Threshold behavior of the cross section for the production of t quarks in e^+e^- annihilation

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The e^+e^- annihilation of a pair of heavy t quarks with $m_t \geqslant M_W$ is analyzed. The total cross section of the process $e^+e^- \to t\bar{t}$ in the near-threshold region is shown to be a calculable function of the mass of the quark, of its width, and of its color constant α_S.

A large B_d^0-$\bar{B}_d^0$ mixing observed by the ARGUS group[1] leads, according to the standard model of electroweak interactions, to the conclusion that the mass of the t quark is large: $m_t \approx 100$–150 GeV ($m_t \geqslant 60$ GeV; see, e.g., Ref. 2; a complete bibliography is given in Ref. 3). Analysis of current experiments on the direct search of t quarks and on the measurement of the gauge boson parameters imposes a constraint[4]: $44 \leqslant m_t \leqslant 190$–$220$ GeV.

The large mass, $m_t \geqslant M_W$, allows us to make several important predictions regarding the properties of t. These properties are related primarily to the predicted large width, $\Gamma_t \approx (m_t/M_W)^3 \times 175$ MeV (Ref. 5), and are determined completely by the semiweak decay $t \to W^+b$ (this also applies to the new-generation quark Q if the semiweak channel is the dominant channel in its decay). We recall that in the case of heavy quarks the nonperturbative effects have only a slight effect on the properties of the bound states with small quantum numbers[6,7] and that the dynamic properties of quarks are determined only by the electroweak and perturbative QCD interactions.[5,8]

Of particular interest is the region of the near-threshold $t\bar{t}$ production, where the interaction reduces to a Coulomb-like, two-body potential, and the total cross section $\sigma_{t\bar{t}}$ of the $e^+e^- \to t\bar{t}$ decay is described by a function of the mass and of the quark width which can be evaluated. Below the threshold the quarks become bound at the distances $r \sim k^{-1} \sim (\alpha_S m_t)^{-1}$ and (at least at the low-lying levels) the quarkonium $t\bar{t}$ forms a standard Coulomb system. The $3S_1$ ground state—toponium T—is localized at the distances $r_1 \sim k_1^{-1}$ $[k_1 = \frac{2}{3}\alpha_S(k_1)m_t]$ and its energy, which is reckoned from $2m_t$, is

$$E_1 = -\frac{4}{9}\alpha_S^2(k_1)m_t . \tag{1}$$

Allowance for the Coulomb effects and for the t-quark width results in the replacement of the standard threshold factor $\rho_V^{(0)} = \frac{3}{2}\beta_t$, which corresponds to the vector contribution to $\sigma_{t\bar{t}}$,

$$\rho_V^{(0)} \to \rho_V = \frac{3}{2}\left(\frac{4\pi}{m_t^2}\mathrm{Im}\ G_{E+i\Gamma_t}(0,0)\right) . \tag{2}$$

Here $G_E(\mathbf{r}, \mathbf{r}')$ is the Green's fuctions of the $t\bar{t}$ system in the singlet state with respect to color, and $E = \sqrt{s} - 2m_t = \beta_t^2 m_t$ is the nonrelativistic quark energy; the axial contribution in the near-threshold region is small: $\rho_A \sim \pi\alpha_S\beta_t^2$ (see Ref. 8 for a detailed discussion).

If the color constant α_S has a fixed value, the expression for $\mathrm{Im}\, G_{E+i\Gamma_t}(0, 0)$ will have the form[8]

$$\mathrm{Im}\, G_{E+i\Gamma_t}(0,0) = \frac{m_t^2}{4\pi}\left[\frac{k_+}{m_t} + \frac{2k_1}{m_t}\arctan\frac{k_+}{k_-} + \sum_{n=1}^{\infty}\frac{2\bar{k}_1^2}{m_t^2 n^4}\,\frac{\Gamma_t\bar{k}_1 n + k_+(n^2\sqrt{E^2+\Gamma_t^2} + \bar{k}_1^2/m_t)}{\left(E + \frac{\bar{k}_1^2}{m_t n^2}\right)^2 + \Gamma_t^2}\right],$$

$$\bar{k}_1 = \frac{2}{3}\alpha_S m_t, \qquad k_\pm = \sqrt{\frac{m_t}{2}\left(\sqrt{E^2+\Gamma_t^2} \pm E\right)}. \tag{3}$$

The first term in square brackets corresponds to the Born approximation which is modified by the width-induced effects, Γ_t, the second term corresponds to a single-loop correction, and the third term is the sum over the bound S-wave states which acquire the width $\Gamma = 2\Gamma_t$.

