Distinctive features of the Shubnikov–de Haas oscillations in 2D systems with a strong spin-orbit coupling and holes at the Si(110) surface

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(Submitted 18 September 1987)
Pis'ma Zh. Eksp. Teor. Fiz. **46**, No. 10, 399–402 (25 November 1987)

A phase change of π has been observed experimentally in the Shubnikov–de Haas oscillations in a rather narrow interval of magnetic fields. The effect is explained on the basis of the energy spectrum {Yu. A. Bychkov and É. I. Rashba, Pis'ma Zh. Eksp. Teor. Fiz. **39**, 66 (1984) [JETP Lett. **39**, 78 (1984)]; Usp. Fiz. Nauk **146**, 531 (1985) [Sov. Phys. Usp. **146**, 632 (1985)]} of 2D electron systems with a strong spin-orbit coupling.

In 2D electron systems in quantizing magnetic fields, minima of the Shubnikov–de Haas oscillations of the diagonal components of the magnetoresistance tensor ρ_{xx} and the magnetoconductivity tensor σ_{xx} are observed[2] as the carriers fill an integer number (N) of magnetic quantum levels, i.e., at values of the magnetic field H_N which satisfy the condition $eH_N N/hc = n_s$. Here n_s is the surface concentration of charge carriers. If the levels are not equidistant, they are frequently resolved as groups[2] of ν levels, and only the maximum energy splitting in a group is manifested experimentally. In this case, the numbers N of neighboring Shubnikov–de Haas oscillations differ by an amount ν, and the period of the oscillations in the reciprocal field is $\delta(1/H) = e\nu/hcn_s$. This picture of Shubnikov–de Haas oscillations apparently persists even at[3,4] $\omega_c\tau \lesssim 1$, where $\omega_c = eH/m^*c$ is the cyclotron frequency, τ is the momentum relaxation time, and m^* is the effective carrier mass.

In the hole channels of silicon field-effect transistors we have observed an unusual transition from even to odd indices of Shubnikov–de Haas oscillations as the magnetic field was increased. The period $\delta(1/H)$ was the same in the two cases and corresponded to $\nu = 2$; this value is equivalent to a change in the oscillation phase by an amount π. This effect can be explained on the basis of the energy spectrum of a 2D electron gas with a strong spin-orbit coupling which was found by Bychkov and Rashba.[1] This is apparently the first successful attempt to use this spectrum to explain the features of Shubnikov–de Haas oscillations. The parameters of the spectrum are estimated by comparing the experimental results with the theory of Ref. 1.

In the measurements we used two silicon field-effect transistors with a hole channel near the (110) surface. We also analyzed the data of Ref. 5, where this effect was actually seen for the first time, although the oscillations were not identified there. von Klitzing *et al.*[5] pointed out that they did not see an explanation for the oscillations in weak magnetic fields. The dimensions of the channels in our samples were 1200×400 μm, and the distance between the potential contacts was 400 μm. The experiment consisted of measuring the resistance as a function of the magentic field at various charge-carrier concentrations in the layer and at the two temperatures 1.5 K and 4.2 K. The magnetic-field interval was 0–8 T (or up to 12 T in a few experiments). In addition, we measured the capacitance between the gate and the channel of the transistor.

Figure 1 shows a typical experimental curve. The positions of the minima of the magnetoresistance, shown at the top, were calculated from the period in the reciprocal field, $\delta(1/H)$. By choosing appropriate values of $\delta(1/H)$, one can bring the experimental positions of the minima into coincidence with the calculated positions, within the experimental error of less than 1%. In order to reconcile the measured carrier concentrations, found from the oscillations, with the capacitance measurements we

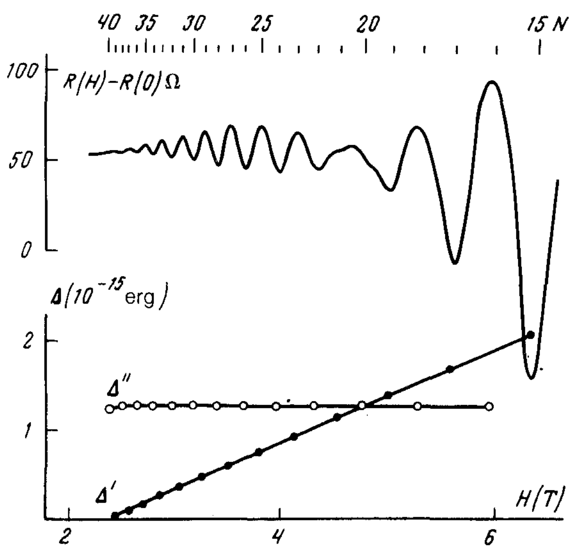

FIG. 1. The magnetoresistance $R(H) - R(0)$ (experimental, sample No. 1) and the energy difference (Δ) between the lower vacant and upper filled magnetic levels (theoretical) versus the magnetic field. Here R is the surface resistivity; $n_s = 2.3 \times 10^{12}$ cm^{-2}; $R(0) = 932\Omega$; and $T = 1.5$ K.

