Displacement current of polarized carriers

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The anomalous Hall effect and the circular surface photovoltaic effect are shown to result from—in addition to an asymmetry of the scattering—a displacement of carriers in real space during quantum transitions. This displacement is possible for particles describable by the Dirac equation and for Bloch carriers. In *p*-Ge, the displacement current also contributes substantially to the circular photoconductivity.

A spin orientation of carriers gives rise to an anomalous Hall effect,[1–3] a circular surface photovoltaic effect, and a circular photoconductivity.[4,5] These effects are believed to result from the appearance of a directed velocity due to an azimuthal asymmetry[6,7] in the scattering of carriers.

However, a current might also be caused by a displacement of carriers in real space. At the time of any quantum transition $l\mathbf{p}\rightarrow l'\mathbf{p}'$, the center of gravity of the wave pocket of the carriers undergoes a displacement

$$\mathbf{R}_{l'\mathbf{p}',l\mathbf{p}}=-\hbar(\nabla_{\mathbf{p}'}+\nabla_{\mathbf{p}})\,\Phi_{l'\mathbf{p}',l\mathbf{p}}+\vec{\Omega}_{l'\mathbf{p}'}-\vec{\Omega}_{l\mathbf{p}}. \tag{1}$$

The indices l and l' may include the spin projection and the band index, the momentum $\mathbf{p}$, or the quasimomentum of the carriers; $\Phi_{l'\mathbf{p}',l\mathbf{p}}$ is the phase of the transition matrix element; $\vec{\Omega}_{l\mathbf{p}}=i\hbar\int d\mathbf{r}u^*_{l\mathbf{p}}\nabla_{\mathbf{p}}u_{l\mathbf{p}}$ is the part of the coordinate matrix element which is diagonal in the momentum; and $u_{l\mathbf{p}}$ is the Bloch amplitude of the an electron in a crystal or a Dirac spinor.

Expression (1) was derived by Belinicher *et al.*,[8] who ignored the spin of the carriers and assumed that a displacement was possible only in crystals lacking an inversion center. Incorporating the spin of the carriers makes a displacement possible in crystals of arbitrary symmetry and in an isotropic medium.

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When a flux of particles is caused by an electric field, by nonuniform illumination, etc., such a displacement leads to (first) a component in the effects[1–5] listed above which is described by the expression

$$\mathbf{j} = e \sum_{l'\mathbf{p}', l\mathbf{p}} W_{l'\mathbf{p}', l\mathbf{p}} \mathbf{R}_{l'\mathbf{p}', l\mathbf{p}}, \tag{2}$$

where $W_{l'\mathbf{p}',l\mathbf{p}}$ is the probability for a quantum transition which incorporates the populations of the states. Formally, current (2) is related to nondiagonal elements of the density matrix, which were taken into account in Refs. 2 and 3. The physical meaning of these terms, however, has not been explained.

Second, the displacements lead to a ballistic component of the current as a result of the directed velocity. This current component is unrelated to an asymmetry of the scattering but is given by the standard expression

$$\mathbf{j} = e \sum_{l\mathbf{p}} \mathbf{v}_{l\mathbf{p}} f_{l\mathbf{p}}^{(as)}. \tag{3}$$

To calculate this component in (for example) the anomalous Hall effect, we need to consider the effect of the scattering on the rate of change of the distribution function in a static electric field. For elastic scattering we would have

$$f_{\mathbf{p}}^{(as)} = \sum_{\mathbf{p}'} W_{\mathbf{p}'\mathbf{p}} e\mathbf{E}\mathbf{R}_{\mathbf{p}'\mathbf{p}} \frac{\partial f_{\mathbf{p}}^{(0)}}{\partial \epsilon_{\mathbf{p}}} \tau_{\mathbf{p}}. \tag{4}$$

The reason for the appearance of $f_{\mathbf{p}}^{(as)}$ is a change $e\mathbf{E}\mathbf{R}_{\mathbf{p}'\mathbf{p}}$ in the potential energy upon the displacement $\mathbf{R}_{\mathbf{p}'\mathbf{p}}$. Since the total energy remains constant, there are corresponding changes in the kinetic energy $\epsilon_{\mathbf{p}}$ and momentum of the carriers and thus in their distribution function. The magnitude of this ballistic component, induced by a displacement of the carriers, is parametrically the same as displacement current (2) and, in general, it is not small in comparison with the effect due to the scattering asymmetry, which arises outside the Born approximation.

