Rectification of the gradient force of resonant radiation pressure

A. P. Kazantsev and I. V. Krasnov
L. D. Landau Institute of Theoretical Physics, Academy of Sciences of the USSR

(Submitted 9 July 1987)
Pis'ma Zh. Eksp. Teor. Fiz. **46**, No. 7, 264–267 (10 October 1987)

A resonant radiation-pressure force of a new type is found. This force can produce ultradeep potential wells or a rotational force field which gives rise to a rotation of particles.

The mechanical effects of laser light on the translational motion of atoms may occur in a variety of regimes. In weak fields, the force of the spontaneous radiation pressure acts, and atoms in a free (unbound) state execute a Brownian motion. The significant cooling effect may cause the localization time of the particles in the field region to be long.[1–3]

In a strong nonuniform field, the gradient force caused by stimulated transitions is dominant.[4,5] In a monochromatic field $E(\mathbf{r})e^{-i\Delta t}$ (Δ is the deviation from resonance, and d is the transition dipole moment) the potential of the gradient force for slow atoms is[6]

$$U(\mathbf{r}) = \frac{\hbar\Delta}{2}\ln\left(1 + \frac{2|V(\mathbf{r})|^2}{\Delta^2 + \gamma^2/4}\right), \qquad V(\mathbf{r}) = dE(\mathbf{r})/\hbar . \tag{1}$$

In the field of the standing wave $V(x) = V\cos kx$ the potential oscillates over space with a period of π/k, and the depth of the potential wells is on the order of $\hbar V$ at $V \sim \Delta \gg \gamma$. Because of the hysteresis set up by the spontaneous relaxation, an atom also

 0021-3640/87/190332-05$01.00

experiences a velocity-dependent friction force.[6–8] Depending on the parameters of the field, the atoms may move at a fixed velocity (bunching in velocity space) or be trapped in potential wells of the standing wave.[4] The bunching effect and a localization (channeling) of atoms in a standing wave have recently been observed experimentally.[11,12]

The oscillatory nature of the gradient force, however, imposes certain restrictions on its utilization. In the present letter we show that in a bichromatic nonuniform field a force on the order of $\hbar kV$ arises; this force changes sign over distances considerably greater than the wavelength of the light. The spatial structure of this force may be either irrotational or rotational. There is accordingly the possibility that atoms will localize in deep potential wells or go into a revolution.

An atom moving at a velocity $\mathbf{v}$ experiences a force[4,5]

$$\mathbf{F} = \hbar p \nabla V^*(\mathbf{r}t) + \text{c. c.}, \qquad \mathbf{r} = \mathbf{v}\, t\,.$$

The dipole moment p (in units of d) and the difference between the level populations, q, are found from the Bloch equations (γ is the decay rate of the upper level)

$$\frac{dp}{dt} + \frac{\gamma}{2} p = - iVq\,; \qquad \frac{dq}{dt} + \gamma(q+1) = 2ipV^* + \text{c. c.} \tag{2}$$

The field takes the form of a superposition of monochromatic fields

$$V(\mathbf{r}t) = (V_0(\mathbf{r}) + V_1(\mathbf{r})e^{-i\Delta_1 t})e^{-i\Delta_0 t}$$

which satisfy the Rabi resonance condition[9]

$$\Delta_1 \gg V_1 \gg V_0\,; \qquad |V_1|^2/\Delta_1 \sim V_0 \sim \Delta_0\,. \tag{3}$$

In this case a Fourier analysis of the Bloch equations becomes a very simple matter: It is sufficient to retain only the terms on the order of $|V_1|^2/\Delta_1$ in (2). After an averaging over the fast oscillations with a frequency Δ_1, we find the following equations for the slow components p, q, and $\mathbf{F}$, for which we retain the earlier notation:

$$\frac{dp}{dt} + \left(\frac{\gamma}{2} - i\Delta(\mathbf{r})\right)p = - iV_0(\mathbf{r})q\,; \qquad \Delta(\mathbf{r}) = \Delta_0 + \frac{2|V_1(\mathbf{r})|^2}{\Delta_1}$$

$$\frac{dq}{dt} + \gamma(q+1) = 2ip^*V_0(\mathbf{r}) + \text{c. c.} \tag{4}$$

$$\mathbf{F} = (\hbar p^* \nabla V_0(\mathbf{r}) + \text{к. c.}) + \frac{1}{2} q \hbar \nabla\Delta(\mathbf{r})\,, \qquad \mathbf{r} = \mathbf{v}\, t\,.$$