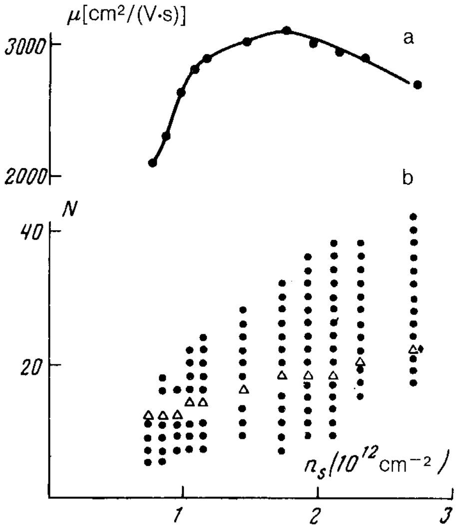

FIG. 2. a—The carrier mobility μ versus the carrier concentration n_s; b—the numbers (N) of Shubnikov–de Haas oscillations observed in the experiments at various concentrations n_s (points). The triangles show the calculated numbers N_0' corresponding to the phase change ($\delta = 0$, $E = 4.0\times10^{-18}$ erg).

should set[2)] $\nu = 2$. This choice of ν is confirmed by the small structural features in the form of oscillations which we observed in the strongest fields at even values of N. We should point out that (as can be seen well in Fig. 1) the transition from even numbers ($N \geqslant 22$) to odd numbers ($N \leqslant 19$) occurs in a narrow interval of these numbers; thus the number at which the phase disruption occurs, N_0', can be determined quite accurately. Figure 2b shows data on the numbers of the oscillations observed at various concentrations in sample No. 1. In sample No. 2, the numbers N_0' turned out to be the same. At $n_s \leqslant 2.7\times10^{12}$ cm^{-2}, only a single energy subband is filled in our samples. We observed the filling of the second subband (see also Ref. 5) at $n_s \gtrsim 3\times10^{12}$ cm^{-2}.

Our explanation of the change in the phase of the Shubnikov–de Haas oscillations is based on the energy spectrum of Ref. 1:

$$\epsilon_s^{\pm} = \hbar\omega_c[\, s \pm (\delta^2 + \gamma^2 s)^{1/2}\,]; \quad \epsilon_0 = \hbar\omega_c\delta. \tag{1}$$

Here $s \geqslant 1$ is an integer; $\gamma^2 = 4E/\hbar\omega_c$; and $\delta = \frac{1}{2}(1 - m^*g/2m)$, where E is a characteristic parameter of the theory of Ref. 1, g is the g-factor, and m is the mass of a free electron. We make the further assumption that only the larger of the adjacent splittings are seen experimentally. Results calculated on the energy difference (Δ) between the lower unfilled level and the upper filled level are shown Fig. 1 for the parameter values $\delta = 0$, $E = 4.0\times10^{-18}$ erg, and $m^* = 0.35$ m. The phase change observed is explained on the basis that as N increases, the energy splitting Δ'' corresponding to even numbers of minima becomes greater than the splitting Δ' corresponding to odd N. The effective mass was assigned a value in accordance with the results of Ref. 5.

This particular set of values of the parameters δ and E leads to correct values of the numbers of the Shubnikov–de Haas oscillations (N_0') at which the phase change occurs for all the concentrations n_s studied (Fig. 2b). We interpret this fact as an argument in favor of the explanation proposed here. This choice of δ and E is not the only possible choice (more on this below). The independence of Δ'' from H in the field interval shown in Fig. 1 is a consequence of the condition $\delta = 0$. Analysis of the results of Ref. 5 (Fig. 1) shows that at $n_s = 2.7 \times 10^{12}$ cm^{-2} odd oscillations were observed there in the number interval 13–25, while even oscillations were observed in the interval 28–32. The calculated value $N'_0 = 26$ is found with $\delta = 0$ and $E = 3.4 \times 10^{-18}$ erg.

