Nonlocal transport of thermal perturbations in a plasma

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An integral equation describing a "wave" transport of heat in a plasma by means of plasma waves is derived. The propagation of thermal perturbations from an instantaneous point source can be more rapid than in the case of diffusion. An example of heat transfer across a magnetic field is examined.

In this letter we demonstrate that the temperatures at different points in a plasma may be related in a nonlocal way. Such a relationship stems from the emission and absorption by particles of plasma waves which have long ranges and which lead to a correlation of the temperature over distances far greater than in the case of ordinary processes of the diffusion type. In this regard the problem is analogous to problems of the radiative transport of an excitation.[1] Since there are some special (natural) wave frequencies in a plasma, this "wave" thermal conductivity is more reminiscent of radiation transport in discrete lines than in a Planckian continuum, as in the case of radiative heat transfer.[2] We should point out that a nonlocal relationship between perturbations can also occur in the case of ordinary collisional transport, by virtue of the long mean free paths of the fast particles[1)] (tail particles).[3] We will ignore such process here, however, as we are justified in doing in the case of (for example) heat transfer across a strong magnetic field.

We begin with equations for the spectral intensity of the plasma waves, I, and the balance equation for the energy carried by these waves:

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$$(\mathbf{s}, \frac{\partial}{\partial \mathbf{r}})\,(I/n^2) = j/n^2 - \alpha I/n^2\,, \qquad (1)$$

$$\frac{\partial q}{\partial t} + \operatorname{div}_{\mathbf{r}}\{\int d\omega\, d\Omega_{\mathbf{s}}\, \mathbf{s}\, I(\omega, \mathbf{s}, \mathbf{r})\} = Q(\mathbf{r}, t), \qquad (2)$$

where $\mathbf{s}$ is the wave propagation direction, q is the electron energy density, Q is an external heat source, $n(\omega,\mathbf{s})$ is the refractive index for the waves, and j and α are the emission and absorption coefficients, which are related to the emissivity η of plasma waves by[4]

$$j = \int \eta(\omega, \mathbf{s}, \mathbf{p}) f(\mathbf{p})\, d\mathbf{p}, \qquad \alpha = -\frac{8\pi^3 c^2}{n^2\omega^2}\int \frac{\partial f}{\partial \epsilon}\eta(\omega, \mathbf{s}, \mathbf{p})\, d\mathbf{p}\ . \qquad (3)$$

Here $f(\mathbf{p})$ is the distribution function of the plasma electrons; $\epsilon = p^2/2m_e$; and in the equation for α it is assumed that f is directionally isotropic.

We are ignoring temporal retardation in Eq. (1) (the retardation due to the finite magnitude of the wave group velocity V_{gr}). This is a legitimate simplification if $V_{gr} \gg V_{fr}$, where V_{fr} is the front velocity of the thermal wave described by Eqs. (1) and (2).

To some extent, system (1) is analogous to the system of transport equations in weak-turbulence theory.[5] There is the distinction that the heat transport over space is caused by plasma waves, rather than by fast particles. A situation of this sort occurs when transport by particles is hindered (e.g., for a transport across a strong magnetic field).

Equations (1) and (2) establish the relationship between generally different functionals j and α in (3). To close system of equations (1) and (2), we thus need to find some additional relations between j and α. We can do this by solving the system of quasilinear equations for the function f (Refs. 5 and 6). If the perturbations are small, however, we can ignore the deviation of the electron distribution function from a locally equilibrium (Maxwellian) function. The functionals j and α are then related by Kirchhoff's law[4]:

$$j(\omega, \mathbf{s}) / \alpha(\omega, \mathbf{s}) = n^2\omega^2 T(\mathbf{r}) / 8\pi^3 c^2, \qquad (4)$$

where $T(\mathbf{r})$ is the electron temperature. Using (4) and substituting formal solution (1) into (2), we find a nonlinear integrodifferential equation for the temperature. Linearizing it in the small parameter $Y \equiv \delta T(\mathbf{r})/T_0$, where δT is the deviation of the temperature from its uniform equilibrium value T_0, we find an equation analogous to the Biberman-Holstein transport equation for resonant radiation[1]:

$$\frac{\partial Y}{\partial t} = -\frac{Y}{\tau} + \frac{1}{\tau}\int Y(\mathbf{r}')\, G(|\mathbf{r} - \mathbf{r}'|, \mathbf{s})\, d\mathbf{r}' + \widetilde{Q}, \quad \mathbf{s} = \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|}, \qquad (5)$$