The high-frequency field $V_1(\mathbf{r})$ gives rise to an effective deviation from resonance which is spatially nonuniform: $\Delta(\mathbf{r})$. For strong fields and large deviations from resonance, V_0, $\Delta \gg \gamma$, kv, we can use the procedure of eliminating p (Ref. 6), and we can put (4) in the form

$$dQ/dt = -\frac{1}{2}\gamma(1+\mu^{-2})Q - \gamma\mu^{-1}; \quad Q = q\mu, \quad \mu = \sqrt{1+\left|\frac{2V_0(\mathbf{r})}{\Delta(\mathbf{r})}\right|^2}$$

$$\mathbf{F} = \frac{1}{2}\,\mathrm{sign}\,\Delta Q \nabla\epsilon(\mathbf{r}); \qquad \epsilon(\mathbf{r}) = \sqrt{\Delta^2(\mathbf{r}) + 4|V_0(\mathbf{r})|^2}\,. \tag{5}$$

The force is proportional to the difference between the populations of the quasienergy states, Q, and the gradient of the quasienergy $\frac{1}{2}\epsilon(\mathbf{r}) = \frac{1}{2}|\Delta(\mathbf{r})|\mu(\mathbf{r})$. For slow atoms in the quasisteady approximation we have $Q = -(2\mu/1+\mu^2)$. In the case of a monochromatic field ($V_1 = 0$, $\Delta = \Delta_0$), Q is a local function of $\epsilon(r)$, and we have $\langle F\rangle = 0$, where the angle brackets mean an average over the field period π/k. In the general case of a bichromatic field, Q is not a local function of $\epsilon(\mathbf{r})$, so that it cannot be written as the gradient of a periodic function. There is accordingly a rectification of the force $\mathbf{F}$: the appearance of a constant-sign component of $\langle\mathbf{F}\rangle$ over large length scales l ($kl \gg 1$).

For low velocities ($kv \ll \gamma$) and a slight saturation [$(|V_0|^2/\Delta_0^2) \ll 1$, $(|V_1|^2/\Delta_1\Delta_0) \ll 1$] we have $\mathbf{F} = \mathbf{F}_0 + \mathbf{F}_1$, where

$$\mathbf{F}_0 = -\hbar\nabla\left(\frac{|V_0(\mathbf{r})|^2}{\Delta_0} + \frac{|V_1(\mathbf{r})|^2}{\Delta_1}\right) + 2\hbar\left|\frac{V_0(\mathbf{r})}{\Delta_0}\right|^4 \nabla\frac{|V_1(\mathbf{r})|^2}{\Delta_1}$$

$$\mathbf{F}_1 = -\frac{4\hbar}{\gamma}\left|\frac{V_0(\mathbf{r})}{\Delta_0}\right|^2 (\mathbf{v}\nabla)\left|\frac{V_0(\mathbf{r})}{\Delta_0}\right|^2 \nabla\left(\frac{|V_0(\mathbf{r})|^2}{\Delta_0} + \frac{|V_1(\mathbf{r})|^2}{\Delta_1}\right). \tag{6}$$

We have ignored the force of the spontaneous radiation pressure, bearing in mind the case in which the fields are moderately weak[4]: $|V_0|^2/\Delta_0 \gg \gamma$.

If $V_1 = 0$, the force $\mathbf{F}_0$ is the gradient of a potential[10] $|V_0(\mathbf{r})|^2/\Delta_0$, while $\mathbf{F}_1$ is a friction force.[6] At finite values of the field V_1, the last term in the expression for $\mathbf{F}_0$ in (6) gives rise to a rectification effect. Specifically, in the one-dimensional case, in the field of two standing waves, $V_0(x) = V_0\cos kx$, $V_1(x) = V_1\cos[(k+\delta k)x + \varphi]$, $\delta k \ll k$, we find

$$\langle F_{0x}\rangle = -\frac{dU}{dx}, \quad U(x) = -\frac{\hbar k}{4\delta k}\left|\frac{V_0}{\Delta_0}\right|^4 \frac{|V_1|^2}{\Delta_1}\cos(2\delta kx + 2\varphi)\,. \tag{7}$$

