Possible violation of the Pauli principle in atoms

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A free field model with an explicit violation of the Pauli principle is constructed. The difficulties which arise in incorporating an electromagnetic interaction and in extending the model to the relativistic case are discussed. Some experiments which might be carried out to search for "non-Pauli" atoms, with an anomalous filling of electron shells or of nucleon levels of the nucleus, are proposed.

Experimental data on the *CP*-violating decays of *K* mesons do not exclude[1,2] a violation of *CPT* invariance. Since one of the basic principles of the *CPT* theorem[3,4] is the relationship between the spin and the statistics, it is reasonable to ask whether there might be small deviations from the Pauli principle in nature. Ignat'ev and Kuz'min[5] have proposed a nonrelativistic, one-level "toy" model in which a violation of the Pauli principle is accompanied by a nonconservation of angular momentum (by an amount of 1/2) and a nonconservation of the number of fermions. Using third-rank matrices as in the model of Ref. 5, we can construct a model with an infinite number of levels, in which the angular momentum and the number of fermions are conserved. We will discuss the difficulties in our model.

Let us assume that the occupation numbers of a given level i for electrons with a given spin projection can be not only 0 and 1 but also 2. We consider the three state vectors

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$$|0\rangle_i = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}_i , \qquad |1\rangle_i = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}_i , \qquad |2\rangle_i = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}_i .$$

We write the operators which create electrons in levels 1,2, . . ., i, . . . in the form

$$a_1^+ = \lambda_1^+ \lambda_2^u \lambda_3^u \dots , \quad a_2^+ = \lambda_1^d \lambda_2^+ \lambda_3^u \dots , \quad a_i^+ = \lambda_1^d \lambda_2^d \dots \lambda_{i-1}^d \lambda_i^+ \lambda_{i+1}^u \lambda_{i+2}^u \dots ,$$

where

$$\lambda^+ = \begin{pmatrix} 0 & \gamma & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} , \quad \lambda^- = \begin{pmatrix} 0 & 0 & 0 \\ \gamma & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} , \quad \lambda^u = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} , \quad \lambda^d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} .$$

The operators a_1^- are found from a_i^+ by replacing λ_i^+ by λ_i^-. It is easy to show, for example, that we have

$$\{a_i^+ , a_k^+\} = 2\gamma \left[\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_i \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}_k + i \leftrightarrow k \right] , \qquad a_i^{+\,2} = \begin{pmatrix} 0 & 0 & \gamma \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_i .$$

A more attractive version is that in which the last equality is retained, but for $i \neq k$ we have $\{a_i^+, a_k^+\} = 0$. Tsypin has called my attention to the circumstance that this situation might be achieved by choosing $\lambda^d = \mathrm{diag}(1, -1, 1)$, $\lambda^u = \mathrm{diag}(1,1,1)$. (In the case $\gamma = 0$ we would find the standard relations; see Ref. 6, for example.)

We introduce $\psi^+(\mathbf{x}) = \Sigma_i a_i^+ \varphi_i(\mathbf{x})$ [for a free particle with a momentum $\mathbf{p}_i$ we would have $\varphi_i(\mathbf{x}) = (1/\sqrt{V})\exp(i\mathbf{p}_i\,\mathbf{x})$, where V is the normalized volume]. Acting with $\psi^+(\mathbf{x}_1)$ on the vacuum, $|\Omega\rangle = |0\rangle_1 |0\rangle_2 \cdots |0\rangle_i \cdots$, we find the single-particle states

$$(\,|1\rangle_1 |0\rangle_2 \dots |0\rangle_i \dots)\,\varphi_1(\mathbf{x}_1) - (\,|0\rangle_1 |1\rangle_2 \dots |0\rangle_i)$$

$$\varphi_2(\mathbf{x}_1) - (-1)^i |0\rangle_1 |0\rangle_2 \dots |1\rangle_i \dots)\,\varphi_i(\mathbf{x}_1) .$$

In a corresponding way, $\psi^+(\mathbf{x}_n)\cdots\psi^+(\mathbf{x}_2)\psi^+(\mathbf{x}_1)$ generates n-particle states. As can be seen even in the example of two particles, some of the states are forbiden by the Pauli principle.

The operator representing the number of particles in level i is

$$N_i = a_i^+ a_i + \frac{2-\gamma^2}{\gamma^2} a_i^+ a_i^+ a_i a_i = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}_i ,$$

and the Hamiltonian of the free particles is $H^0 = \Sigma_i N_i E_i$, where E_i is the energy of the level. This expression agrees with that corresponding to the standard case: $[H^0, a_i^+] = a_i^+$.

If we incorporate an interaction with an electromagnetic field in the standard gauge approach, we run into several serious difficulties: 1) The quadratic term in the Hamiltonian (which is nonlocal in x space) contributes a nonlocal interaction. 2) The repulsion between pairs of electrons violates the relation $[H, a_i^+] = a_i^+$. 3) In order to avoid a quartic creation of electrons and positrons, we would have to set $\gamma = 0$ for the positron operators b and b^+ (Ref. 7, for example). This step would of course violate *CPT*. 4) In the relativistic generalization of this model, the renormalizability of QED must be violated, and it is not clear whether one can, by reducing γ, preserve the prevailing agreement between the radiation corrections and experiment and preserve the (essentially) massless photon.

Without reference to a particular model, it would be interesting to search for two types of effects associated with a possible violation of the Pauli principle in atoms.

1. A search for transitions of the $2p$-$1s$ type in ordinary stable atoms, which would result in the appearance of a third electron in the $1s$ shell. According to Koval'chuk,[8] a lower limit on the time for a transition of this sort in the iodine atom would be 3×10^{22} yr. We should emphasize, however, that if the Hamiltonian has exchange symmetry, a spontaneous conversion of an ordinary atom into a "non-Pauli" atom would be impossible (Ref. 9, for example).

2. A search for stable non-Pauli atoms. Such atoms might be of cosmological origin, if not all of the 10^{80} electrons in the universe are antisymmetrized. The chemical properties of multielectron atoms with three electrons in the $1s$ shell should be similar to those of their "junior" neighbors in the periodic table. For example, non-Pauli Na would be similar to Ne. In terms of physical properties (spectral lines), in contrast, these atoms would be quite peculiar. It would also be interesting to search for "magnetic" ortho-helium.

Promising approaches for testing the Pauli principle for nucleons would be a mass-spectroscopic or NMR search for anomalous non-Pauli nuclei with masses a few tens of MeV smaller than those of the ordinary isotopes with the same number of nucleons.

The special place enjoyed by the Pauli principle in modern theoretical physics does not mean that this principle does not require further and exhaustive experimental tests. On the contrary, it is specifically the fundamental nature of the Pauli principle which would make such tests, over the entire periodic table, of special interest.

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