Interestingly, at the time at which the electric field is turned on, the carrier acquires a velocity $\delta \mathbf{v}_{l\mathbf{p}} = -e\Sigma_{l\mathbf{p}}[\mathbf{E}\times \mathrm{curl}_{\mathbf{p}} \vec{\Omega}_{l\mathbf{p}}]$, which would lead in the absence of scattering to a current

$$\mathbf{j} = -e^2 \sum_{l\mathbf{p}} f_{l\mathbf{p}} [\mathbf{E}\times \mathrm{curl}_{\mathbf{p}} \vec{\Omega}_{l\mathbf{p}}]. \tag{5}$$

Current (5) is equal in magnitude and opposite in direction to the sum of displacement-current component (2) and the ballistic-current component determined by expressions (3) and (4), which contain the difference $\vec{\Omega}_{l'\mathbf{p}'} - \vec{\Omega}_{l\mathbf{p}}$. All are consequences of the initial conditions, and they do not contribute to a steady-state current.

As a result, the effect is due to only the displacement components in real space which contain a gradient of the phase of transition matrix elements. The invariance of the current under the choice of the phase of the wave functions is preserved, and it can be established by means of a kinetic equation, since all the quantum transitions form a closed cycle in the steady state.

During the scattering of carriers, they are displaced in the direction perpendicular to the spin of the carriers, **s**, and perpendicular with respect to the momentum transferred to an impurity or to a phonon. In the case of scattering by a Coulomb center, for example, the displacement of a particle describable by the Dirac equation is

$$\mathbf{R}_{\mathbf{p}'\mathbf{p}} = \hbar[\mathbf{s} \times (\mathbf{p}' - \mathbf{p})]/2m^2c^2. \qquad (6)$$

In Kane's model, the displacement $\mathbf{R}_{\mathbf{p}'\mathbf{p}}$ for an electron is given by an expression which differs from (6) in that $(mc)^2$ is replaced by $(4/3)\gamma P_{cv}^2/E_g^2$, where E_g is the band gap, P_{cv} is the interband matrix element, and the constant γ depends on the relation between E_g and the spin-orbit splitting of the valence band, Δ:

$$\gamma = 1, \text{ if } \Delta \gg E_g \text{ or } \gamma = 2\Delta/E_g \text{ if } \Delta \ll E_g.$$

This displacement contributes a component to the anomalous Hall effect involving polarized electrons in semiconductors. At a low impurity concentration, $Na_B^3 < 0.02$ (corresponding to the experimental conditions of Ref. 9), this component is an order of magnitude smaller than the ballistic component[1] due to the scattering asymmetry.

During optical transitions between branches of a degenerate valence band, however, the displacement components are important. In such transitions, a hole is displaced in a direction perpendicular to its momentum and to the angular momentum transferred by the photon, by an amount on the order of its wavelength:

$$\mathbf{R}_{2M\mathbf{p},1M'\mathbf{p}} = -3\gamma_2^2 \hbar[\vec{\kappa} \times \mathbf{p}]\,\delta_{M,M'\pm 1}/|(\mathbf{ep})_{2M\mathbf{p},1M'\mathbf{p}}|^2. \qquad (7)$$

Here **e** is the polarization vector of the light; (**ep**) is the matrix element of the transition between subbands; γ_2 is the constant of the Luttinger Hamiltonian; the pseudovector $\vec{\kappa} = P_{\text{circ}}\mathbf{q}/q$ determines the degree of circular polarization of the light, P_{circ}; **q** if the photon wave vector; and M and M' are the projections of the angular momentum onto the momentum **p**. In the case of the illumination of p-Ge by the beam from a CO_2 laser ($\hbar\omega = 117$ meV), at $T = 300$ K, a calculation of the component resulting from displacement (7) yields a current $j_{\text{disp}} = 10^{-9}\,A/(\text{W}\times\text{V/cm})$. A directed velocity of carriers does not arise during direct transitions, and a ballistic current arises under these conditions during transitions between branches involving optical phonons. In other words, it stems from the quantum corrections in the parameter $(\hbar/\tau_{\mathbf{p}})/E_{\text{kin}}$ ($\tau_{\mathbf{p}}$ is the momentum relaxation time, and E_{kin} is a characteristic energy of the holes). Since it is also proportional to $\tau_{\mathbf{p}}$, its magnitude is parametrically no different from j_{disp}, so the displacement component substantially determines the magnitude of the circular photoconductivity in this situation.

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