Average force (7) is $\hbar kV_0$ in order of magnitude, and it oscillates with a period $l = \pi/\delta k$, which is several orders of magnitude greater than the wavelength of the light. Correspondingly, the depth of the potential wells of the "rectified" gradient force, $U \sim \hbar kV/\delta k$, becomes larger than the ordinary resonant potential $\hbar V$ by a factor on the order of the parameter $k/\delta k \gg 1$. For Na atoms in a field with an intensity of 1 W/cm^2, with $\delta k/k \sim 10^{-3}$–10^{-4}, for example, we would have $U \sim 2$–20 K. The rectification effect is illustrated schematically in Fig. 1. Figure 1a shows the potential of the ordinary gradient force, with an amplitude $\hbar V$ and a period π/k. Figure 1b shows the average potential U (the solid line) and the unaveraged potential (dashed line), which contains small-scale oscillations with an amplitude $\hbar V$ and a period π/k.

To examine the rectification effect in the two-dimensional case, we use the follow-

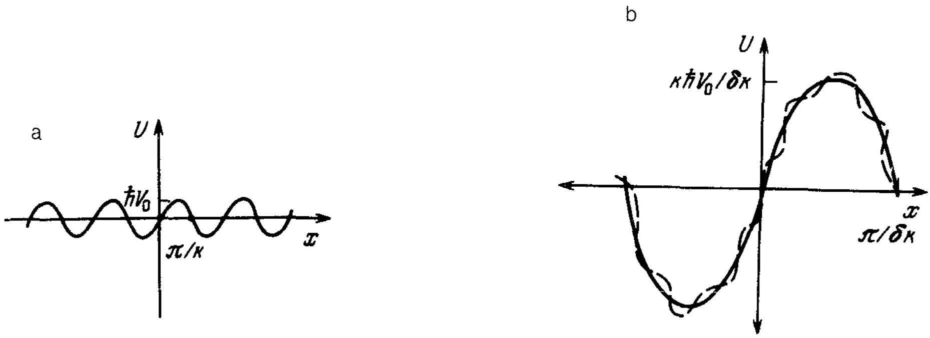

FIG. 1.

ing superposition of fields:

$$V_0(\mathbf{r}) = V_0[\exp(i\mathbf{k}_0\mathbf{r}) + a\exp(i\mathbf{k}_1\mathbf{r}) + a\exp(i\mathbf{k}_2\mathbf{r})]$$

$$V_1(\mathbf{r}) = V_1[\exp(i\mathbf{q}_0\mathbf{r}) + b\exp(i\mathbf{q}_1\mathbf{r} + i\varphi_1) + b\exp(i\mathbf{q}_2\mathbf{r} + i\varphi_2)],$$

where $|\mathbf{k}_i| = k$, $|\mathbf{q}_i| = k + \delta k$, $\delta k \ll k$, $i = 0,1,2$.

We direct the vectors $\mathbf{k}_i$ along the bisectors of the first, second, and fourth quadrants of a Cartesian coordinate system with unit vectors $\mathbf{e}_x$ and $\mathbf{e}_y$. In other words, we set $\mathbf{k}_0 = (1/\sqrt{2})(\mathbf{e}_x + \mathbf{e}_y)k$, $\mathbf{k}_1 = (1/\sqrt{2})(\mathbf{e}_x - \mathbf{e}_y)k$, $\mathbf{k}_2 = -\mathbf{k}_1$. The vectors $\mathbf{q}_i = \mathbf{k}_i + \delta\mathbf{k}_i$ differ only slightly from $\mathbf{k}_i$ ($|\delta\mathbf{k}_i| \ll k$). For simplicity, we assume that the coefficients a and b are small. The average force then becomes

$$\langle \mathbf{F}_0 \rangle = f\{\sin[(\delta\mathbf{k}_2 - \delta\mathbf{k}_0)\mathbf{r} + \varphi_2]\mathbf{e}_x + \sin[(\delta\mathbf{k}_1 - \delta\mathbf{k}_0)\mathbf{r} + \varphi_1]\mathbf{e}_y\}$$
$$f = 8\sqrt{2}ab\hbar k\left|\frac{V_0}{\Delta_0}\right|^4\frac{|V_1|^2}{\Delta_1}. \tag{8}$$

By varying the direction of the vectors $\mathbf{q}_i$ with respect to $\mathbf{k}_i$ slightly, we can control the vector deviations $\delta\mathbf{k}_i$. There are two qualitatively different situations here.

