(Plane wave)-point diffractive focusing of x radiation by a biaxially curved crystal

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It is shown by means of geometric-optics theory that it is possible in principle to focus x radiation during the Bragg diffraction of a monochromatic plane wave by a biaxially curved crystal. The crystal curvature conditions are found. The position of the focal point in vacuum is found.

The refractive index of a material for x radiation with a wavelength $\lambda \sim 10^{-10}$ m deviates from unity by no more than 10^{-5}–10^{-6} (Ref. 1), so the standard approach to the development of focusing optics cannot be taken. The only practical path is to make use of the diffraction of x rays in crystals. Several diffractive-focusing methods, using

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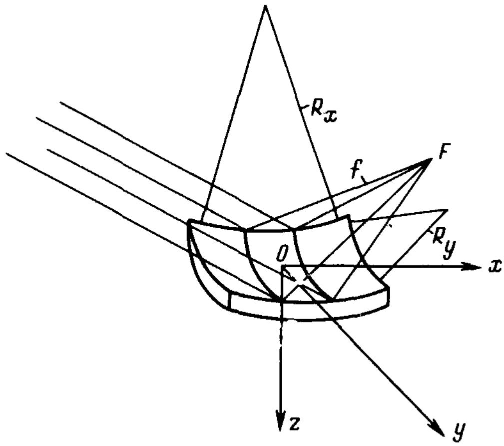

FIG. 1. (Plane wave)-point diffractive focusing during reflection.

either plane or curved crystals, have been proposed. It has been shown[2–6] (see also the literature cited in the reviews[7,8]) that crystals curved around a single axis focus the radiation in the diffractive-scattering plane in a process accompanied by the formation of a line focus.

A problem whose solution is of fundamental importance for the development of x-ray optics is that of the three-dimensional focusing of a diffracted beam to a point by means of a biaxially curved crystal. For example, Berreman *et al.*[9] have generalized the Johann focusing arrangement, in which the radiation source and the focal point are in the plane of the diffractive scattering of a curved crystal, and the line which connects these points is simultaneously the axis around which the crystal is curved, in the plane perpendicular to the diffractive-scattering plane. Kushnir *et al.*[10] have proposed a method for focusing by means of a biaxially curved crystal under conditions of three-wave diffraction.

In the present letter we would like to discuss the possibility of a diffractive focusing during the Bragg reflection of a monochromatic plane wave from a crystal curve in two mutually perpendicular directions, x and y [the (x,z) plane is the diffractive scattering plane]. We assume that for the incident wave the Bragg condition holds at the origin of the coordinate system $(0, 0, 0)$, which is placed at the surface of the curved crystal, and we assume that the wave vectors of the incident wave $(\mathbf{k}_0)$ and of the Bragg-reflected wave $(\mathbf{k}_h)$ have the following respective components (symmetric Bragg reflection; Fig. 1):

$$\mathbf{k}_0 = \mathbf{k}\ (\cos\theta,\ 0,\ \sin\theta\ ),\ \ \mathbf{k}_h = (\ \cos\theta, 0,\ -\sin\theta\ ), \tag{1}$$

where $k = 2\pi/\lambda$, and θ is the Bragg angle.

The displacement vector $\mathbf{u}(\mathbf{x})$, which describes the positions of the reflecting planes of the biaxially curved crystal, has the components $(|\partial u_i/\partial x_j| \ll 1)$

$$\mathbf{u} = (\ 0,\ 0,\ u_z\ )\ , \qquad u_z = -x^2/\ 2R_x\ - y^2/\ 2R_y, \tag{2}$$

where R_x and R_y are the radii of curvature of the crystal (Fig. 1).

It is easy to show[8] that in the diffractive-scattering plane, which corresponds to the $y=0$ cross section, the wave which has undergone Bragg reflection from the crystal is focused at a distance

$$L_h = \sin\theta\, R_x / 2, \qquad (3)$$

which is reckoned from the origin, $\mathbf{x}_0 = (0, 0, 0)$, along the direction of the wave vector $\mathbf{k}_h$.

We now consider the ray paths of waves reflected in cross sections $y \neq 0$. Within terms $|\partial u_i/\partial x_j| \ll 1$ inclusively, the local reciprocal-lattice vector corresponding to the selected system of reflecting planes of the curved crystal is

$$\mathbf{h} = \mathbf{h}_0 - \nabla(\mathbf{h}_0\mathbf{u}), \qquad (4)$$

where $\mathbf{h}_0$ is the reciprocal-lattice vector of the ideal (uncurved) crystal.

As the wave emerges from the crystal into vacuum, the tangential component of the wave vector $\mathbf{k}_h$ is continuous, given by

$$\mathbf{k}_{ht} = \mathbf{k}_{0t} + \mathbf{h}_t. \qquad (5)$$

According to (2) and (4), the vector $\mathbf{h}_t$ is

$$\mathbf{h}_t = -2\sin\theta\, k\,(x/R_x,\ y/R_y). \qquad (6)$$

Using energy conservation

$$k_h^2 = k^2,$$

we find from (5) and (6), within small terms $x^2/R_x^2, y^2/R_y^2 \ll 1$, the following expression for the local values of the wave vector at the surface of the curved crystal:

$$\mathbf{k}_h = k\,(\cos\theta - 2\sin\theta\, x/R_x,\ -2\sin\theta\, y/R_y,\ -\sin\theta - 2\cos\theta\, x/R_x). \qquad (7)$$

We find the corresponding ray path equations in vacuum to be

$$(x_p - x)/(\cos\theta - 2\sin\theta\, x/R_x)$$

$$= (y_p - y)/(-2\sin\theta\, y/R_y) = z_p/(-\sin\theta - 2\cos\theta\, x/R_x).$$

It follows immediately that if the condition

$$R_y = \sin^2\theta\, R_x = 2\sin\theta\, L_h \qquad (8)$$

holds for a biaxially curved crystal, the diffracted radiation will be focused in vacuum at the point with the coordinates

$$\mathbf{x}_f = L_h\,(\cos\theta, 0, -\sin\theta). \qquad (9)$$

The diffractive broadening of focal point (9) along the x and y axes is found through a direct calculation based on dynamic diffraction theory, (8), to be, respectively,

$$\delta x_f \sim 2 \cot \theta\Lambda, \quad \delta y_f \sim \lambda R_y / \sin\theta \, l_y, \tag{10}$$

where Λ is the x-radiation extinction length corresponding to the given Bragg reflection, and l_y is the dimension of the crystal along the y axis. For the (333) reflection from a silicon crystal and for Mo Kα radiation with the wavelength $\lambda = 0.7 \times 10^{-10}$ m, for example, expressions (10) with $l_y \sim 10^{-2}$ m and $R_y \sim 1$ m yield $\delta x_f \sim \delta y_f \sim 10^{-6}$ m.

In estimating the sharpness of the focusing we must also consider the collimation blurring and the chromatic blurring of the focus which result from the finite angular width and the finite spectral width of the incident radiation. Their effects on the focusing will be studied in a separate paper.

In summary, this analysis demonstrates the possibility in principle of a three-dimensional (plane wave)-point focusing during the Bragg reflection of x radiation from a biaxially curved crystal. The particular crystal curvature conditions under which the crystal focuses the radiation in the manner of a Fresnel diffraction lens in ordinary objects have been found [conditions (8)]. The focusing method described above has obvious analogs in the Bragg reflection of thermal neutrons and Mössbauer γ rays.

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