We will use relation (3) to illustrate the evolution of the levels of the quarkonium $t\bar{t}$ and the threhsold behavior of the continuous spectrum with changing m_t. Taking into account the QED effects due to the emission of γ rays by the initial electrons see, e.g., Ref. 9), we can use the following expression to describe the ratio $R_{t\bar{t}} = \sigma_{t\bar{t}}/(\sigma_{\mu^+\mu^-})_{\mathrm{QED}}$:

$$R_{t\bar{t}} = \frac{9}{2}\frac{4\pi}{m_t^2}\left\{Q_t^2 + \frac{v_t^2}{\kappa\left(1 - \frac{M_Z^2}{4m_t^2}\right)^2}\right\}\frac{\left(1 + \frac{3}{4}\beta\right)}{\left(1 - P(4m_t^2)\right)^2}$$

$$\times \int_0^1 dx\, \beta x^{\beta-1}\, \mathrm{Im}\, G_{E-m_t x+i\Gamma_t}(0,0), \tag{4}$$

where $Q_t = 2/3$ is the t-quark charge, $v_t = 1 - (8/3)\sin^2\theta_W$ is the axial constant of the t quark, $\varkappa = (16\sin^2\theta_W\cos^2\theta_W)^2$, $\beta = (4\alpha/\pi)(\ln(2m_t/m_e) - 1/2)$, and $P(s)$ is the real part of the γ-ray polarization operator.

We have constructed several plots of the E dependence of $R_{t\bar{t}}$ for various values of m_t and α_S. Figure 1 shows plots for $m_t = 100$ GeV, Fig. 2 shows plots for $m_t = 140$ GeV, and Fig. 3 corresponds to $m_t = 200$ GeV. It was assumed that $P(4m_t^2) \approx 0.07$ for all values of m_t. The solid curves correspond to $\alpha_S = 0.150$ and the dashed curves illustrate the simplest way of taking into acount α_S versus the characteristic virtuali-

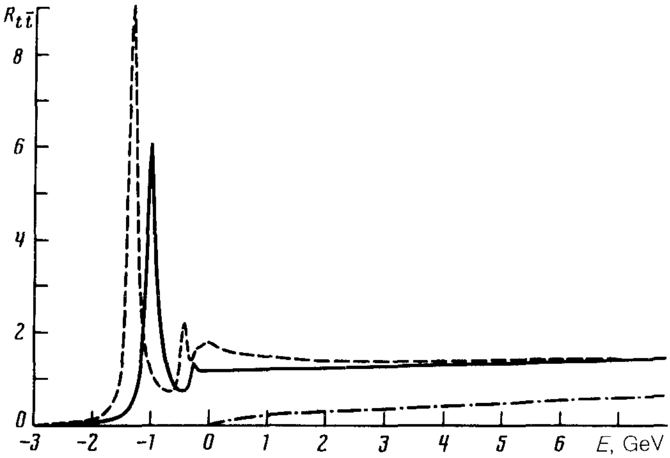

FIG. 1. $R_{t\bar{t}}$ versus E for $m_t = 100$ GeV. Solid curve—$\alpha_S = 0.150$; dashed curve—changing α_S.

ties $\alpha_S = 4\pi\{7.67 \ln[m_t (E^2 + \Gamma_t^2)^{1/2}/\Lambda^2]\}^{-1}$ for $\Lambda = 100$ MeV. The dot-dashed curves correspond to the case in which there is no interaction between t and $\bar{t}$ (cf. Ref. 5).

Analysis of the behavior of $\sigma_{t\bar{t}}$ in the near-threshold region allows us to draw the following conclusions.