where the kernel $G(|\mathbf{r} - \mathbf{r}'|,\mathbf{s})$ describes the probability for the absorption at the point $\mathbf{r}$ of heat radiated at the point $\mathbf{r}'$ in the direction $\mathbf{s}$. It is expressed in terms of the probability $T(\rho)$, which is the probability for the traversal of a distance no less than ρ

by a plasmon without absorption:

$$G(\rho, s) = -\frac{1}{\rho^2}\frac{\partial T(\rho, s)}{\partial \rho}, \quad T(\rho, s) = \frac{1}{J}\int j(\omega, s)\, e^{-\alpha(\omega, s)\rho}\, d\omega d\Omega_s. \tag{6}$$

The time scale τ in (5),

$$1/\tau = \int d\omega d\Omega_s j(\omega, s) / \frac{3}{2} n_e T_0 \equiv J / \frac{3}{2} n_e T_0, \tag{7}$$

describes the "lifetime" of the temperature due to the radiative loss of waves. The dependence of the kernel G on the direction **s** incorporates the presence of an external source of an asymmetry (which we assume below is a magnetic field).

We seek the law of motion of the front of the "thermal wave" from the source $Q \propto \delta(t)\delta(x)\delta(y)$ across the magnetic field $\mathbf{B}\|Z$ through the emission and absorption of Bernstein modes.[4,6] Working from an analysis of various results of radiation transport theory,[7,8] we can show that in this case the time scale for the propagation of the thermal wave over a distance $R = (x^2 + y^2)^{1/2}$ from the point of the source, $R = 0$, is, for $t \gg \tau$,

$$t^{-1}(R) \sim \frac{1}{\tau}\int d\Omega_s T\left(\frac{R}{\sin\theta}, s\right) \equiv \int d\omega\, d\Omega_s j(\omega, s) \exp\left[-\frac{R}{\sin\theta}\alpha(\omega, s)\right] / \frac{3}{2} n_e T_0, \tag{8}$$

where θ is the axial angle made with the Z axis. In specific calculations of time (8) we use the dispersion law for Bernstein modes which are propagating essentially across a magnetic field.[4] We also make use of the independence of the heat transport in the various harmonics at $\omega_p \lesssim \omega_B$ (ω_p and ω_B are respectively the plasma frequency and cyclotron frequency of the electrons). Assuming a Doppler mechanism for the broadening of an individual radiated mode, and estimating the integrals in (8) for $\widetilde{R} \equiv (R\omega_B \delta / v_0) \gg 1$, we find

$$t(R) \sim \frac{T_0 v_0}{e^2 \omega_B^2}\frac{\widetilde{R}\ln\widetilde{R}}{\delta^2}, \quad \delta = (\omega_p/\omega_B)^2, \tag{9}$$

where $v_0 = \sqrt{2T_0/m_e}$ is the electron thermal velocity.

We see that the law of motion of the front is nearly uniform and is different from a diffusion law. The time $t(R)$ is dominated (at $t \gg \tau$, $\widetilde{R} \gg 1$) by a region of the parameters ω and **s** in which the function j (and also α) is small in comparison with its maximum value (the "wings of the line"). The behavior of j and α at the wings turns out to be of such a nature that a formal expansion of Eq. (5) in $|\mathbf{r} - \mathbf{r}'|$ leads (in an unbounded space) to a differential equation with an infinite diffusion coefficient. This is the "nondiffusion-nature" singularity of the wave transport of perturbations in a homogeneous medium (in contrast with the usual estimate[9] of the contribution of oscillations to the effective diffusion coefficient).

Let us find the distance $\mathbf{r}^*$, over which the front [see (9)] of a thermal perturbation carried by waves overtakes a front which is undergoing a diffusion motion and which is caused by a collisional electron thermal conductivity in a magnetized plasma

[Eq. (59, 28) in Ref. 10]. An estimate yields (Λ is the Coulomb logarithm)

$$r^* \omega_B / v_0 \sim \Lambda \ln \Lambda . \quad (10)$$

We see that the wave thermoconductivity "overtakes" the collisional conductivity over distances on the order of 10^1–10^2 Larmor radii.

This analysis demonstrates that there is the possibility in principle of a substantial acceleration of heat transport processes by virtue of the nonlocal nature of the propagation of plasma waves. To determine the significance of these effects in real fusion systems, we would need to allow for the various factors which substantially influence the transport: variations in the plasma temperature and density, a deviation from a Maxwellian electron distribution, the contribution of many harmonics, and so forth.

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