puzzle. Why does this constant slope of the trajectories persist down to the very smallest values of the angular momentum?

The quantization of an ideal string leads to a functional dependence $M^2 = 2\pi\sigma I$ down to $I = 1$, while for an unbending rod I^2 would be replaced by $I(I+1)$. This result can be interpreted as an energy increase which occurs because a transverse stiffness comes into play. It may be that this difficulty can be removed by the circumstance that the transverse stiffness of a gluon string connecting quarks is small: The energy of a curved string is apparently proportional not to the square but to the fourth power of the ratio of the transverse dimension to the radius of curvature.

There is yet another, and possibly more convincing, argument in favor of elongated hadrons. The region in which the gluon field and thus the quarks are enclosed has dimensions on the order of the confinement radius R_c (the length scale over which the Coulomb interaction between quarks gives way to a σR law), which is $R_c \cong (600\text{–}700\text{ MeV})^{-1} \simeq 0.3$ F, according to numerical simulations. This value is considerably smaller than the electromagnetic "radius" of a nucleon, which is determined by the pion cloud ($R_E \cong 0.8$ F). The kinetic energy of the quarks in this volume makes a large contribution to the mass: $2.04\, N_c/R_c \cong 4$ GeV for baryons and $2.04\cdot 2/R_c = 2.7$ GeV for mesons (for the MIT boundary conditions). The masses should decrease to the observed value after the interaction of valence quarks with the quark condensate is taken into account, i.e., as a result of the excitation of a pion field. A baryon consisting of three quarks interacts directly with a pion field, since it is possible to construct a pseudoscalar generating a pion field ($\sim\sigma\nabla\pi$). A spherical meson, however, could generate a pion field only through fluctuations of the quark density. Its mass would have to be approximately equal to the value given above. If a meson is stretched, on the other hand, these fluctuations grow, and the quark and the antiquark generate a "dipole" pion field. The interaction of the meson with the quark condensate intensifies, and it may lower the mass to the observed value.

Even without an interaction with a pion field, however, it would seem that an elongated shape of a hadron would be advantageous. Specifically, an analysis of the appearance of a gluon string[1,2] leads to the following conclusion: A necessary condition for the existence of a string is rapid growth of the gluon energy with increasing minimum length scale ρ of the gluon field (e.g., the transverse dimension of a string) near $\rho \simeq R_c$. The zero-point vibrations of the shape of the hadron thus occur asymmetrically. The amplitude of the spherical vibrations is far lower than that of the vibrations accompanied by the formation of a string. In the former case the minimum length scale increases in comparison with R_c, while in the latter case the transverse dimension remains equal to R_c, and only the length of the string increases. The asymmetry of the vibrations leads to an effective elongation. Furthermore, one might expect that even in the equilibrium state an elongated shape would be preferable.

The "string" interpretation of Regge trajectories does indeed lead to the conclusion that a spherical shape of a hadron would be unstable: A minimal rotation would stretch the hadron [at a point as early as $I = 1$, $l = 2.6\,R_c$, according to (2)]. We can clarify the situation with a simple calculation, which is simply a guideline, and by no means a proof.

Let us assume that a hadron is an ellipsoid with semiaxes a and b. The mass of a baryon can then be written

$$M = N_c \frac{1.44}{a}\left\{1 + \frac{0.72}{\xi} + \frac{0.3}{\xi^2}\right\}^{1/2} + \sigma\xi a\,, \tag{3}$$

where $\xi = b/a$. The first term is the kinetic energy of the quarks (see Ref. 3 and the bibliography there), while the second is the energy of the gluon field. The mass of a meson is found by replacing N_c by 2. We assume that a minimization of the energy in terms of the transverse dimension corresponds to the value $a = R_c$. The shape of the hadron is then determined by minimizing expression (3) with respect to the major semiaxis.

Substituting the values $\sigma = (400\ \text{MeV})^2$ and $R_c = (600\ \text{MeV})^{-1}$, we find $\xi = 2.2$ for a baryon and $\xi = 1.8$ for a meson. Until a theory of the hadron is derived, it will be difficult to determine how the elongation will change when the pion field is incorporated in a systematic way.

This estimate has been based on the representation of a hadron as a bag in which a quark is moving. In the case of massless quarks, their motion is determined by the motion of the gluon field which they produce. A quark marks the point at which a string ends, and an uncertainty in its coordinate corresponds to quantum fluctuations of the string. The mass of a hadron is thus determined by the energy of the zero-point vibrations of a string-like gluon field, and it does not require adding the kinetic energy of the quarks in this volume.

The elongation of a nucleon of course does not give rise to an electric quadrupole moment, since the quadrupole-moment operator constructed from Pauli matrices vanishes identically. In the singlet state of the deuteron, the orientation of the elongation of the neutron and the proton is arbitrary, but in the triplet state a correlation between these directions should arise. This correlation may lead to a difference between the interaction forces in the triplet and singlet states. The elongation of hadrons should be manifested in experiments on the scattering of nucleons by nucleons and in meson-nucleon scattering.

In our opinion, the arguments presented above are convincing enough to warrant an experimental search for manifestations of an elongation of hadrons, without waiting for the derivation of a systematic theory of hadrons (whose completion by no means lies on the horizon).

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