1. In the case $\delta\mathbf{k}_2 - \delta\mathbf{k}_0 = \delta q\mathbf{e}_x$ and $\delta\mathbf{k}_1 - \delta\mathbf{k}_0 = \delta q\mathbf{e}_y$, an irrotational force field arises with a potential

$$U(r) = \frac{f}{\delta q}[\cos(\delta qx + \varphi_2) + \cos(\delta qy + \varphi_1)]. \tag{9}$$

The depth of the potential wells in (9) is on the order of $f/\delta q \gg \hbar V_0$, and the period is $2\pi/\delta q$.

2. In the case $\delta\mathbf{k}_1 - \delta\mathbf{k}_0 = \delta\kappa\mathbf{e}_x$ and $\delta\mathbf{k}_2 - \delta\mathbf{k}_0 = -\delta\kappa\mathbf{e}_y$ we are dealing with a rotational force field ($\mathrm{div}\langle\mathbf{F}_0\rangle = 0$) with a vector potential $\mathbf{A} \equiv (0,0,A)$, $\langle\mathbf{F}_0\rangle = \mathrm{curl}\,\mathbf{A}$:

$$A = \frac{f}{\delta\kappa}[\cos(\delta\kappa x - \varphi_1) + \cos(\delta xy + \varphi_2)]. \tag{10}$$

A force of this sort can cause the particles to revolve at some characteristic frequency which can be found from the linearized equation of motion of the particle near the nodes of the force $\langle \mathbf{F}_0 \rangle$:

$$m\ddot{\mathbf{r}} = -f\frac{k\mathbf{v}}{\gamma} + f\delta\kappa\,[\mathbf{e}_z\mathbf{r}]\,. \tag{11}$$

Here we have assumed $V_0^2/\Delta_0 < V_1^2/\Delta_1$. In a moderately weak field, $f \gg f_c = \delta\kappa m(\gamma/k)^2$, a rotation of the type $x + iy \sim \exp(i\Omega t)$ arises, where $\Omega = \Omega_0(1 - if_c/f)$, and $\Omega_0 = \delta\kappa\gamma/k$. The particle rotates with a frequency Ω_0, and the rotation radius increases slowly if $f > 0$.

A radiation pressure force of a new type thus arises in a nonuniform bichromatic field. This force might be utilized in particular to scatter particles in atomic beams and to spatially localize atoms. The lifetime of atoms trapped in the superdeep potential wells would be exponentially long, $\sim \exp[(k/\delta k)\mathrm{const}]$; i.e., this time would be essentially unbounded.

[1]A. L. Migdal, J. V. Prodan, W. D. Phillips, T. H. Bergman, and H. J. Metcalf, Phys. Rev. Lett. **54**, 2596 (1985).

[2]S. Chu, J. E. Bjorkholm, A. Ashkin, and A. Cable, Phys. Rev. Lett. **57**, 314 (1986).

[3]D. E. Pritchard, E. L. Raab, V. Bagnato, C. E. Wieman, and R. N. Watts, Phys. Rev. Lett. **57**, 310 (1986).

[4]A. P. Kazantsev, G. A. Ryabenko, G. I. Surdutovich, and V. P. Yakovlev, Phys. Rep. **129**, 1606 (1985).

[5]S. Stenholm, Rev. Mod. Phys. **58**, 699 (1986).

[6]A. P. Kazantsev, Zh. Eksp. Teor. Fiz. **66**, 1599 (1974) [Sov. Phys. JETP **39**, 784 (1974)].

[7]J. P. Gordon and A. Ashkin, Phys. Rev. A **21**, 1606 (1980).

[8]J. Dalibard and C. Cohen-Tannoudji, JOSA **2**, 1707 (1985).

[9]I. V. Krasnov, Zh. Eksp. Teor. Fiz. **89**, 420 (1985) [Sov. Phys. JETP **62**, 238 (1985)].

[10]G. A. Askar'yan, Zh. Eksp. Teor. Fiz. **42**, 1567 (1962) [Sov. Phys. JETP **15**, 1088 (1962)].

[11]A. Aspect, J. Dalibard, A. Heidmann, C. Salomon, and C. Cohen-Tannoudji, Phys. Rev. Lett. **57**, 1686 (1986).

[12]C. Salomon, J. Dalibard, A. Aspect, H. Metcalf, and C. Cohen-Tannoudji, Phys. Rev. Lett. (to be published).

Translated by Dave Parsons