The model which has been proposed predicts an alternation of groups of Shubnikov–de Haas oscillations with even and odd numbers as the field is varied, so that the phase change observed experimentally is simply a fragment of the overall picture. It is easy to show that the transition from odd to even numbers with increasing N occurs at the point $\Delta' = \Delta''$, where we have

$$\Delta' = \epsilon^-_{s+2k+1} - \epsilon^+_s = \frac{E_0}{2s+2k+1}\Big\{2k+1-[\delta^2+\lambda^2(2s+2k+1)(s+2k+1)]^{1/2} - [\delta^2+\lambda^2 s(2s+2k+1)]^{1/2}\Big\}$$

$$\Delta'' = \epsilon^+_{s+1} - \epsilon^-_{s+2k+1} = \frac{E_0}{2(s+k+1)}\Big\{-2k+[\delta^2+2\lambda^2(s+1)(s+k+1)]^{1/2} + [\delta^2+2\lambda^2(s+k+1)(s+2k+1)]^{1/2}\Big\}.$$

Here $E_0 = 2\pi\hbar^2 n_s/m^*$; $k \geqslant 0$ is an integer; and $\lambda^2 = 4E/E_0$. We have made use of the circumstance that n_s is fixed, and we have calculated Δ' and Δ'' for magnetic field values $H' = hcn_s/e(2s+2k+1)$ and $H'' = hcn_s/2e(s+k+1)$, respectively. Solving the equation $\Delta' = \Delta''$ under the conditions $s+k \gg 1$ and $\lambda^2 \ll 1$, we find $\lambda^2 N'^2_k = 2(k+\frac{1}{4})^2 - 2\delta^2$, where $N'_k \cong 2(s+k)$ is the number of the oscillation at which the phase change occurs. In a corresponding way, we can show that the transition to the group of odd numbers of Shubnikov–de Haas oscillations occurs at values of N''_k which satisfy the condition $\lambda^2 N''^2_k = 2(k+\frac{3}{4})^2 - 2\delta^2$.

Since only a single phase change is observed experimentally, we have $N_{max} < N''_k$ and $N_{min} > N''_{k-1}$ (if $k \geqslant 1$), where N_{max} and N_{min} are respectively the maximum and minimum numbers of the Shubnikov–de Haas oscillations observed experimentally. On the basis of the experimental results we can estimate E and δ: $E \lesssim 1 \times 10^{-16}$ erg and $|\delta| - 0.25 \lesssim k \lesssim -0.1 + \sqrt{\delta^2 + 0.1}$. The calculated results in Fig. 1 correspond to $k = 0$.

Taken together, the results obtained at the various concentrations and mobilities of the carriers contradict the suggestion that the phase change results from a transition from the inequality $\omega_c \tau \gtrsim 1$ to the opposite inequality (Fig. 2). The explanation of the experimental results proposed here is not the only possible explanation. The same effect might result from a complicated energy spectrum of the holes in silicon[2] when higher-lying energy subbands are taken into account. However, the occurrence of a phase change in this case would apparently be possible only for special relations among the parameters of the spectrum.

In the absence of a magnetic field, the spectrum of 2D systems with a strong spin-orbit coupling contains a term which is linear in the wave vector and in which the coefficient α is related to E: $\alpha = \hbar(2E/m^*)^{1/2}$ (Ref. 1). For the parameter values which we used we find $\alpha = 1.0\times10^{-10}$ eV·cm; i.e., this coefficient is comparable in magnitude to the coefficient found in Ref. 1 through an analysis of the results of measurements of cyclotron and combined resonances at GaAs-Al_xGa_{1-x}As heterojunctions and of the anomalous magnetoresistance of a 2D hole gas in structures based on silicon on sapphire.

We wish to thank É. I. Rashba, Yu. A. Bychkov, and V. T. Dolgopolov for useful discussions.

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[2)] Oscillations with odd indices N and with $\nu = 2$ had been observed previously in the same samples in strong magnetic fields.[6]

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[1]Yu. A. Bychkov and É. I. Rashba, Pis'ma Zh. Eksp. Teor. Fiz. **39**, 66 (1984) [JETP Lett. **39**, 78 (1984)]; Usp. Fiz. Nauk **146**, 531 (1985) [Sov. Phys. Usp. **146**, 632 (1985)].
[2]T. Ando, A. B. Fowler, and F. Stern, Rev. Modern Phys. **54**, 437 (1982).
[3]T. Ando, J. Phys. Soc. Jpn. **37**, 1233 (1974).
[4]A. Isihara and L. Smrčka, J. Phys. **C19**, 6777 (1986).
[5]K. von Klitzing, G. Landwehr, and G. Dorda, Solid State Commun. **14**, 387 (1974).
[6]G. M. Gusev, Z. D. Kvon, I. G. Neizvestnyĭ, V. N. Ovsyuk, and P. A. Cheremnykh, Pis'ma Zh. Eksp. Teor. Fiz. **39**, 446 (1984) [JETP Lett. **39**, 541 (1984)].

Translated by Dave Parsons