1. At $m_t < 100$ GeV we see the formation of a standard system of Coulomb-like levels and at $m_t = 100$ GeV the width of the ground level of T is $\Gamma_T = 2\Gamma_t \approx 170$

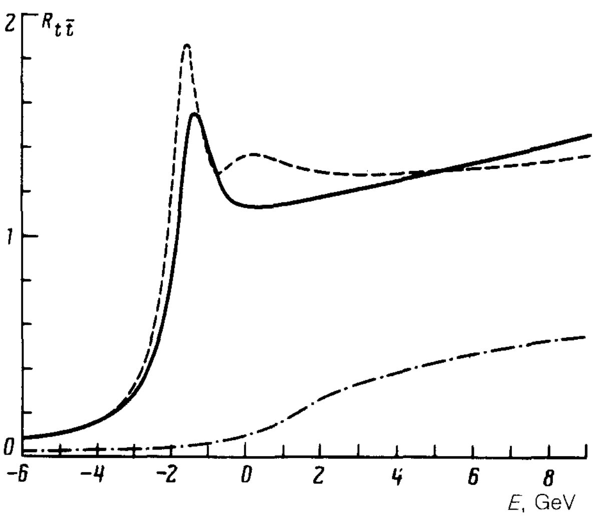

FIG. 2. $R_{t\bar{t}}$ versus E for $m_t = 140$ GeV. Solid curve—$\alpha_S = 0.150$; dashed curve—changing α_S.

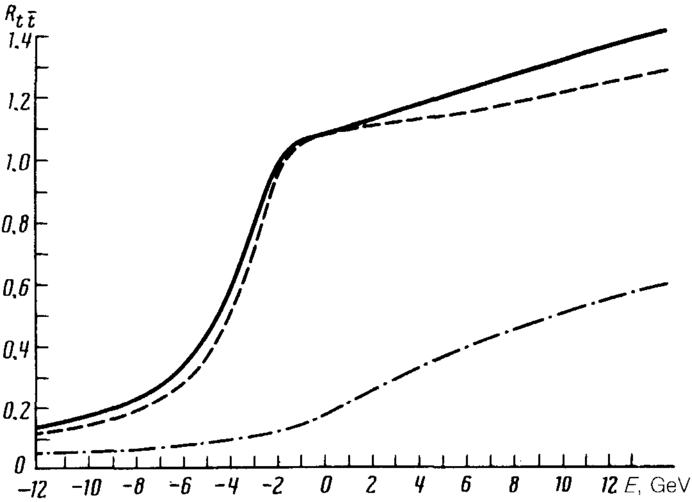

FIG. 3. $R_{t\bar{t}}$ versus E for $m_t = 200$ GeV. Solid curve—$\alpha_S = 0.150$; dashed curve—changing α_S.

MeV; this width is much smaller than the amount by which T shifts with respect to $2m_t$: $\Delta E_1 \approx 1$ GeV when $\alpha_S = 0.15$ [see Eq. (1)].

The cross section for the resonance maximum of toponium is

$$R_{\rm T} = \frac{\sigma\,(e^+ e^- \to {\rm T} \to {\rm all})}{(\sigma_{\mu^+\mu^-})_{\rm QED}} \approx \alpha_S^3 (k_1)\, \frac{m_t}{\Gamma_t}\,. \tag{5}$$

2. With an increase in m_t the height of the resonance maximum decreases, its width increases, and the continuous spectrum shifts to the left. At $m_t = 150$ GeV the lifetime of toponium, $\tau_{\rm T} = 1/\Gamma_{\rm T}$, becomes comparable to $1/\Delta E_1 = (1.5\ {\rm GeV})^{-1}$, i.e., to characteristic time reversal, t_R, of the bound $t\bar{t}$ state, $\tau_{\rm T} \approx 1/\Delta E_1 \approx t_R \approx 0.13$ fm.

3. Upon further increase in m_t the $t\bar{t}$ levels overlap each other and merge with the nonresonant background (at $m_t = 200$ GeV and $\alpha_S = 0.13$ we have $\Delta E_1 = 1.5$ GeV and $\Gamma_{\rm T} \approx 5$ GeV).

The behavior of the process $e^+e^- \to t\bar{t}$ in the near-threshold region is therefore a dynamic function of m_t, Γ_t, and α_S which can be evaluated. An experimental study of this region will make it possible to determine the mass m_t and the width Γ_t (and therefore to check the predictions of the lifetime based on the standard model). In principle, the value of α_S can also be measured under more favorable conditions from the viewpoint of theoretical interpretation. A comparison of a precise measurement of M_Z with the value predicted by the standard model, which takes into account the effects due to the large mass m_t, is of considerable interest from the standpoint of a critical test of the standard model.

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